12140

Sam G. Drake

1843.

12140

Sam G Drake R.P.
1843.

Ed James

Saml G. Drake
1843.

In "The new Practical Navigator by John Hamilton Moore", "the first American, from the thirteenth English Edition", "improved by the introduction of several new Tables," &c. by a Skilful Mathematician & Navigator," 8° Newburyport, 1799, it is said in the Preface, "The Editor is also indebted to Nicholas Pike, A.M. Member of the American Academy of Arts & Sciences."

I have exchanged numerous copies of this work, as they came into the store, retaining always the best. I now exult in the possession of this, which is the best I ever saw. As to the editions of this work, none are so good as the first.

There is some account of the author in the valuable collections of Farmer & Moore.

Pike 335, q. 8

Put x = time all the cocks will fill the vessel. Now while A is filling the vessel 2 times, B will fill it 4 times & C 3 times, consequently if they all run together for the whole time x, it will be thus represented:

$$2x + 4x + 3x = 9x \text{ times full}$$

in the time x, $= 9x$ cisterns.

Hence, $9x : x :: 1 : \frac{1x}{9x} = \frac{1}{9}$ hour, if the time be taken in hours.

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ib. 337 – 1

Put x = yds. damask, then $x - 15$ = lining;

hence $x + \overline{x-15} = 2x - 15$ = whole.

$\therefore 8x + 3x - 45 = 3£.\ 10s. = 70s.$

or, $5x = 25$

&, $x = 5$ yds. damask

[ib. 2]

Put x = income, then $x + \frac{x}{8}$ = As in 1 year.

$x - 30£$ for year, & for 8 years $x - 240$

hence $x + 40 = 240$

& $x = 200$ = income, Ans.

21. Two notes, one of 120 dollars, payable in 6 months, and the other of \$150, payable in 9 months, were discounted for \$8,50. What rate of interest were they discounted at ; Let x denote the interest of one dollar for 12 months ; then the amount of \$1 in 6 months being $1+\frac{1}{2}x$, and in 9 months $1+\frac{3}{4}x$ the present value of the bill due at the end of 6 mos. will, $\therefore$ be $\frac{120}{1+\frac{1}{2}x}$; and that of the bill, due at the end of 9 mos, $\frac{150}{1+\frac{3}{4}x}$; $\therefore \frac{120}{1+\frac{1}{2}x}+\frac{150}{1+\frac{3}{4}x} = (120+150-8{,}50) = 261.50$, by the question, and reduction $120+90x+150+75x = 261.5(1+\frac{5}{4}x+\frac{3}{8}x^2)$, or $270+165x = \frac{1}{8}\{261.5\times(8+10x+3x^2)\}$; $\therefore \frac{21600}{2615}+\frac{13200}{2615}x = (3x^2+10x+8)$, $\therefore x^2+\frac{2590}{1569}x = \frac{136}{1569}$; hence, by art. 70, case 1, we find $x = \sqrt{\{\frac{136}{1569}+(\frac{1295}{1569})^2\}}-\frac{1295}{1569} = \frac{\sqrt{[(136\times1569)+(1295\times1295)]}-1295)}{1569}$ $=0.05093$ and $\times 100 = \$5.093$ cts. Ans.

1. If 9 gentlemen and 15 ladies will eat up 16 apples in 4 hours, and 15 gentlemen and 15 ladies can eat up 47 apples of a similar size in 12 hours, the apples growing uniformly, how many boys will eat up 188 apples in 60 hours, admitting that 112 boys can eat the same number as 18 gentlemen and 26 ladies.

lad gent apples — apples Ans.

$$\text{If } \left.\begin{matrix} 15 : 4 :: 16 \\ 40 \quad 12 \end{matrix}\right\} = \frac{40\times12\times16}{15\times4} = \frac{10\times4\times16}{5} = 2\times4\times16 = 128$$

which 40 ladies will eat in 12 hours, admitting the apples do not grow. But 40 ladies eat only 47 apples in 12 hours; $(12-4) = 8$ hours, 47 apples by the growth of the apples, are equivalent to 128 apples. Hence $128-47=81$ apples, the increase upon 47 apples in $(12-4)=8$ hours. Now the real quantity which 15 ladies will eat in 4 hours without any increase was 17 apples. Hence the time of increase upon 188 apples is only $(60-4)=56$ hours. ap. ho. in.

$$\text{And if } \left.\begin{matrix} 47 : 8 :: 81 \\ 188 : 56 :: \end{matrix}\right\} = \frac{188\times81\times56}{47\times8} = \frac{4\times81\times56}{8} = 4\times81\times7$$

$=226$ apples, the increase on 188 apples in 56 hours. Hence $2268+188=2456$ apples, which is to support the required number of ladies for 60 hours.

$$\text{Again } \left.\begin{matrix} 15 : 4 :: 16 \\ 60 \quad 2456 \end{matrix}\right\} = \frac{188\times81\times56}{60\times16} = \frac{2456\times15}{15\times16} = \frac{2456}{16} =$$

$153\frac{1}{2}$ ladies to eat the required number of apples. Lastly,

If 9 gen. : 15 la. :: 15 gen : 25 lad. and $25+15=40$ lad.

Then if 56 : 112 boys :: $153\frac{1}{2}$ ladies : 307 boys, Answer.

2. If 12 oxen will eat up $3\frac{1}{3}$ or $3\frac{1}{2}$ acres of grass in 4 weeks, and 21 oxen will eat up 10 acres in 9 weeks, how many oxen will eat up 24 acres in 18 weeks, the grass being allowed to grow uniformly?

94. A trader maintained himself for three years, at the expense of \$50 a year; and in each of those years, augmented that part of his stock which was not so expended by one third thereof. At the end of the third year his original stock was doubled. What was that stock?

Let $x =$ the number of dollars required; then $x-50$ $=$ the sum not expended; and with this he traded; $\therefore$ $\frac{1}{3}(x-50) =$ his gain the first year, and $\frac{4}{3}(x-50) =$ the sum he had at the end of the first year; $\therefore$ $\frac{1}{3}(4x-200)$ $-50 = \frac{1}{3}(4x-350) =$ the sum he traded with the second year; $\therefore$ $\frac{4}{3} \cdot \frac{4x-350}{3} = \frac{16x-1400}{9} =$ the sum he had at the end of the second year; and $\frac{16x-1400}{9} - 50 =$ $\frac{1}{9}(16x-1850) =$ the sum he traded with at the end of the third year. And the sum he had at the end of the third year $= \frac{4}{3} \cdot \frac{16x-1850}{9}$; whence $\frac{4}{3} \cdot \frac{16x-1850}{9} = 2x$, and $32x-3700 = 27x$; by transposition, $5x = 3700$, and $x = 740$. Ans.

OLD AND NEW STILE.

The Roman year from the old institution of Numa, was lunar, borrowed from the Greeks, amongst whom it consisted of three hundred and fifty-four days: Numa added one more to them, to make the whole number odd, which was thought the more fortunate; and, to fill up the deficiency of his year, to the measure of the solar course, inserted likewise, or intercalated, after the manner of the Greeks, an extraordinary month of twenty-two days every second year, and twenty three every fourth, between the twenty third and twenty-fourth day of February: the care of intercalating this month, and this supernumerary day, was committed to the college of priests, who, in a process of time, partly by a negligent, partly a superstitious, but chiefly by an arbitrary abuse of their trust, used either to drop or insert them, as it was found most convenient to themselves, or their friends, to make the current year longer or shorter. Thus Cicero, when harrassed by a perpetual course of pleading, prayed, that there might be no intercalation to lengthen his fatigue; and when proconsul of Cilicia, pressed Atticus to exert all his interest to prevent any intercalation within the year, that it might not protract his government, and retard his return to Rome. Curio, on the contrary, when he could not persuade the priests to prolong the year of his tribunate, by an intercalation, made that a pretence for abandoning the senate and going over to Cæsar.

This licence of intercalating introduced the confusion above mentioned, in the computation of their time, so that the order of all their months was transposed from their stated seasons, the winter months carried back into autum, the autumnal into summer, till Cæsar resolved to put an end to this disorder, by abolishing the source of it, the use of *intercalations*; and instead of the Lunar to establish the Solar year, adjusted to the exact measure of the sun's revolution in the Zodiac, or to that period of time in which it returns to the point from which it set out; and as this, according to the astronomers of that age, was supposed to be *three hundred and sixty-five days and six hours*, so he divided the days into twelve artificial months: and to supply the deficiency of six hours, by which they fell short of the sun's complete course, he ordered a day to be intercalated after every four years, between the twenty-third and twenty-fourth of February.

But to make this new year begin, and proceed regularly, he was forced to insert into the current year two extraordinary months, between November and December, the one of thirty three, the other of thirty four days, besides the ordinary intercalary month of twenty three days, which fell into it of course, which were all necessary to fill up the number of days that were lost to the old year, by the omission of intercalations, and to replace the months in their proper seasons. All this was effected by the care and skill of Sosigenes, a celebrated astronomer of Alexandria, whom Cæsar had brought to Rome for that purpose; and a *new kalender* was formed upon it by Flavius, a scribe, digested according to the order of the Roman festivals, and the old manner of computing their days, by Kalends, Ides, and Nones, which was published and authorished by the dictator's edict, not long after his return from Africa. This year, therefore, was the longest that Rome had ever known, consisting of fifteen months, or four hundred and forty five days, and is called the last of the confusion; because it introduced *the Julian or Solar year*, with the commencement of the ensuing January, which continues in use to this day in all Christian countries, without any other variation than that of *the old and new stile*.

This difference *of the old and new stile* was occasioned by a regulation made by Pope Gregory, A. D. 1582; for it having been observed, that the computation of *the Vernal Equinox* was fallen back ten days from the time of *the Council of Nice*, when it was found, to be on the 21st of March, according to which all the festivals of the church were then solemnly settled, Pope Gregory, by the advice of astronomers, caused ten days to be entirely sunk and thrown out of the current year, between the 4th and 15th of October.

The first who published an express treatise on Decimals was Simon Stevinus, about the year 1582. Ramus in his Arithmetic, written about 1550, & published by Lazarus Schonerus in 1586, had used decimal periods in carrying on the square & cube roots to fractions. Buckley & Record had done the same thing before.

Encyc. Perthensis.

Regiomontanus, alias John Müller, alias Kings Mountain the place of his residence, revived Algebra in Germany & was the first who made use of Decim.l Arith.c in his table of Sines. Montanius died in 1476.

Encyc.a Amer.a

A NEW AND COMPLETE SYSTEM OF ARITHMETIC,

COMPOSED FOR THE

USE OF THE CITIZENS

OF THE

UNITED STATES:

By NICOLAS PIKE, *A. M.*

K

QUID MUNUS REIPUBLICÆ MAJUS MELIUSVE AFFERRE POSSUMUS, QUAM SI JUVENTUTEM DOCEMUS, ET BENE ERUDIMUS?

——E VARIIS SUMENDUM EST OPTIMUM.

Cicero.

NEWBURY-PORT:

PRINTED AND SOLD BY JOHN MYCALL.

MDCCLXXXVIII.

State of South-Carolina } I HEREBY Certify that in pursuance of an Act of the
Secretary's Office. } Legislature of this State passed the twenty-sixth day of March Anno Domini One Thousand seven Hundred and Eighty-four, entitled "An Act for the encouragement of Arts and Sciences." I have registered a Work entitled "A new and Complete System of Arithmetic, composed for the use of the United States, by Nicholas Pike of Newbury-port in the State of Massachusetts.

GIVEN Under my hand this fourteenth day of February Anno Domini One Thousand seven hundred and Eighty-seven and in the Eleventh year of the Sovereignty and Independence of the United States of America.

PETER FRENEAU, Dep. Sec'y.

STATE of PENNSYLVANIA.

I JONATHAN BAYARD SMITH Prothonotary of the Court of Common Pleas of Philadelphia County do certify that Nicholas Pike has on this 26th day of October 1786 registered with me his name as author and proprietor of a book entitled "*A new and complete system of Arithmetic composed for the use of the Citizens of the United States, by Nicholas Pike, A. M.*" agreeable to act of Assembly.

J. B. SMITH.

Secretary's Office of the State of New-York, June 21st. 1787.

I DO hereby certify that in the Miscellaneous Book of Records remaining in this office, and at the 71st page thereof, the name of, Nicholas Pike, is, agreeable to a law of this State passed the 29th of April 1786, Registered as Author of "a new and complete System of Arithmetic composed for the use of the Citizens of the United States."

ROBERT HARPUR, Dep. Sec'y.

Commonwealth of Massachusetts. *In the* HOUSE *of* REPRESENTATIVES.

ON the petition of *Nicholas Pike*, Esq. praying that he may be exempted from Excise Duties, in the publication of his System of Arithmetic, which he has prepared for the public.

Resolved, That as the said System may essentially serve the present and future generations, the prayer of the petitioner be granted, and that he be and hereby is exempted from all Excise Duties in the necessary publications relative to the said Treatise, and that the Collectors of Excise be and hereby are directed to govern themselves accordingly.

Sent up for concurrence. ARTEMAS WARD, Speaker.

In *Senate May* 1, 1787. Read and concurred.

SAMUEL PHILLIPS, jun. President. By the Governor. Approved.

JAMES BOWDOIN.

True Copy. Attest. JOHN AVERY, jun. Secretary.

*** The Author begs leave to inform the Public in general, and Printers and Booksellers in particular, that he has complied with the Requisitions of the several Acts throughout the United States, "for the encouragement of Literature," he therefore hopes they will not incur the penalties of those Statutes, by *publishing* or *vending* any spurious copies of this book.

RECOMMENDATIONS.

Dartmouth University A. D. 1786.

AT the request of Nicolas Pike, Esq. we have inspected his System of Arithmetic, which we cheerfully recommend to the public as easy, accurate and complete. And we apprehend there is no treatise of the kind extant, from which so great utility may arise to Schools.

B. WOODWARD, Math. and Phil. Prof.

JOHN SMITH, Professor of the Learned Languages.

I do most sincerely concur in the preceding recommendation.

J. WHEELOCK, *President of the University.*

RECOMMENDATIONS.

Providence, State of Rhode-Iſland 1785.

WHOEVER may have the peruſal of this treatiſe on Arithmetic may naturally conclude I might have ſpared myſelf the trouble of giving it this recommendation, as the work will ſpeak more for itſelf than the moſt elaborate recommendation from my pen can ſpeak for it: but as I have always been much delighted with the contemplation of mathematical ſubjects, and at the ſame time fully ſenſible of the utility of a work of this nature, was willing to render every aſſiſtance in my power to bring it to the public view: and ſhould the ſtudent read it with the ſame pleaſure with which I peruſed the ſheets before they went to the Preſs, am perſuaded he will not fail of reaping that benefit from it which he may expect, or wiſh for, to ſatiſfy his curioſity in a ſubject of this nature. The Author, in treating on numbers, has done it with ſo much perſpicuity and ſingular addreſs, that I am convinced the ſtudy thereof will become more a pleaſure than a taſk.

The arrangement of the work, and the method by which he leads the *Tyro* into the firſt principles of numbers, are novelties I have not met with in any book I have ſeen. Wingate, Hatton, Ward, Hill, and many other Authors, whoſe names might be adduced, if neceſſary, have claimed a conſiderable ſhare of merit, but when brought into a comparative point of view with this treatiſe, they are inadequate and defective. This volume contains, beſides what is uſeful and neceſſary in the common affairs of life, a great fund for amuſement and entertainment. The Mechanic will find in it much more than he may have occaſion for; the Lawyer, Merchant and Mathematician will find an ample field for the exerciſe of their genius; and I am well aſſured it may be read to great advantage by ſtudents of every claſs, from the loweſt ſchool, to the Univerſity. More than this need not be ſaid by me, and to have ſaid leſs, would be keeping back a tribute juſtly due to the merit of this Work.

BENJAMIN WEST.

Univerſity in Cambridge, A. D. 1786.

HAVING, by the deſire of Nicolas Pike, Eſq. inſpected the following volume in manuſcript, we beg leave to acquaint the Public, that in our opinion it is a work well executed, and contains a complete ſyſtem of Arithmetic. The rules are plain, and the demonſtrations perſpicuous and ſatisfactory; and we eſteem it the beſt calculated, of any ſingle piece we have met with to lead youth, by natural and eaſy gradations, into a methodical and thorough acquaintance with the ſcience of figures. Perſons of all deſcriptions may find in it every thing, reſpecting numbers, neceſſary to their buſineſs; and not only ſo, but if they have a ſpeculative turn and mathematical taſte, may meet with much for their entertainment at a leiſure hour.

We are happy to ſee ſo uſeful an American production, which, if it ſhould meet with the encouragement it deſerves, among the inhabitants of the United States, will ſave much money in the country, which would otherwiſe be ſent to Europe, for publications of this kind.

We heartily recommend it to ſchools, and to the Community at large, and wiſh that the induſtry and ſkill of the Author may be rewarded, for ſo beneficial a work, by meeting with the general approbation and encouragement of the public.

JOSEPH WILLARD, D. D. Preſident of the Univerſity.
E. WIGGLESWORTH, S. T. P. Hollis.
S. WILLIAMS, L. L. D. Math. et Phil. Nat. Prof. Hollis.

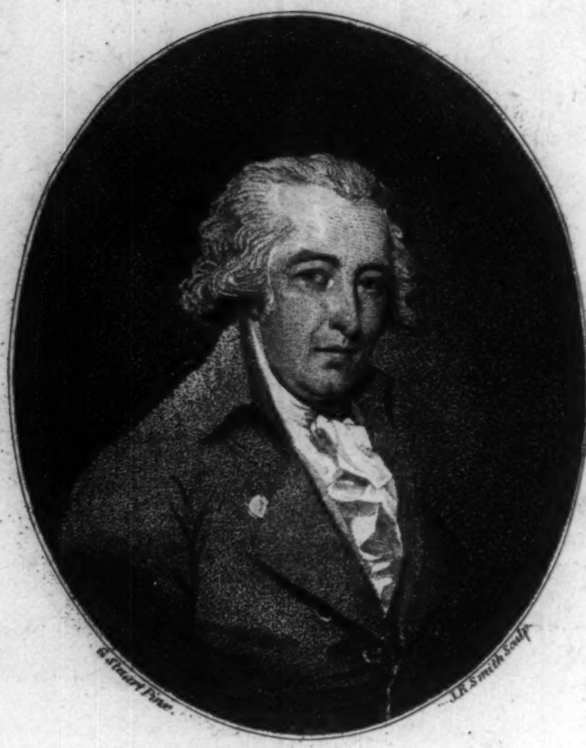
Yale-College, 1786.

UPON examining Mr. Pike's Syſtem of Arithmetic and Geometry in Manuſcript, I find it to be a Work of ſuch Mathematical Ingenuity, that I eſteem myſelf honored in joining with the Reverend Preſident Willard, and other learned Gentlemen, in recommending it to the Public as a Production of Genius, interſperſed with Originality in this Part of Learning, and as a Book ſuitable to be taught in Schools—of Utility to the Merchant, and well adapted even for the Univerſity Inſtruction.—I conſider it of ſuch Merit, as that it will probably gain a very general Reception and Uſe throughout the Republic of Letters. EZRA STILES, Preſident.

Boſton, 1786.

FROM the known character of the Gentlemen, who have recommended Mr. Pike's Syſtem of Arithmetic, there can be no room to doubt, that it is a valuable performance; and will be, if publiſhed, a very uſeful one. I therefore wiſh him ſucceſs in its publication.

JAMES BOWDOIN.



Hon: Jas. Bowdoin.

E James

TO HIS EXCELLENCY

James Bowdoin, *Esquire*,

GOVERNOR AND COMMANDER IN CHIEF

OF THE

COMMONWEALTH OF *MASSACHUSETTS*

AND

PRESIDENT

OF THE

American Academy of Arts and Sciences.

MAY IT PLEASE YOUR EXCELLENCY,

THE Author of this Syſtem, anxious to procure for it a favorable Reception from his Fellow-citizens, takes the liberty of ſoliciting the Honor of your Excellency's Patronage.

As this Work is the firſt of the kind compoſed in America, he feels himſelf entitled to the candid indulgence of the Learned in general—and from your Excellency's zeal for the advancement of the Sciences, and attachment to the Republic of Letters, he reſts aſſured

that the Public will pardon him the ambition of inſcribing your Name to this Literary Attempt.

That your Excellency may long continue the Ornament of your Country and the Delight of your Friends, is the ardent wiſh of

May it Pleaſe your Excellency,

Your Excellency's much Obliged,

moſt Obedient

and very Humble Servant

NICOLAS PIKE.

Newbury-port, Commonwealth of Maſſachuſetts, June 1ſt. 1786.

PREFACE.

IT may, perhaps, by ſome be thought needleſs, when Authors are ſo multiplied, to attempt publiſhing any thing further on Arithmetic, as it may be imagined there can be nothing more than the repetition of a Subject already exhauſted. It is however the Opinion of not a few, who are conſpicuous for their Knowledge in the Mathematics, that the books, now in uſe among us, are generally deficient in the Illuſtration and Application of the rules; of the truth of which, the general Complaint among Schoolmaſters is a ſtrong Confirmation. And not only ſo, but as the United States are now an independent Nation, it was judged that a Syſtem might be calculated more ſuitable to our Meridian, than thoſe heretofore publiſhed.

Although I had ſufficient reaſon to diſtruſt my Abilities for ſo arduous a Taſk, yet not knowing any one, who would take upon himſelf the trouble, and apprehending I could not render the public more eſſential Service, than by an attempt to remove the difficulties complained of, with diffidence I devoted myſelf to the Work.

I have availed myſelf of the beſt Authors which could be obtained, but have followed none particularly, except Bonnycaſtle's Method of Demonſtration.

Although I have arranged the Work in ſuch order as appeared to me the moſt regular and natural, the Student is not obliged to pay a ſtrict adherence to it; but may paſs from one Rule to another, as his Inclination, or Opportunity for Study, may require.

The Fœderal Coin, being purely decimal, moſt naturally falls in after Decimal Fractions.

I have given several Methods of extracting the Cube Root, and am indebted to a learned Friend, who declines having his Name made public, for the Investigation of two very concise Algebraic Theorems for the extraction of all Roots, and of a particular Theorem for the Sursolid.

And here I would observe that, in the Extraction of the Square Root, when more than half of the Root is found, the remaining figures of it may be found by division, making use of the last divisor, and taking care to bring down so many of the next Figures *of the Resolvend, as there were* Periods *to come down, when you began the division.*

Among the miscellaneous Questions, I have given some of a philosophical nature, as well with a view to inspire the Pupil with a relish for Philosophical Studies, as to the usefulness of them in the common Businesses of life.

The short Introduction to Algebra, which is subjoined, was abstracted principally from Bonnycastle, and that of Conic Sections, from Emerson's Works.

Being sensible the following Treatise will stand or fall, according to its real Merit or Demerit, I submit it to the Judgment of the CANDID.

With pleasure I embrace this Opportunity, to express my Gratitude to those learned Gentlemen, Who have honored this Treatise with their Approbation, as well as to such Gentlemen as have encouraged it by their Subscriptions; and to request the Reader to excuse any errors he may meet with; for although great pains have been taken in correcting, yet it is difficult to prevent errors from creeping into the Press, and some may have escaped my own observation; in either case, a hint from the candid will much oblige their

most Obedient,
and humble Servant

 THE AUTHOR.

CONTENTS.

B *Table*

CONTENTS.

Permutations

CONTENTS.

Prob. 2.

CONTENTS.

To

CONTENTS.

Art. 32

CONTENTS.

Explanation of the CHARACTERS *made use of in this Treatise.*

$=$	THE sign of Equality: As, 12 Pence $=$ 1 Shilling, signifies that 12 Pence are equal to 1 Shilling: and in general, that whatever precedes it is equal to what follows.
$+$	THE Sign of Addition: As $5+5=10$, that is, 5 added to 5 is equal to 10.—-Read 5 plus 5, or 5 more 5 equal to 10.
$-$	THE Sign of Subtraction: As, $12-4=8$, that is, 12 lessened by 4 is equal to 8, or 4 from 12 and 8 remains—Read 12 minus 4, or 12 less 4 equal to 8.
$\times$	THE Sign of Multiplication: As $6\times5=30$, that is, 6 multiplied by 5 is equal to 30—-Read 6 into 5 equal to 30.
$\div$ *or* 6)30(	THE Sign of Division: As, $30\div5=6$, that is, 30 divided by 5 is equal to 6—Read 30 by 5 equal to 6.
$\frac{875}{25}$	NUMBERS, placed fraction-wise, do likewise denote division, the Numerator or upper number being the dividend, and the Denominator or lower number, the divisor, thus, $\frac{875}{25}$ is the same as $875\div25=35$.
$:\ ::\ :$	THE Sign of Proportion, thus, $2:4::8:16$, that is, As 2 is to 4 so is 8 to 16.
$\overline{9-2}+6=13$	SHEWS that the difference between 2 and 9 added to 6 is equal to 13—Read 9 minus 2 plus 6 equal to 13;—and that the line a-top (called a *Vinculum*) connects all the numbers over which it is drawn.
$12-\overline{3+5}=4$	SIGNIFIES that the sum of 3 and 5 taken from 12 leaves or is equal to 4.
$\overline{\quad}\vert^{2}$	SIGNIFIES the second Power, or Square.
$\overline{\quad}\vert^{3}$	SIGNIFIES the third Power, or Cube.
$\overline{\quad}\vert^{m}$	SIGNIFIES any Power in general, as $\overline{6}\vert^{2}=$ Square of 6; and $\overline{50}\vert^{3}=$ Cube of 50, &c. thus *m* signifies either the Square or Cube, or any other Power.
$\surd$, *or* $\overline{\quad}\vert^{\frac{1}{2}}$	PREFIXED to any number or quantity, signifies that the Square Root of that number is required. It likewise (as also the Character for any other root) stands for the expression of the root of that number or quantity to which it is prefixed—As $\sqrt{36}=6$, and $\sqrt{108+36}=12$, *or* $\overline{36}\vert^{\frac{1}{2}}=6$, *&c.*

PREFIXED

$\sqrt[3]{\ }$, or $\overline{\ \ }|^{\frac{1}{3}}$ — PREFIXED to any number, signifies that the Cube Root of that number is required, or expressed. As $\sqrt[3]{216} = 6$, and $\sqrt[3]{\overline{513+216}} = 9$, &c.—or $\overline{216}|^{\frac{1}{3}} = 6$, &c.

$\sqrt[m]{\ }$, or $\overline{\ \ }|^{\frac{n}{m}}$ — SIGNIFIES any Root in general. As $\overline{36}|^{\frac{1}{2}} =$ Square Root, $\overline{216}|^{\frac{1}{3}} =$ Cube Root, &c. Thus $\frac{n}{m}$ signifies either the Square Root, Cube Root, or any other Root whatever.

a b c d — WHEN several Letters are set together, they are supposed to be multiplied into each other; as those in the margin are the same as $a \times b \times c \times d$, and represent the continual product of quantities or numbers.

$\frac{1}{a}$ Is the reciprocal of a, and $\frac{a}{b}$ is the reciprocal of $\frac{b}{a}$

IF a be the root, then $a \times a = aa$ or a^2 is the square of a, and $a \times a \times a = aaa$ or a^3 is the Cube of a, &c.

Note, The figure a-top is called the Index of the Power.

IT is usual to write Shillings at the left-hand of a stroke, and Pence at the right; thus, 13/4 is thirteen shillings and four-pence.

Note, The use of these Characters must be perfectly understood by the Pupil, as he may have occasion for them.

ARITHMETIC.

ARITHMETIC is the Art or Science of computing by numbers, and consists both in Theory and Practice.——The Theory considers the nature and quality of numbers, and demonstrates the reason of practical operations.——The practice is, that which shews the method of working by numbers, so as to be most useful and expeditious for business, and is comprised under five principal or fundamental Rules, viz. NOTATION or NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION; the knowledge of which is so necessary that, scarcely any thing in life, and nothing in trade can be done without it.

NUMERATION

TEACHETH the different value of figures by their different places, and to read or write any sum or number by these ten characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.—0 is called a cypher, and all the rest are called figures or digits. The names and significations of these characters, and the origin or generation of the numbers they stand for, are as follow; 0 nothing; 1 one, or a single thing called an unit; $1+1=2$, two; $2+1=3$, three; $3+1=4$, four; $4+1=5$, five; $5+1=6$, six; $6+1=7$, seven; $7+1=8$, eight; $8+1=9$, nine; $9+1=10$, ten, which has no single character; and thus, by the continual addition of one, all numbers are generated.

2. BESIDE the simple value of figures, as above noted, they have, each, a local value, according to the following law; *viz.* In a

combination of figures, reckoning from right to left, the figure, in the first place, represents its primitive simple value; that in the second place, ten times its simple value, and so on; the value of the figure, in each succeeding place, being ten times the value of it, in that, immediately preceding it.

3. The values of the places are estimated according to their order: the first is denominated the place of units; the second, tens; the third, hundreds, and so on, as in the table. Thus in the number—5293467: 7, in the first place, signifies only seven; 6, in the second place, signifies 6 tens, or sixty; 4, in the third place, four hundred; 3, in the fourth place, three thousand; 9, in the fifth place, ninety thousand; 2, in the sixth place, two hundred thousand; 5, in the seventh place, is five millions; and the whole, taken together, is read thus; five millions, two hundred and ninety three thousand, four hundred and sixty-seven.

4. A cypher, though it is of no signification, itself, yet, it possesses a place, and, when set on the right hand of figures, in whole numbers, increases their value in the same tenfold proportion; thus, 9 signifies only nine; but, if a cypher is placed on its right hand, thus, 90, it then becomes ninety; and, if two cyphers be placed on its right, thus, 900, it is nine hundred; &c.

To enumerate any parcel of figures, observe the following Rule.

First, commit the words at the head of the Table, viz. units, tens, hundreds, &c. to memory; then, to the simple value of each figure, join the name of its place, beginning at the left hand, and reading towards the right.——*More particularly*—1. Place a dot under the right-hand figure of the 2d, 4th, 6th, 8th, &c. half-periods and the figure over such dot will, universally, have the name of thousands.——2. Place the figures 1 2 3 4, &c. as indices, over the 2d, 3d, 4th, &c. period: These indices will then shew the number of times the millions are involved—the figure under 1, bearing the name of millions, that under 2, the name of billions (or millions of millions) that under 3, trillions (or millions of millions of millions.)

EXAMPLE.

Sextillions		Quintilli.		Quatrill.		Trillions		Billions		Millions		Units	
th.	un.	th.	un.	th.	un.	th.	un.	th.	un.	th.	un.	c.x.t	c.x.u
	6		5		4		3		2		1		
913,	208;	000,	341;	620,	057;	219,	356;	809,	379;	120,	406;	129,	763
Thousands		*Thousands*		*Thousands*		*Thousands*		*Thousands*		*Thousands*		*Thousands*	

Note 1. Billions is substituted for millions of millions—Trillions

ons, for millions of millions of millions—Quatrillions, for millions of millions of millions of millions.

Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c. anſwer to millions ſo often involved as their indices reſpectively denote.

NOTE 2. The right-hand figure of each half-period has the place of units of that half-period; the middle one, that of tens, and the left-hand one, that of hundreds.

The APPLICATION.

Write down, in proper figures, the following numbers.

Fifteen.
Two hundred and ſeventy-nine.
Three thouſand, four hundred and three.
Thirty-ſeven thouſand, five hundred and ſixty ſeven.
Four hundred, one thouſand and twenty-eight.
Nine millions, ſeventy-two thouſand and two hundred.
Fifty-five millions, three hundred, nine thouſand and nine.
Eight hundred millions, forty-four thouſand, and fifty five.
Two thouſand, five hundred and forty-three millions, four hundred and thirty-one thouſand, ſeven hundred and two.

Write down, in words at length, the following numbers.

8	437	709040	3476194	7584397647
17	3010	879066	84094007	49163189186
129	76506	4091875	690748591	500098400700

Notation by Roman Letters.

I. One.	XV. Fifteen.	CC. Two hundred.
II. Two.	XVI. Sixteen.	CCC. Three hundred.
III. Three.	XVII. Seventeen.	CCCC. Four hundred.
IV. Four.	XVIII. Eighteen.	D or IↃ. Five hundred.
V. Five.	XIX. Nineteen.	DC. Six hundred.
VI. Six.	XX. Twenty.	DCC. Seven hundred.
VII. Seven.	XXX. Thirty.	DCCC. Eight hundred.
VIII. Eight.	XL. Forty.	DCCCC Nine hundred.
IX. Nine.	L. Fifty.	M or CIↃ. One thouſand.
X. Ten.	LX. Sixty.	IↃↃ. Five thouſand.
XI. Eleven.	LXX. Seventy.	IↃↃↃ. Fifty thouſand.
XII. Twelve.	LXXX. Eighty.	IↃↃↃIↃↃↃ. Five hundred thouſ.
XIII. Thirteen.	XC. Ninety.	MDCCLXXXVIII. One thouſand
XIV. Fourteen.	C. Hundred.	ſeven hundred and eighty-eight.

A leſs literal number, placed after a greater, always augments the value of the greater; if put before, it diminiſhes it. Thus, VI is 6: IV is 4. XI is 11; IX is 9. &c.

ADDITION

ADDITION

Is the putting together two or more numbers, or sums, to make them one total, or whole sum.

SIMPLE ADDITION.

Is the adding of several integers or whole numbers together, which are all of one kind, or sort; as 7 pounds, 12 pounds, and 20 pounds, being added together, their aggregate, or sum total is 39 pounds.

Rule.

Having placed units under units, tens under tens, &c. draw a line underneath, and begin with the units: after adding up every figure in that column, consider how many tens are contained in their sum, and, placing the excess under the units, carry so many, as you have tens, to the next column, of tens:—proceed in the same manner through every column, or row, and set down the whole amount of the last row.*

Proof. Begin at the top of the sum, and reckon the figures downwards, in the same manner as they were added upwards, and if it be right, this aggregate will be equal to the first.——Or, cut off the upper line of figures, and find the amount of the rest; then, if the amount and upper line, when added, be equal to the sum total, the work is supposed to be right.

Addition

* This Rule, as well as the method of Proof, is founded on the known Axiom, "The whole is equal to the Sum of all its parts." The method of placing the numbers, and carrying for the tens, is evident from the nature of notation; for any other disposition of the numbers would alter their value; and carrying one, for every ten, from an inferior to a superior column, is, evidently, right, because one unit in the latter case is equal to the value of ten units in the former.

Beside the method of proof, here given, there is another, by casting out the nines; thus:

1. Add the figures in the upper row together, and find how many nines are contained in their sum.
2. Reject the nines, and set down the remainder, directly even with the figures, in the row.
3. Do the same with each of the given numbers, and set all the excesses of nines in a column, and find their sum; then, if the excess of nines in this sum, found as before, is equal to the excess of nines in the sum total; the question is supposed to be right.

Example.

	Excess of nines.
5738	5
9156	3
8471	2
5324	5
28689	6

This method depends upon a property of the number 9, which, except 3, belongs to no other digit whatever; *viz.* that any number, divided by 9, will leave the same remainder, as the sum of its figures, or digits, divided by 9: which may be thus demonstrated.

Demonstration. Let there be any number, as 5432; this, separated into its several parts, becomes $5000+400+30+2$; but $5000=5\times1000=5\times999+1=5\times999+5$. In like manner $400=4\times99+4$, and $30=3\times9+3$. Therefore $5432=5\times999+5,+4\times99+4,+3\times9+3+2=5\times999+4\times99+3\times9+5+4+3+2$. And $\frac{5432}{9}=\frac{5\times999+4\times99+3\times9+5+4+3+2}{9}$; but $5\times999+4\times99+3\times9$ is di-

visible

Addition and Subtraction TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	5	6	7	8	9	10	11	12	13	14
3	5	6	7	8	9	10	11	12	13	14	15
4	6	7	8	9	10	11	12	13	14	15	16
5	7	8	9	10	11	12	13	14	15	16	17
6	8	9	10	11	12	13	14	15	16	17	18
7	9	10	11	12	13	14	15	16	17	18	19
8	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	16	17	18	19	20	21
10	12	13	14	15	16	17	18	19	20	21	22

When you would add two numbers, Look one of them in the left hand column, and the other a-top, and in the common angle of meeting, or, at the right hand of the first, and under the second, you will find the sum.—as, 5 and 8 is 13.

When you would subtract: find the number to be subtracted in the left-hand column, run your eye along to the right-hand till you find the number *from* which it is to be taken, and right over it, a-top, you will find the difference.—as, 8, taken from 13, leaves 5.

1	2	3	4	5	6
£.	℔.	Cwt.	Miles.	Yards.	£.
1	12	123	1234	12345	987654321
2	34	456	5678	67890	123456789
3	56	789	9098	98765	234567891
4	78	12	7654	43210	345678910
5	90	345	3210	12345	456789123
6	1	678	69	67890	567879287
7	23	901	4713	74100	678900028
8	45	234	131	64786	789400690
9	67	567	9128	19876	548769138

7	8	9	10
1234567	1234567	67	1234567
2345678	723456	123	9876543
3456789	34565	4567	2102865
4567890	4566	89098	4321234
5678209	333	654321	5682098
6789098	90	1234567	6543218

SUBTRAC-

visible by 9; therefore 5432, divided by 9, will leave the same remainder as 5+4+3+2, divided by 9; and the same will hold good of any other number whatever.

The same property belongs to the number 3: however, this Inconveniency attends this method, that, although the work will always prove right, when it is so; it will not, always, be right, when it proves so; I have therefore given this demonstration more for the sake of the curious, than for any real advantage.

SUBTRACTION

TEACHETH to take a less number from a greater, to find a third, shewing the inequality, excess or difference between the given numbers; and it is both simple and compound.

SIMPLE SUBTRACTION

TEACHETH to find the difference between any two numbers, which are of a like kind.

RULE.

PLACE the larger number uppermost, and the less underneath, so that units may stand under units, tens under tens, &c. then, drawing a line underneath, begin with the units, and subtract the lower from the upper figure, and set down the remainder; but if the lower figure be greater than the upper, borrow ten, and subtract the lower figure therefrom; to this difference, add the upper figure, which, being set down, you must add one to the ten's place of the lower line for that which you borrowed; and thus proceed through the whole.‡

PROOF.

IN either simple or compound Subtraction, add the remainder and the less line together, whose sum, if the work be right, will be equal to the greater line:—Or, subtract the remainder from the greater line, and the difference will be equal to the less.

EXAMPLES.

	1	2	3	4	5	6
	£.	£.	Miles.	Yards.	Feet.	Cwt.
From	25	305	4670	58934	879647	9187641
Take	12	103	4020	6182	164348	91843
Rem.						
Proof.						

7	8	9
1002003004005006007008009OO	10000	1000000
980760540320110230450670089	9999	1

MULTI-

‡ *Dem.* WHEN all the figures of the least number are less than their correspondent figures in the greatest, the difference of the figures, in the several like places, must, all taken together, make the true difference sought; because, as the sum of the parts is equal to the whole; so must the Sum of the differences, of all the similar parts, be equal to the difference of the whole.

2. WHEN any figure in the greatest number is less than its correspondent figure in the least, the ten, which is added by the Rule, is the value of an unit in the next higher place, by the nature of notation; and the one which is added to the next place of the least number, is to diminish the correspondent place of the greatest, accordingly;

MULTIPLICATION

MAY be accounted the most serviceable Rule in Arithmetic: It teacheth how to increase the greater of two numbers given, as often as there are units in the less;—performeth the work of many additions in the most compendious manner——brings numbers of great denominations into small, as pounds into shillings, pence or farthings, &c. and, by knowing the value of one thing, we find the value of many.

IT consists of three parts.

1. THE Multiplicand, or number given to be multiplied, and, commonly, the largest number.
2. THE Multiplier, or number to multiply by, commonly, the least number.
3. THE Product is the result of the work, or the answer to the question.

SIMPLE MULTIPLICATION

Is the multiplying any two numbers together, without having regard to their signification; as 7 times 8 is 56, &c.

MULTIPLICATION and DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

To learn this Table, for Multiplication: Find your multiplier in the left hand column, and your multiplicand a-top, and in the common

ingly; which is only taking from one place, and adding as much to another, whereby the total is never changed: and, by this mean, the greater is resolved into such parts, as are, each, greater than, or equal to, the similar parts of the less; and the difference of the correspondent figures, taken together, will, evidently, make up the difference of the whole.

THE truth of the method of proof is evident; for the difference of two numbers, added to the least, is, manifestly, equal to the greater.

mon angle of meeting, or, against your multiplier, along at the right-hand, and under your multiplicand, you will find the product, or answer.

To learn it for Division: Find the divisor in the left-hand column, and run your eye along the Row to the right-hand 'till you find the dividend, then, directly over the dividend, a-top, you will find the quotient, shewing how often the divisor is contained in the dividend.

CASE I.

WHEN the multiplier is not more than 12, always placing the greatest number uppermost, set the multiplier underneath, units under units, &c. and begin as the Table directs, setting down the unit-figure under units, and carrying the tens to the next place, in all respects as in simple addition.‡

PROOF.

MULTIPLY the multiplier by the multiplicand.

EXAMPLES.

	1	2	3	4
	37934	769308	4980076	763896
	2	3	4	5
Product				

5	6	7	8
67589	503764	3918295	9164785
6	7	8	9

9	10	11
4879567	5864734	8593478649
10	11	12

CASE 2.

WHEN the multiplier consists of more places than one, multiply each figure in the multiplicand by every figure in the multiplier, beginning with the units, and placing the first figure of each product exactly under its multiplier: lastly, add these several products together,

‡ *Dem.* WHEN the multiplier is a single digit, it is plain that we find the Product; for, by multiplying every figure, that is, every part of the multiplicand, we multiply the whole; and, the writing down the products, which are less than ten, or the excess of tens, in the places of the figures multiplied, and carrying the number of tens to the product of the next place, is only gathering together the similar parts of the respective products, and is therefore the same in effect, as though we wrote down the multiplicand as often as the multiplier expresses, and added them together; for the sum of every column is the product of the figures in the place of that column; and the products, collected together are evidently equal to the whole required product.

ther, in the same order as they stand, and their sum will be their total product.‡

EXAMPLES.

1.	2.	3.
6357534	8324629	46293845
47	59	76
44502738		
25430136		
Prod. 298804098		

4.	5.	6.
647906	760483	91867584
4873	9152	6875
3157245938	6959940416	631589640000

D CASE 3.

‡ IF the multiplier be a number, made up of more than one figure; after we have found the product of the multiplicand by the first figure of the multiplier, as above, we suppose the multiplier divided into parts, and, after the same manner, find the product of the multiplicand by the second figure of the multiplier; but as the figure, by which we are multiplying, stands in the place of tens, the product must be ten times its simple value, and, therefore, the first figure in this product must be noted in the place of tens, or, which is the same, directly under the figure we are multiplying by. And, proceeding in the same manner with all the figures of the multiplier, separately, it is evident we shall multiply all the parts of the multiplicand by all the parts of the multiplier; therefore, these several products being added together, will be equal to the whole required product.

THE Reason of the method of proof, depends upon this proposition, that if two numbers are to be multiplied together, either of them may be made the multiplier or multiplicand, and the product will be the same.

A small attention to the nature of numbers will make this truth evident; for $5 \times 9 = 45 = 9 \times 5$; and in general, $2 \times 3 \times 4 \times 5 \times 6$, &c. $= 3 \times 2 \times 6 \times 5 \times 4$, &c. without any regard to the order of the terms; and this is true of any number of factors whatever.

N. B. BY factors are meant the multiplier and multiplicand.

THE following examples are subjoined, to make the reason of the Rule appear as clearly as possible.

64753		
5		
15	=	3×5
25	=	50×5
35	=	700×5
20	=	4000×5
30	=	60000×5
323765	=	64753×5

237956		
3728		
1903648	=	8 times the multiplicand.
475912	=	20 times ditto.
1665692	=	700 times ditto.
713868	=	3000 times ditto.
887099968	=	3728 times ditto.

MULTIPLICATION may also be proved, by casting out the nines; but is liable to the inconvenience before mentioned.

IT may likewise be, very naturally, proved by division; for the product, being divided by either of the factors, will, evidently, give the other; and it might not be amiss for the pupil to be taught division, at the same time with multiplication, as it not only serves for proof; but also gives him a readier knowledge of them both; but it would have been contrary to good method to have given this rule in the text, because the Pupil is supposed, as yet, to be unacquainted with division.

CASE 3.

WHEN the multiplier is a compoſite number, that is, when it is produced by the multiplication of any two numbers in the Table, multiply the multiplicand by one of thoſe figures, firſt, and that product by the other : and the laſt product will be the total required. §

EXAMPLES.

	1.	2.	3.
Mult.	59375 by 35.	39187 by 48.	91632 by 56.

$$\begin{array}{lr} & 7 \\ \hline 7\times5=35 & 415625 \\ & 5 \\ \hline & 2078125 \\ \hline \end{array}$$

	4.	5.	6.
Mult.	91738 by 81.	35963 by 72.	847396 by 99.

	7.	8.	9.
Mult.	38462 by 108.	749357 by 121.	9043278 by 144.

CASE 4.

WHEN there are cyphers on the right-hand of, either the multiplicand, or multiplier, or both, neglect thoſe cyphers ; then place the ſignificant figures under one another, and multiply by them only ; add them together, as before directed, and place to the right-hand as many cyphers as there are in both the factors.

EXAMPLES.

§ THE reaſon of this method is obvious : for any number, multiplied by the component parts of another number, muſt give the ſame product, as though it were multiplied by that number at once : thus, in example firſt, 5 times the product of 7, multiplied into the given number, makes 35 times that given number as plainly, as 5 times 7 make 35.

EXAMPLES.

	1.	2.	3.
	67910	956700	930137000
	5600	320	9500
Prod.	380296000	306144000	883630150000

	4.	5.	6.
	359260	8196000	623000
	7364	59180	.589000
Prod.	2645590640	485039280000	366947000000

CASE 5.

WHEN there are cyphers between the significant figures of the multiplier, omit multiplying by them, and place the first figure of each product of the significant figures, exactly under that figure by which you multiply; lastly, add them together, and their sum will be the total product.

EXAMPLES.

	1.	2.	3.
	5397	85630	48976850
	8009	7005	400030
	48573		
	43176		
Prod.	43224573	599838150	19592209305500

CASE 6.

To multiply by 10, 100, 1000, &c. set down the multiplicand underneath, and join the cyphers in your multiplier to the right-hand of them.

EXAMPLES.

	1.	2.	3.	4.
	57935	84935	613975	8473965
	10	100	1000	10000
Prod.	579350			

CASE 7.

CASE 7.

To multiply by 99, 999, &c. in one line; place as many dots at the right-hand of the multiplicand, as there are figures of 9 in your multiplier, which dots suppose to be cyphers, then, beginning with the right-hand dot, subtract the *given* multiplicand from the *new* one, and the remainder will be the total product.†

EXAMPLES.

1.	2.	3.
6473..	857389...	5384976....
99	999	9999
640827	856531611	53844375024

THAT these Examples may appear as clear as possible, I will illustrate them by giving another.

Mult.	371967...	According to the rule, it will stand thus	371967...	Minuend.
by	999		371967	Subtrahend
	371595033		371595033	Rem. or total Prod.

CASE 8.

To multiply by 13, 14, 15, &c. to 19 *inclusively*, at one multiplication.

RULE I.

MULTIPLY the multiplicand by the unit-figure of the multiplier, and add to the product of each multiplication that figure which stands next on the right hand of that which you multiplied, and, to the last figure in the multiplicand, add what you carry.

EXAMPLES.

† HERE it may easily be seen that, if you multiply any sum by 9, the product will be but 9 tenths of the product of the same sum, multiplied by 10; and, as the annexing a dot, or cypher, to the right hand of the multiplicand, supposes it to be increased tenfold; therefore, subtracting the given multiplicand from the ten-fold multiplicand, it is evident that the remainder will be nine-fold the said given multiplicand, equal to the product of the same by 9; and the same will hold true of any number of nines.

Note, WHEN the multiplicand has a fraction added to it, as one fourth, one half, &c. add such a part of the multiplier as the fraction makes, to the last product:—But when such fraction belongs to the multiplier, add to the last product such a part of the multiplicand as the fraction denotes.

EXAMPLES.

1.	2.	3.	4.	5.	6.	7.
65497	84916	19345	7398	9108	6173	54937
13	14	15	16	17	18	19
Prod. 851461						

In example first, I say, 3 times 7 is 21; I put down 1 and carry 2; saying, 3 times 9 is 27, and 2, that I carried, makes 29, and 7, the right-hand figure to 9, makes 36; I put down 6, and carry 3: then 3 times 4 is 12, and, 3 which I carried, makes 15, and 9, its right-hand figure, makes 24; therefore I put down 4, and carry 2; saying, 3 times 5 is 15, and 2, which I carried, makes 17, and 4, its right-hand figure, makes 21; I, therefore, put down 1 and carry 2; saying, 3 times 6 is 18, and 2, which I carried, makes 20, and 5, its right-hand figure, makes 25, I therefore set down 5, and carry 2; lastly, the 2 which I carry, and 6, the last figure in the multiplicand, make 8, which gives the total product.

RULE 2.

To multiply by 13, 14, 15, &c. to 19: place your multiplier at the right of the multiplicand, with the sign of multiplication between them, and multiply with the unit-figure, only, of the multiplier, removing the product one figure to the right-hand of the multiplicand: then add all together, and their sum will be the total product.

EXAMPLES.

1.	2.	3.	4.	5.
75964 × 13	7598 × 14	76013 × 15	8196 × 16	3179 × 17
227892				
Prod. 987532				

CASE 9.

To multiply by 111, 112, 113, &c. to 119, so as to have the product in one line: multiply the multiplicand by the unit-figure, only, of the multiplier, and add to each multiplication the *two* figures which stand next on the *right*-hand to that which you multiplied, and to the *two* last figures, separately, add what you carry.

EXAMPLES.

EXAMPLES.

1.	2.	3.	4.	5.
52976	58975	89193	76435	781572
111	112	113	114	115

Prod.

6.	7.	8.	9.
43958	647358	499789	9417
116	117	118	119
			1120623

In the last example I say, 9 times 7 is 63, I set down 3 and carry 6; then, 9 times 1 is 9, and 6 I carried makes 15, and 7, its right-hand figure, makes 22, I set down 2 and carry 2, then, 9 times 4 is 36, and 2 I carry is 38, and its right-hand figures, 1 and 7, make 46; I set down 6 and carry 4; then, 9 times 9 is 81, and 4 I carried, is 85, and 4 and 1, its right-hand figures, make 90, I put down 0, and carry 9, which I add to 9 and 4, the two last figures, and they make 22, I then put down 2 and carry 2; lastly, this 2 and 9 make 11, which I set down, and the product is complete.

CASE 10.

To multiply by 101, 102, 103, &c. to 109, so as to have the product in one line:

RULE 1.

Multiply the multiplicand by the unit-figure of the multiplier, and add to it the next right-hand figure, but one, to that which you multiplied, remembering to add to the two last figures in your multiplicand, separately, what you carry.

EXAMPLES.

1.	2.	3.	4.	5.
57691	89726	75964	84975	635918
101	102	103	104	105

Prod.

6.	7.	8.	9.
64791	58493	8493294	6479
106	107	108	109

Prod. | | | | 706211 |

In

In the last example I say, 9 times 9 is 81, I set down 1 and carry 8; then, 9 times 7 is 63, and 8 I carry is 71, I set down 1 and carry 7; then, 9 times 4 is 36, and 7 I carry is 43, and 9, its right-hand figure but one, makes 52: I set down 2 and carry 5, saying, 9 times 6 is 54, and 5 I carry is 59, and 7, its next right-hand figure but one, makes 66: I set down 6 and carry 6, which I add to 4, the last figure but one in the multiplicand, and it makes 10; I set down 0, and carry 1, which I add to 6, the last figure, and it makes 7, which I set down, and I have the whole product.

Rule 2.

To multiply by 101, 102, 103, &c. to 109: multiply by the right-hand figure, only, of the multiplier, removing the product two figures to the right-hand of the multiplicand: add all together, and the sum will be the total product.

Examples.

	1.	2.	3.
	64795 ×101	79164 ×102	37598 ×103
	64795		
Product.	6544295		

	4.	5.	6.
	73967 ×104	84973 ×105	748794 ×106
Prod.			

7.	8.	9.
87958 ×107	395867 ×108	5916379 ×109

Case II.

To multiply by 21, 31, 41, &c. to 91, in one line:

Rule.

First, bring down the unit-figure of the multiplicand, which will, always, be the unit-figure of the product, then multiply every figure of the multiplicand by the ten's figure of the multiplier, and to each product add the figure, which stands next on the left-hand to that which you multiplied.

Examples.

EXAMPLES.

1.	2.	3.	4.
6493587	935846	3584956	716298543
21	31	41	51
Prod.			

5.	6.	7.	8.
537984	326478	938467	4793
61	71	81	91
Prod.			436163

IN the last example, I first bring down the unit-figure 3, of the multiplicand, for the unit-figure of the product; and, then, I say, 9 times 3 is 27, and 9, its left-hand figure, makes 36, I set down 6 and carry 3: then 9 times 9 is 81, and 3 I carry is 84, and 7, its left-hand figure is 91; I set down 1 and carry 9, saying, 9 times 7 is 63, and 9 I carry is 72, and 4, its left-hand figure, is 76: I set down 6 and carry 7; lastly, 9 times 4 is 36, and 7 I carry makes 43, which I set down, and have the product complete.

RULE 2.

To multiply by 21, 31, 41, &c. to 91: multiply by the ten's figure, only, of the multiplier, and set the unit-figure of the product under the place of tens; add them all together, and their sum will be the total product.

EXAMPLES.

1.	2.	3.	4.
73918 ×21	56934 ×31	86789 ×41	759846 ×51
147836			
1552278			

5.	6.	7.	8.
37954 ×61	73958 ×71	84937 ×81	54937 ×91

CASE 12.

To multiply by 22, 23, 24, &c. to 29; so as to have the product in one line; multiply every figure of the multiplicand by the unit-figure

figure of the multiplier, and add to each product *twice* that figure which ſtands next on the right-hand of that figure you multiplied; and to *twice* the laſt figure of the multiplicand, add what you carry.

EXAMPLES.

	1.	2.	3.	4.
	649378	46795	64839	83964723
	22	23	24	25
Prod.				

	5.	6.	7.	8.
	73758	91357	849358	7657
	26	27	28	29
Prod.				222053

In the laſt example, I ſay, 9 times 7 is 63, I ſet down 3 and carry 6; then, 9 times 5 is 45, and 6 I carry is 51, and 7, its right-hand figure, added *twice*, makes 65, I therefore ſet down 5 and carry 6, ſaying 9 times 6 is 54, and 6 I carry is 60, and 5, its right-hand figure, added *twice*, makes 70; I ſet down 0 and carry 7; then, 9 times 7 is 63, and 7 I carry is 70, and 6, its right-hand figure, added twice, makes 82; I ſet down 2 and carry 8; laſtly, I add this 8 to *twice* the laſt multiplicand-figure, and they make 22, and the whole product ſtands as above.

To multiply any number, viz. whole or Decimal, by any number, giving only the Product.

Put down the Product-figure of the firſt figure of the multiplicand by the FIRST of the multiplier—To the product of the SECOND of the multiplicand by the FIRST of the multiplier, add the number to be carried, and the product of the FIRST of the multiplicand by the SECOND of the multiplier; then, carrying for the tens in the ſum, put down the reſt. To the product of the THIRD of the multiplicand by the FIRST of the multiplier add the number to be carried, and the product of the SECOND of the multiplicand by the SECOND of the multiplier, alſo the product of the FIRST of the multiplicand by the THIRD of the multiplier, carry the tens, and put down the reſt, and ſo proceed till you have multiplied the HIGHEST of the multiplicand by the loweſt of the multiplier. Then multiply the HIGHEST of the multiplicand by the SECOND of the multiplier; add the number to be carried, and the product of the laſt but one of the multiplicand by the THIRD of the multiplier, and the product of the laſt but TWO of the multiplicand by the FOURTH of the multiplier, &c. Then to the product of the LAST BUT ONE of the multiplicand by the FOURTH of the multiplier; and ſo proceed till you have multiplied the LAST of the multiplicand by the laſt of the multiplier, which finiſhes the work.

 Example.

Example.	
Mult.	5321415
By	2354
Prod.	12526610910

Explanation.

$$\overline{5\times4}=20$$
$$\overline{1\times4}+2+\overline{5\times5}=31$$
$$\overline{4\times4}+3+\overline{1\times5}+\overline{5\times3}=39$$
$$\overline{1\times4}+3+\overline{4\times5}+\overline{1\times3}+\overline{5\times2}=40$$
$$\overline{2\times4}+4+\overline{1\times5}+\overline{4\times3}+\overline{1\times2}=31$$
$$\overline{3\times4}+3+\overline{2\times5}\times\overline{1\times3}+\overline{4\times2}=36$$
$$\overline{5\times4}+3+\overline{3\times5}+\overline{2\times3}+\overline{1\times2}=46$$
$$\overline{5\times5}+4+\overline{3\times3}+\overline{2\times2}=42$$
$$\overline{5\times3}+4+\overline{3\times2}=25$$
$$\overline{5\times2}+2=12$$

DIVISION.

Teacheth to separate any number, or quantity given, into any number of parts assigned; or to find how often one number is contained in another;—or from any two numbers given, to find a third, which shall consist of so many units, as the one of those given numbers is comprehended in the other; and is a concise way of performing several Subtractions.

There are four principal parts to be noticed in Division, *viz.*

1. The Dividend, or number given to be divided.
2. The Divisor, or number given to divide by.
3. The Quotient, or answer to the question, which shews how often the divisor is contained in the dividend.
4. The Remainder (which is always less than the Divisor, and of the same name with the Dividend) is very uncertain, as there is sometimes a Remainder, and sometimes none.

Division is both simple and compound.

Proof.

Multiply the divisor and quotient together, and add the remainder, if there be any, to the product;—If the work be right, that Sum will be equal to the dividend.

SIMPLE DIVISION

Is the dividing of one number by another, without regard to their values: as, 56, divided by 8, produces 7 in the quotient: that is, 8 is contained 7 times in 56.¶

Case

¶ According to the rule, we resolve the dividend into parts, and find, by trial, the number of times the divisor is contained in each of those parts; and the only thing which remains to be proved, is, that the several figures of the quotient, taken as one number, according to the order, in which they are placed, is the true quotient of the whole dividend by the divisor; which may be thus demonstrated. Dem.

CASE I.

RULE. First, seek how many times the divisor is contained in a competent number of the first figures of the dividend ;—when found, place the figure in the quotient ;—multiply the divisor by this quotient-figure, place the product under the left-hand figures of the dividend ; then subtract it therefrom, and bring down the next figure of

Dem. THE complete value of the first part of the dividend, is, by the nature of notation, 10, 100, 1000, &c. the value of what is taken in the operation ; accordingly, as there are 1, 2 or 3, &c. figures standing before it ; and, consequently, the true value of the quotient-figure, belonging to that part of the dividend, is also 10, 100, 1000, &c. times its simple value ; but the true value of the quotient-figure, belonging to that part of the dividend, found by the rule, is also 10, 100, 1000, &c. times its simple value ; for there are as many figures set before it, as the number of remaining figures in the dividend ; therefore, this first quotient-figure, taken in its complete value from the place it stands in, is the true quotient of the divisor in the complete value of the first part of the dividend. For the same reason, all the rest of the figures of the quotient, taken according to their places, are, each, the true quotient of the divisor, in the complete value of the several parts of the dividend belonging to each ; because, as the first figure, on the right hand of each succeeding part of the dividend, has a less number of figures standing before it, so ought their quotients to have ; and so they are actually ordered : consequently taking all the quotient-figures in order, as they are placed by the rule, they make one number, which is equal to the sum of the true quotients of all the several parts of the dividend ; and is therefore the true quotient of the whole dividend by the divisor.

THAT no obscurity may remain, in this demonstration, it is illustrated by the following example.

Divisor 25)74503 Dividend		
1st. part of the dividend is =	74000	
25 x 2000 =	50000	— — 2000 the 1st. quotient.
1st. remainder =	24000	
add	500	
2d. part of the dividend =	24500	
25 x 900 =	22500	— — 900 the 2d quotient.
2d. remainder =	2000	
add	00	
3d. part of the dividend =	2000	— — 80 the third quotient.
25 x 80 =	2000	
	00	
add	3	
4th. part of the dividend =	3	
25 x 0	0	— — 0 the 4th quotient.
last remainder =	3	— 2980 = Sum of all the quotients, or, the Answer.

Explanation. IT is evident the dividend is resolved into these parts, 74000+500+00+3 for the first part of the dividend is considered only as 74 ; but yet it is, truly, 74000 ; and therefore its quotient, instead of 2, is 2000, and the remainder 24000 : and so of the rest ; as may be seen in the operation.

WHEN there is no remainder to a division, the quotient is the absolute and perfect answer to the question ; but where there is a remainder, it may be observed, that it goes so

of the dividend to the right-hand of the remainder :—If when you have brought down a figure to the remainder, it is ſtill leſs than the diviſor, a cypher muſt be placed in the quotient, and another fi-
gure

ſo much towards another time as it approaches the diviſor ; thus, if the remainder be half the diviſor, it will go half of a time more, and ſo on ; in order therefore to complete the quotient, put the laſt remainder to the end of it, above a line, and the diviſor below it.

IT is ſometimes difficult to find how often the diviſor may be had in the numbers of the ſeveral ſteps of the operation : the beſt way will be to find how often the firſt figure of the diviſor may be had in the firſt, or two firſt figures of the dividend, and the anſwer, made leſs by one or two, is, generally, the figure wanted ; but if, after ſubtracting the product of the diviſor and quotient from the dividend, the remainder be equal to, or exceed the diviſor, the quotient-figure muſt be increaſed accordingly.

THE reaſon of the method of proof is plain ; for ſince the quotient is the number of times the dividend contains the diviſor, the product of the quotient and diviſor, muſt, evidently, be equal to the dividend.

THERE are ſeveral other methods made uſe of to prove diviſion ; as follow, *viz.*

RULE 1.

SUBTRACT the remainder from the dividend ; divide this number by the quotient, and the quotient, found by this diviſion, will be equal to the former diviſor, when the work is right.

RULE 2.

ADD the remainder and all the products of the ſeveral quotient-figures multiplied by the diviſor together, according to the order in which they ſtand in the work, and the ſum, when the work is right, will be equal to the dividend.

HERE, the numbers to be added are the products of the diviſor by every figure of the quotient, ſeparately ; and each, by its place, poſſeſſes its complete value ; therefore the ſum of the parts, together with the remainder, muſt be equal to the whole. I will illuſtrate the whole by an example proved according to the ſeveral different methods,

```
79)987654321(12501953
   79*                      79 + 34 remainder.
   ---                ------------
   197                 112517577
   158*                87513671
   ----                     + 34
   .396               ------------
   .395*               987654321 Proof by multiplication.
   .-----
   ...154                          987654321
   ...  79*                              - 34
   ...  ----                      -----------
   ....753           12501953)987654287(79 Proof by division.
   ....711*                   87513671
   ....-----                  -----------
   .....422                   112517577
   .....395*                  112517577
   .....-----                 -----------
   ......271
   ......237*
   ......-----
   .......34*
   ----------
   987654321 Proof by Addition.
```

WE need only to refer to the example ; except for the proof of addition ; where, it may be remarked, that the *Aſteriſms* ſhew the numbers to be added, and the dotted lines their order.

gure be brought down; after which, you must seek, multiply and subtract, till you have brought down every figure of the dividend.

EXAMPLES.

1.

```
Divisor. Dividend. Quotient.
   3)175817(58605
     15
     --
      25
      24
      --
       18
       18
       --
         17
         15
         --
          2 Rem.
  Proof
  58605 Quotient
     ×3 Divisor + 2
  ------
 175817
```

In this example, I find that 3, the divisor, cannot be contained in the first figure of the dividend, therefore, I take two figures, *viz.* 17, and enquire how often 3 is contained therein, which finding to be 5 times, I place the 5 in the quotient, and multiply the divisor by it, setting the first figure of the multiplication under the 7 in the dividend, &c. I then subtract 15 from 17, and find a remainder of 2, to the right hand of which I bring down the next figure of the dividend, viz. 5, then, I enquire how often the divisor 3, is contained in 25, and, finding it to be 8 times, I multiply by 8, and proceed as before, till I bring down the 1, when, finding I cannot have the divisor in 1, I place 0 in the quotient, and bring down 7 to the 1, and proceed as at the first.

Observe, that in multiplying by 3, I add in the 2.

```
        2.                          3.
 29)153598(5296         6493)91876375(14150
    145                      6493
    ---                      -----
      85                     26946
      58                     25972
      --                     -----
      279                      9743
      261                      6493
      ---                      ----
       188                     32507
       174                     32465
       ---                     -----
        14                       425
```

```
      4.                 5.                  6.
 28)503775(          35)197184(          85(994466(

      7.                 8.                  9.
236)3798567(     3479)483956795(     5679)19647394(
```

EXAMPLES.

EXAMPLES.

10. 38473)119184693(

11. 641976)9187642959(

12. 5823789)791822376496(

13. 123456789)121932631112635269(

CASE 2.

WHEN there is one cypher, or more, at the right hand of the divisor, it or they must be cut off; also, cut off the same number of figures from the dividend, and then proceed as in Case first: But the figures which were cut off from the dividend must be placed at the right-hand of the remainder.‡

EXAMPLES.

1.

```
65|00)3794326|75(58374
      325
      ---
       544
       520
       ---
        243
        195
        ---
         482
         455
         ---
          276
          260
          ---
          1675 Rem.
```

2. 5193|000)8937643|893(

3. 917|0)476583(

4. 875000)917647894300000(

5. Quot. 1|0)9584|6 Rem.

6. Quot. Rem. 1|00)76495|80

7. Quot. Rem. 1|000)93751839|462

Note. In dividing by 10, 100, 1000, &c. when you cut off as many figures from the dividend, as there are cyphers in the divisor, your work is done; those figures, cut off at the right hand, are the remainder, and those on the left, the quotient, as above.

CASE

‡ THE reason of this contraction is easy to conceive; for the cutting off the same figures from each, is the same as dividing each of them by 10, 100, 1000, &c. and it is evident, that, as often as the whole divisor is contained in the whole dividend, so often must any part of the divisor be contained in the like part of the dividend; this method is only to avoid a needless repetition of Cyphers, which would happen in the common way.

CASE 3.

To perform division without setting down the multiplication: First, seek how often the divisor may be contained, as before directed; place the figure in the quotient, and multiply it by the divisor: subtract the unit-figure of the multiplication from the dividend, and, if you are obliged to borrow in subtracting, you must add one extraordinary to the next multiplication, and proceed as before.¶

EXAMPLES.

1.

```
756)647395(856
    ----
    4259
    ----
    4795
    ----
     259
```

2.

```
8197)9167846(
```

3.

```
293)5017846(
```

4.

```
5840)89764399(
```

5.

```
75000)9840000(
```

In the first example I find the first quotient-figure to be 8; then, I say, 8 times 6 is 48; subtract the unit-figure 8 from the 3 in the dividend, and 5 remains; then, as I was obliged to borrow 1 in subtracting, I carry 5, that is, 4 tens in 48, and one ten I borrowed, make 5 tens; saying, 8 times 5 is 40, and 5 I carried, makes 45; then, 5 from 7 and 2 remains: I now carry only 4, as I borrowed none in subtracting; next, I say 8 times 7 is 56, and 4 I carry, is 60, which, subtracted from 64, leaves 4; I now bring down the next figure, and proceed, in the same manner, through the whole.

CASE 4.

SHORT Division is, when the divisor does not exceed 12.

RULE.

FIRST, seek how often the divisor can be had in the first figure, or figures, of the dividend; which, when found, place in the quotient; then,

¶ THE reason of this rule is the same as that of the general Rule, page 34.

then, *mentally*, multiply your divisor by the figure placed in the quotient, and subtract the product from the like number of the left-hand-figures of your dividend, and the units which remain, must be accounted so many tens, which you must suppose to stand at the left-hand of the next figure in the dividend, and to be reckoned with it; then, seek how often you can have your divisor in those two figures; but, if nothing remain, you must then seek how often your divisor is contained in the next figure, or figures, and thus proceed till you have done.

EXAMPLES.

Divisor. Dividend.	2.	3.	4.	5.
2)71935	3)51903	5)633795	6)8471937	7)193847
Quot. 35967—1				

6.	7.	8.	9.
8)5437846	9)459637 84	11)91843756	12)1196437847536

CASE 5.

WHEN the Divisor is such a number, that any two, or more, figures in the Table, being multiplied together, will produce it, divide the given dividend by one of those figures;—the quotient, thence arising, by the other, and so on; and the last quotient will be the answer.†

EXAM-

† THIS follows from the contraction in case 3d. of simple multiplication, of which it is only the reverse; for the fourth part of the half of any thing is evidently the same as the eighth part of the whole; and so of any other number.

As the learner, at present, is supposed to be unacquainted with the nature of fractions, and as the quotient is incomplete without the remainder; I shall here give a rule for finding the true remainder, without having recourse to fractions.

RULE.

MULTIPLY the quotient by the divisor: subtract the product from the dividend, and the result will be the true remainder.

THE Rule which is most commonly made use of, when the divisor is a composite number, is

RULE II.

MULTIPLY the last remainder by the preceding divisor, or last but one, and to the product add the preceding remainder; multiply this sum by the next preceding divisor, and to the product add the next preceding remainder; and so on, till you have gone thro' all the divisors and remainders, to the first.

EXAMPLES.

EXAMPLES.

1st. method.	2d. method.	3d. method.
9)196473	8)196473	72)196473(2728 Quot.
8)21830	9)24559	144
Quot. 2728—57	Quot. 2728—57	524
		504
		207
		144
		633
		576
		57 Remainder.

I have wrought the above question three ways, that the learner may understand the method of finding the true remainder, according to this case.—In the *first*, in dividing by 9, 3 remains, and, by 8, 6 remains; which, being the last remainder, I multiply it by the first divisor 9, and add in the first remainder 3, and they make 57, the true remainder.—In the *second* method, dividing by 8, 1 remains, and by 9, 7 remains, I therefore multiply 7, the last remainder, by 8, adding in the 1, and they make 57, as before.—The *third* method is self-evident, and shews that the other remainders are true.

2.	3.	4.	5.
36)79638	25)197835	84)93975	54)9373876 4

6.	7.	8.
121)75323939	132)38473692	144)891376429732

 Supplement

EXAMPLE.

6)85397 divided by 150.

*5)14232—5

5)2846—2

569—1

Ans. 569$\frac{47}{150}$.

	1	the last remainder.
multiply by	*5	the last divisor but one.
	5	
add	2	the second remainder.
	7	
multiply by	6	the first divisor.
	42	
add	5	the first remainder.
	47	the true remainder.

To explain this rule from the example, we may observe that every unit in the first quotient may be looked upon as containing 6 of the units in the given dividend, consequently, every unit, which remains, will contain the same; therefore, this remainder must be multiplied by 6, to find the units it contains of the given dividend. Again, each unit in the next quotient will contain 5 of the preceding ones, or 30 of the first, that is, 6 times 5; therefore, what remains must be multiplied by 30, or, which is the same

Supplement to Contractions in Multiplication.

1. The shortest method of multiplication, when the multiplier is any even part of 100, 1000, &c. is by division: for if the multiplicand be increased by a number of cyphers equal to the places in the multiplier, and a part of that product taken for the same proportion, which the multiplier bears to 1, and the same number of cyphers annexed to it, the quotient will be the true product.

1. Multiply 39756 into 125.
$125 = \frac{1}{8}$ of 1000, wherefore,
8)39756000
4969500 Product.

2. Multiply 57638 by $33\frac{1}{3}$.
$33\frac{1}{3} = \frac{1}{3}$ of 100, therefore,
3)5763800
$1921266\frac{2}{3}$ Product.

3. Multiply 91378 by $333\frac{1}{3}$.
$333\frac{1}{3} = \frac{1}{3}$ of 1000, therefore
3)91378000
$30459333\frac{1}{3}$ Product.

2. From the aforementioned property peculiar to the digit 9, it follows, that whatever other digit, with any number of cyphers annexed, is divided by it, the quotient will consist, wholly, of such digits, and so many 9ths of an unit over; hence the following method, of multiplying by repetends of any of the digits.

	1.		2.		3.
	645 by 8888.		5394 by 66666.		3798 by 444
	80000		600000		4000
	9)51600000		9)3236400000		9)15192000
	5733333		359600000		1688000
Subtract	573	Subt.	3596	Subt.	1688
Product.	5732760	Prod.	359596404	Prod.	1686312

TABLES in COMPOUND ADDITION.

1. Money.

				marked.
Note,	4 Farthings	make one	Penny	*d.*
	12 Pence		Shilling	*s.*
	20 Shillings		Pound	£.

Farthings

same thing, by 6 and 5 continually: Now, this is the same as the Rule; for, instead of finding the remainders, separately, they are reduced from the bottom, upwards, step by step, to one another, and the remaining units, of the same Class, taken as they occur.

Farthings.

4 = 1 Penny.
48 = 12 = 1 Shilling.
960 = 240 = 20 = 1 Pound.

Pence Tables.

d.		s.	d.	d.		s.	d.	s.		d.	s.		d.
20	=	1	8	120	=	10		2	=	24	12	=	144
30	=	2	6	130	=	10	10	3	=	36	13	=	156
40	=	3	4	140	=	11	8	4	=	48	14	=	168
50	=	4	2	150	=	12	6	5	=	60	15	=	180
60	=	5		160	=	13	4	6	=	72	16	=	192
70	=	5	10	170	=	14	2	7	=	84	17	=	204
80	=	6	8	180	=	15		8	=	96	18	=	216
90	=	7	6	190	=	15	10	9	=	108	19	=	228
100	=	8	4	200	=	16	8	10	=	120	20	=	240
110	=	9	2					11	=	132			

2. *Troy Weight.* ‡

24 Grains	make one	Pennyweight,	*marked* grs. pwt.
20 Penny-weights		Ounce,	oz.
12 Ounces		Pound	℔ or lb.

Grains 24 = 1 Penny-weight.
480 = 20 = 1 Ounce.
5760 = 240 = 12 = 1 Pound.

3. *Avoirdupois Weight.* *

16 Drams	make one	Ounce,	*marked* dr. oz.
16 Ounces		Pound,	℔
28 Pounds		quarter of a hundred weight,	qr.
4 Quarters		hundred weight or 112 pounds,	Cwt.
20 Hundred wt.		Ton.	T.

Drams

‡ By this weight are weighed Gold, Silver, Jewels, Electuaries, and all Liquors.

An Ounce of Gold is divided into 24 parts, called carats, and an Ounce of Silver, into 20 parts, called pennyweights; therefore, to distinguish fineness of metals, such Gold as will abide the fire without loss, is accounted 24 carats fine: If it lose 2 carats in trial, it is called 22 carats fine, &c.

A pound of Silver, which loses nothing in trial, is 12 ounces fine; but, if it lose 3pwts. it is 11oz. 17pwts. fine, &c.

Alloy is some base metal with which Gold or Silver is mixed, to abate its fineness. Twenty-two carats of Gold, and 2 carats of copper are esteemed the true Standard for gold coin in England, the alloy being one eleventh of the fine gold: and 11oz. 2pwts. of fine Silver, melted with 18pwts. of copper, make the true Standard for Silver coin.

Note. 175 Troy Ounces are precisely equal to 192 Avoirdupois Ounces, and 175 Troy pounds are equal to 144 Avoirdupoise. 1 lb. Troy = 5760 grains, and 1 lb. Avoirdupoise = 7000 grains.

* By Avoirdupois are weighed all coarse and drossy goods, grocery and chandlery wares; bread, and all metals, except Gold and Silver.

A Barrel of Pork weighs 220lb.—a Barrel of Beef, 220lb.—a Quintal of Fish, 1 Cwt. avoird.—12 particular things make 1 dozen; 12 doz. 1 gross, and 144 dozen 1 great gross. —20 particular things make 1 score.

A

Drams					
16 =	1 Ounce.				
256 =	16 =	1 pound.			
7168 =	448 =	28 =	1 quarter.		
28672 =	1792 =	112 =	4 =	1 hund. weight.	
573440 =	35840 =	2240 =	80 =	20 =	1 Ton.

4. *Apothecaries Weight.* §

20 Grains	make 1	Scruple.	*marked* gr. ℈
3 Scruples	make 1	Dram.	ʒ
8 Drams	make 1	Ounce.	℥
12 Ounces	make 1	Pound.	℔

20 Grains	1 Scruple.			
60 =	3 =	1 Dram.		
480 =	24 =	8 =	1 Ounce.	
5760 =	288 =	96 =	12 =	1 pound.

5. *Cloth Measure.* ‡

2 Inches, and one fifth	make 1	Nail	*marked* in. na.
4 Nails, or 9 Inches	make 1	Quarter of a yard,	qr.
4 Quarters of a yard, or 36 Inches	make 1	Yard	yd.
3 Quarters of a yard, or 27 Inches	make 1	Ell-Flemish	E. Fl.
5 Quarters of a yard, or 45 Inches	make 1	Ell-English	E. E.
6 Quarters of a yard, or 54 Inches	make 1	Ell-French	E. Fr.
4 Quarters, 1 Inch & one fifth, or 37 Inches and one fifth	make 1	Ell-Scotch	E. Sc.
3 Quarters and two thirds	make 1	Spanish Var.	

Nails	4 =	1 Quarter.	
	16 =	4 =	1 Yard.
	12 =	3 =	1 Flemish Ell.
	20 =	5 =	1 English Ell.
	24 =	6 =	1 French Ell.

6.

			lb.		lb.
A Firkin of	Foreign	Butter	56	A Stone of Iron Shot, or horseman's weight.	14
——	——	Soap	94	—— Butcher's Meat.	8
A Barrel of	——	Anchovies	30	A Gallon of Train Oil	$7\frac{1}{2}$
——	——	Soap	256	A Tod is — —	28
——	——	Raisins	112	A Weigh — —	182
A Puncheon of	——	Prunes	1120	A Sack — —	364
A Fother of	——	Lead	$19\frac{1}{2}$ *Cwt.*	A Last — —	4368

§ All the weights now used by Apothecaries, above grains, are Avoirdupois.

The Apothecaries' pound and ounce, and the pound and ounce Troy are the same, only differently divided and subdivided.

‡ All Scotch and Irish linens are bought by the English or American yard, which is the same, and all Dutch linens by the Ell-Flemish; but are all sold in America by the American yard; though the Dutch linens are sold in England by the Ell-English, and the Scotch and Irish linens, as in America.

The Scotch allow one English yard in every score yards.

6. *Long Measure.* ¶

3 Barley-corns	make 1	Inch	*marked* bar. in.
12 Inches		Foot	feet.
3 Feet		Yard	yd.
$5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ feet		Rod, Perch, or Pole	pol.
40 Poles		Furlong	fur.
8 Furlongs		Mile	mile.
$69\frac{1}{2}$ Statute miles, *nearly*,		Degree of a great Circle.	deg.
360 Degrees		A great Circle of the Earth.	

Or in Measuring Distances,

$7\frac{92}{100}$ Inches	make	1 Link.
25 Links		1 Pole.
100 Links		1 Chain.
10 Chains		1 Furlong.
8 Furlongs		1 Mile.

Barley-corns. 3 =	1 Inch.					
36 =	12 =	1 Foot.				
108 =	36 =	3 =	1 Yard.			
594 =	198 =	$16\frac{1}{2}$ =	$5\frac{1}{2}$ =	1 Pole.		
23760 =	7960 =	660 =	220 =	40 =	1 Furlong.	
190080 =	63360 =	5280 =	1760 =	320 =	8 =	1 Mile.

Inches.	Links.				
$7\frac{92}{100}$ =	1 Poles or Perch				
198 =	25 =	1 Chain.			
792 =	100 =	4 =	1 Furlong.		
7920 =	1000 =	40 =	10 =	1 Mile.	
63360 =	8000 =	320 =	80 =	8 =	1

7. *Time.* ‡

60 Seconds	make one	Minute,	*marked* s. m.
60 Minutes		Hour	h.
24 Hours		Day	d.
7 Days		Week	w.
4 Weeks		Month	mo.
13 Months, 1 day & 6 hours		Julian year	yr.

Seconds

¶ The use of long measure is to measure the distance of places, or any other thing, where length is considered, without regard to breadth.

Note. 60 Geometrical miles make a Degree.—4 Inches a Hand.—5 Feet a geometrical pace. 6 Points make 1 Line, 12 Lines an Inch, 12 Inches a Foot, and 6 Feet one French Toise, or Fathom, equal to 6 Feet 4 Inches 8,812875 Lines, English measure. 1 English foot equal to 11 Inches, 3,1154 Lines, French.——66 Feet, or 4 Poles make a Gunter's Chain.—3 Miles make a League.

‡ By the Calendar, the year is divided in the following manner.

Thirty Days hath September, April, June and November;
February, twenty-eight alone, and all the rest have thirty-one.

When you can divide the year of our Lord by 4, without any remainder, it is then Bissextile, or Leap-Year, in which February has 29 days.

Seconds 60 =	1 Minute.				
3600 =	60 =	1 Hour.			
86400 =	1440 =	24 =	1 Day.		
604800 =	10080 =	168 =	7 =	1 Week.	
2419200 =	40320 =	672 =	28 =	4 =	1 Month.

			d. h.	w. d. h.	
31557600 =	525960 =	8766 =	365 6 =	52 1 6 =	1 Julian year.‖

			d. h. m. s.	
31558154 =	525969 =	8766 =	365 6 9 14 =	1 Periodical year. ¶
31556937 =	525948 =	8765 =	365 5 48 57 =	1 Tropical year.†

8. *Motion*.

60 Seconds	make 1	Prime minute	marked ″ ′
60 Minutes		Degree	°
30 Degrees		1 Sign	s
12 Signs, or 360 degrees		The whole great circle of the Zodiac.‡	

Seconds 60 =	1 Minute.			
3600 =	60 =	1 Degree.		
108000 =	1800 =	30 =	1 Sign.	
1296000 =	21600 =	360 =	12 =	Zodiac.

9. *Land, or Square Measure*.

144 Inches	make 1	Square foot.
9 Feet		—— Yard.
30¼ Yards, or 272¼ Feet		—— Pole.
40 Poles		—— Rood.
4 Roods, or 160 Rods, or 4840 yards		—— Acre.
640 Acres		—— Mile.

Inches 144 =	1 Foot.					
1296 =	9 =	1 Yard.				
39204 =	272¼ =	30¼ =	1 Pole.			
568160 =	10890 =	1210 =	40 =	1 Rood.		
6272640 =	43560 =	4840 =	160 =	4 =	1 Acre.	
4014489600 =	27878400 =	3097600 =	102400 =	2560 =	640 =	1 Mile.

10.

‖ The civil Solar year of 365 days being short of the true by 5h. 48m. 57s. occasioned the beginning of the year to run forwards through the seasons nearly one day in four years, on this account Julius Cæsar ordained that one day should be added to February, every fourth year, by causing the 24th day to be reckoned twice; and because this 24th day was the sixth (sextilis) before the kalends of March, there were, in this year, two of these sextiles, which gave the name of Bissextile to this year, which, being thus corrected, was, from thence, called the Julian year.

¶ A just and equal measure of the year is called the periodical year, as being the time of the Earth's period about the Sun; in departing from any fixed point in the heavens, and returning to the same again.

† The several points of the Ecliptic having a retrograde, or backward motion, the Equinox will, as it were, meet the Sun; by which mean the Sun will arrive at the Equinox, or first point of Aries, before his revolution is completed, and this space of time is called the tropical year.

‡ The Zodiac is a great Circle of the Sphere, containing the 12 Signs, through which the Sun passes.

10. *Solid Measure.* ‡

1728 Inches	make 1	Foot.
27 Feet	make 1	Yard.
40 Feet of round Timber, or 50 feet of hewn Timber	make 1	Ton or Load.
128 Solid Feet, i.e. 8 in length, 4 in breadth, and 4 in heighth	make 1	Cord of Wood.

11. *Wine Measure.* †

			marked
2 Pints	make 1	Quart	pts. qts.
4 Quarts	make 1	Gallon	gal.
10 Gallons	make 1	Anchor of Brandy	anc.
18 Gallons	make 1	Runlet	run.
31½ Gallons	make 1	Half an hogshead	½hhd.
42 Gallons	make 1	Tierce	tier.
63 Gallons	make 1	Hogshead	hhd.
2 Hogsheads	make 1	Pipe or Butt	P. or B.
2 Pipes	make 1	Tun	Tun.

Cubic Inches.

$28\frac{7}{8}$ = 1 Pint.
$57\frac{3}{4}$ = 2 = 1 Quart.
231 = 8 = 4 = 1 Gallon.
9702 = 336 = 168 = 42 = 1 Tierce.
14553 = 504 = 252 = 63 = $1\frac{1}{2}$ = Hogshead.
19404 = 672 = 336 = 84 = 2 = $1\frac{1}{3}$ = 1 Puncheon.
29106 = 1008 = 504 = 126 = 3 = 2 = $1\frac{1}{2}$ = 1 Pipe.
58212 = 2016 = 1008 = 252 = 6 = 4 = 3 = 2 = 1 Tun.

12. *Ale or Beer Measure.* ¶

			marked
2 Pints	make 1	Quart	pts. qts.
4 Quarts	make 1	Gallon	gal.
8 Gallons	make 1	Firkin of Ale in London	A. fir.
8½ Gallons	make 1	Firkin of Ale or Beer.	
9 Gallons	make 1	Firkin of Beer in London	B. fir.
2 Firkins	make 1	Kilderkin	Kil.
2 Kilderkins	make 1	Barrel	Bar.
1½ Barrel, or 54 Gallons	make 1	Hogshead of Beer	hhd.
2 Barrels	make 1	Puncheon	pun.
3 Barrels, or 2 Hogsheads	make 1	Butt	Butt.

Cubic

‡ By Solid measure are measured all things that have length, breadth and depth.

† All Brandies, Spirits, Perry, Cyder, Mead, Vinegar, Honey, and Oil are measured by Wine-measure: Honey is, commonly, sold by the pound avoirdupois.

¶ Milk is sold by the Beer-quart.

A Barrel of Mackarel, and other barrelled fish, by an act of this Commonwealth, is to contain not less than 30 Gallons.

In England, a Barrel of Salmon or Eels is 42 Gallons, and a Barrel of Herrings 32 Gallons. The Gallon appointed to be used for measuring all kinds of Liquids in *Ireland*, is *two hundred and seventeen* Cubic Inches, and *sixth tenths*.

BEER.

Cubic Inches.

$35\frac{1}{4}$ =	1 Pint.								
$70\frac{1}{2}$ =	2 =	1 Quart.							
282 =	8 =	4 =	1 Gallon.						
2538 =	72 =	36 =	9	1 Firkin.					
5076 =	144 =	72 =	18 =	2 =	1 Kilderkin.				
10152 =	218 =	144 =	36 =	4 =	2 =	1 Barrel.			
15228 =	432 =	216 =	54 =	6 =	3 =	$1\frac{1}{2}$ =	1 Hogshead.		
20304 =	576 =	288 =	72 =	8 =	4 =	2 =		1 Puncheon.	
30456 =	864 =	432 =	108 =	12 =	6 =	3 =		2 =	1 Butt.

ALE.

Cubic Inches.

$35\frac{1}{4}$ =	1 Pint.						
$70\frac{1}{2}$ =	2 =	1 Quart.					
282 =	8 =	4 =	1 Gallon.				
2256 =	64 =	32 =	8 =	1 Firkin.			
4512 =	128 =	64 =	16 =	2 =	1 Kilderkin.		
9024 =	256 =	128 =	32 =	4 =	2 =	1 Barrel.	
13536 =	384 =	192 =	48 =	6 =	3 =	$1\frac{1}{2}$ =	1 Hogshead.

13. *Dry Measure.* †

2 Pints	make 1	Quart	*marked* pts. qts.
2 Quarts		Pottle	pot.
2 Pottles		Gallon	gal.
2 Gallons		Peck	pk.
4 Pecks		Bushel	bu.
2 Bushels		Strike	str.
2 Strikes		Coom	co.
2 Cooms		Quarter	qr.
4 Quarters		Chaldron	ch.
$4\frac{1}{2}$ Quarters		Chaldron in London.	
5 Quarters		Wey	wey.
2 Weys		Last	Last.

Cubic Inches.

$268\frac{4}{5}$ =	1 Gallon.							
$537\frac{3}{5}$ =	2 =	1 Peck.						
$2150\frac{2}{5}$ =	8 =	4 =	1 Bushel.					
$4300\frac{4}{5}$ =	16 =	8 =	2 =	1 Strike.				
$8601\frac{1}{3}$ =	32 =	16 =	4 =	2 =	1 Coom.			
$17203\frac{1}{3}$ =	64 =	32 =	8 =	4 =	2 =	1 Quarter.		
86106 =	320 =	160 =	40 =	20 =	10 =	5 =	1 Wey.	
172032 =	640 =	320 =	80 =	40 =	20 =	10 =	2 =	1 Last.

COM-

† THIS measure is applied to all dry goods, as Corn, Seed, Fruit, Roots, Salt, Sand, Oysters and Coals.

A Winchester Bushel is $18\frac{1}{2}$ Inches diameter, and 8 Inches deep.

COMPOUND ADDITION

Is the adding of ſeveral numbers together, having different denominations, as Pounds, Shillings, Pence, &c. Tons, Hundreds, Quarters, &c.

Rule.¶

1. Place the numbers ſo that thoſe of the ſame denomination may ſtand directly under each other.

2. Add the firſt Column or denomination together as in whole numbers; then divide the ſum by as many of the ſame denomination as make one of the next greater, ſetting down the remainder under the column, added, and carry the quotient to the next ſuperior denomination, continuing the ſame to the laſt, which add as in ſimple addition.

1. Money.

Examples.

1. £.	s.	d.		2. £.	s.	d.	qr.		3. £.	s.	d.	qr.		4. £.	s.	d.	qr.
9	16	10		47	17	6	2		847	11	11	3		915	10	10	2
7	10	9		3	9	10	3		491	19	6	1		64	8	9	1
	18	6		75	13	9	1		59	6	10	-		5	16	11	3
5	11	11		4	11	11	-		747	16	1	2		419	2	10	2
6	—	8		—	16	8	2		849	12	11	3		491	19	11	3
5	9	10		17	6	2	1		741	17	8	2		762	17	6	1

5. £.	s.	d.	qr.		6. £.	s.	d.	qr.		7. £.	s.	d.	qr.		8. £.	s.	d.	qr.
479	11	11	2		764	13	10	2		7	17	10	3		584	19	10	3
64	17	8	3		43	9	8	1		60	6	8	-		765	14	8	1
912	16	10	-		59	17	11	2		7	15	11	2		91	17	10	2
497	5	4	2		817	16	9	3		18	19	9	3		18	19	6	3
69	10	11	3		762	19	10	1		91	16	8	2		847	13	8	2
917	6	9	2		419	17	6	2		918	17	10	3		918	17	11	-

French Money.

Note. 12 Deniers, or pence	make 1	Sol or Shilling.
20 Sols, or Shillings		Livre, or Pound.
3 Livres, or pounds		Crown of exchange.
6 Livres, or pounds		Real Crown, or ecu d'argent.

¶ The Reaſon of this Rule is evident from what has been ſaid in Simple Addition: For, in addition of money, as 1, in the pence, is equal to 4 in the farthings; 1, in the ſhillings, to 12 in the pence, and 1, in the pounds, to 20 in the ſhillings; therefore, carrying as directed, is the arranging the money, ariſing from each column properly in the ſcale of denominations; and this reaſoning will hold good in the addition of compound numbers, of any denomination whatever.

Cr. of exc.	*liv.*	*sols.*	*den.*
976	2	17	10
379	1	12	6
491	1	—	11
592	2	15	8

Ecu d'ar.	*liv.*	*sols.*	*den.*
567	5	13	11
389	1	19	6
548	4	17	10
632	3	11	9

DUTCH MONEY.

Note. 8 Phennings, 2 Groats, or 16 Phennings, 20 Stivers make 1 Groat. Stiver. Guilder or Florin.

8 Phennings	make 1	Groat.
2 Groats, or 16 Phennings	make 1	Stiver.
20 Stivers	make 1	Guilder or Florin.

ALSO.

12 Groats, or 6 Stivers	make 1	Schilling.
20 Schillings or 6 Guilders	make 1	Pound.

Guild.	*stiv.*	*gr.*	*ph.*
197	17	1	7
348	12	0	6
491	13	1	3
749	19	1	4

Guild.	*stiv.*	*ph.*
549	19	14
317	16	12
859	13	8
467	10	15

£.	*sch.*	*grs.*
357	18	11
508	12	6
497	13	10
618	17	8

2. TROY WEIGHT.

1.

lb.	*oz.*	*pwts.*	*gr.*
767	10	17	22
39	6	9	17
417	11	16	18
935	9	17	19
478	10	17	22
387	9	16	15

2.

lb.	*oz.*	*pwts.*	*gr.*
649	11	19	20
32	9	6	5
840	10	11	19
473	9	17	23
764	11	8	9
165	6	10	19

3.

lb.	*oz.*	*pwts.*	*gr.*
859	9	15	20
437	10	17	22
641	11	6	—
738	9	12	18
49	—	16	17
584	10	—	9

3. AVOIRDUPOIS WEIGHT.

1.

lb.	*oz.*	*dr.*
19	13	12
21	9	6
4	15	15
22	10	5
18	13	12
6	11	10

2.

Cwt.	*qrs.*	*lb.*
17	3	19
18	1	27
9	2	9
14	3	16
12	-	6
15	2	—

3.

T.	*Cwt.*	*qrs.*	*lb.*
59	13	2	17
6	17	1	21
45	11	3	25
57	16	2	19
75	17	3	17
6	19	-	26

4.

T.	*Cwt.*	*qrs.*	*lb.*	*oz.*	*dr.*
91	17	2	25	13	15
19	9	-	17	10	12
14	13	2	00	9	11
47	11	3	19	14	—
69	—	1	—	—	12
77	19	3	27	15	11

4. APOTHECARIES

4. Apothecaries' Weight.

1.			2.				3.					4.				
ʒ	℈	gr.	℥	ʒ	℈	gr.	℔	℥	ʒ	℈	gr.	℔	℥	ʒ	℈	gr.
9	1	17	10	7	2	19	12	11	6	1	15	5	9	3	2	13
3	2	19	6	3	-	12	4	9	1	-	12	4	8	6	-	19
6	1	17	7	6	1	17	91	10	7	2	16	9	10	5	2	12
4	-	6	9	5	2	12	4	8	1	2	19	6	5	6	1	17
5	2	12	6	1	-	16	6	—	-	1	10	8	9	4	-	—
8	1	10	9	3	2	19	4	9	2	1	6	7	1	-	1	17

5. Cloth Measure.

1.			2.			3.			4.			5.		
Yd.	*qr.*	*n.*	*E.E.*	*qr.*	*n.*	*E.Fl.*	*qr.*	*n.*	*E.Fr.*	*qr.*	*n.*	*Yds.*	*qr.*	*n.*
76	2	3	91	3	2	75	2	1	49	3	3	914	2	3
3	3	1	49	4	3	7	1	3	19	5	2	49	2	1
42	3	3	6	2	3	84	-	2	24	2	1	561	3	-
57	2	2	84	4	1	76	2	3	67	4	3	84	-	2
16	3	3	7	-	-	48	2	2	48	2	2	549	3	1
49	2	2	61	2	1	9	2	3	6	3	3	617	1	3

6. Long Measure.

1.			2.			3.			4.			5.						
Ft.	*in.*	*bar.*	*Yd.*	*ft.*	*in.*	*Pol.*	*ft.*	*in.*	*Mil.*	*fur.*	*pol.*	*Deg.*	*mi.*	*fur.*	*pol.*	*ft.*	*in.*	*br.*
9	11	2	7	2	11	12	11	10	9	7	36	759	50	6	29	15	10	2
6	9	1	4	1	6	9	16	9	7	3	19	317	39	1	36	11	6	1
7	0	2	6	-	10	8	22	11	4	1	24	497	63	7	24	9	8	1
8	10	-	7	2	9	7	15	6	6	5	12	562	17	-	11	13	11	-
9	6	2	8	1	10	4	14	9	4	6	9	64	48	5	17	9	4	2
7	10	2	9	2	11	5	11	11	5	1	10	764	52	4	19	15	11	1

7. Time.

1.					2.				3.			4.						
W.	*d.*	*h.*	*m.*	*s.*	*Mo.*	*d.*	*h.*	*m.*	*Y.*	*mo.*	*d.*	*Y.*	*mo.*	*w.*	*d.*	*h.*	*m.*	*s.*
3	6	22	57	42	5	24	19	45	19	10	19	57	11	3	6	23	29	55
1	5	19	31	28	4	27	21	35	7	9	27	4	8	1	1	19	45	38
2	3	17	9	15	9	18	00	12	4	8	16	29	9	2	3	17	18	19
3	-	9	17	58	4	19	23	19	1	11	14	46	10	2	5	11	50	13
1	1	16	19	10	8	11	12	13	17	6	9	19	9	2	1	16	18	17
2	2	20	53	48	9	10	8	29	12	5	20	45	9	3	5	18	17	59

8.

8. Motion.

1.			2.			3.			
17°	55′	48″	25°	49′	51″	9s	29°	35′	53″
2	37	51	4	21	36	10	00	18	31
28	19	45	19	47	18	4	17	13	42
19	19	37	25	25	39	6	19	50	—

9. Land or Square Measure.

1.			2.			3.				
Pol.	*feet.*	*in.*	*Yds.*	*ft.*	*in.*	*Acres.*	*rood.*	*pol.*	*ft.*	*in.*
36	179	137	28	7	119	756	3	37	245	228
19	248	119	9	3	75	29	1	28	93	25
12	96	75	29	6	120	416	2	31	128	119
18	110	122	4	8	12	37	1	19	218	20
9	269	24	9	1	119	61	-	—	92	103
25	221	143	8	3	43	191	1	25	129	136

10. Solid Measure.

1.			2.			3.		
Ton.	*ft.*	*in.*	*Yds.*	*ft.*	*in.*	*Cord.*	*ft.*	*in.*
29	36	1229	75	22	1412	37	119	1015
12	19	64	9	26	195	9	110	159
8	11	917	3	19	1091	48	127	1071
19	8	1001	28	15	1110	8	111	956
5	—	523	49	24	218	21	9	27
17	39	1119	18	17	1225	9	28	1091

11. Wine Measure.

1.				2.				3.			
Tierce.	*gal.*	*qts.*	*pts.*	*Hhd.*	*gal.*	*qts.*	*pts.*	*Ton.*	*hhd.*	*gal.*	*qts.*
37	39	3	1	51	53	1	1	37	2	37	2
9	17	2	1	27	39	3	-	19	1	59	1
4	28	-	-	9	18	-	1	28	2	—	-
32	19	1	1	—	9	2	1	19	-	47	1
9	—	3	1	16	24	1	1	37	1	17	3
12	40	1	1	5	—	3	-	14	2	48	2

12. Ale *and* Beer Measure.

1.

A.B.	*fir.*	*gal.*
49	3	7
26	2	5
9	-	4
17	3	-
27	1	6
19	3	7

2.

B.B.	*fir.*	*gal.*
29	1	8
19	3	5
16	-	3
9	1	8
14	2	-
17	1	5

3.

Hhd.	*gal.*	*qts.*
379	53	3
19	—	1
121	37	2
467	19	1
591	16	-
75	—	2

13. Dry Measure.

1.

Qrs.	*bu.*	*p.*	*qt.*
64	7	3	7
9	4	1	5
19	6	2	1
4	-	2	-
17	3	-	6
9	5	3	4

2.

Bus.	*p.*	*qt.*	*pt.*
37	2	5	1
19	3	7	-
16	2	-	1
5	1	6	1
9	-	3	-
19	3	-	1

3.

Ch.	*bu.*	*p.*	*qts.*
37	27	3	5
6	29	1	7
15	30	-	-
4	11	3	-
5	—	1	-
2	—	2	1

COMPOUND SUBTRACTION

Teaches to find the difference, inequality, or excess between any two sums of diverse denominations.

Rule. ¶

Place those numbers under each other, which are of the same denomination, the less being below the greater; begin with the least denomination, and, if it exceed the figure over it, borrow as many units as make one of the next greater; subtract it therefrom; and to the difference add the upper figure, remembering, always, to add one to the next superior denomination, for that which you borrowed.

1. Money.

1.

	£.	s.	d.	qr.
Borrowed	349	15	6	1
Paid	195	11	8	1
Remains to pay	154	3	10	0
Proof.				

2.

	£.	s.	d.	qr.
Lent	791	9	8	1
Received	197	16	4	2
Due to me				

3.

¶ The Reason of this Rule will readily appear, from what was said in Simple Subtraction; for, the borrowing depends upon the same principle, and is only different, as the numbers to be subtracted are of different denominations.

	3. £.	s.	d.	qr.	4. £.	s.	d.	qr.	5. £.	s.	d.	qr.
From	439	9	10	1	843	12	1	3	569	7	5	2
Take	196	-	10	3	746	15	-	2	508	16	4	-
Rem.												

6.

	£.	s.	d.	qr.
Borrowed	19372	12	6	
Paid at sundry times.	293	16	8	
	74	9	7	2
	9413	11	-	1
	1994	—	10	3
	3914	19	—	-
	1064	17	9	1
Paid in all				
Remains to be paid				

7.

	£.	s.	d.	qr.
Lent	27109	5	8	3
Received at several times.	5196	15	10	
	384	17	6	2
	4187	18	11	1
	1649	16	8	-
	917	9	10	3
	3196	—	2	1
Rec. in all				
Remains due				

2. Troy Weight.

	1. lb.	oz.	pwt.	gr.	2. lb.	oz.	pwt.	gr.	3. lb.	oz.	pwt.	gr.
Bought	749	5	13	16	379	8	12	10	543	3	9	13
Sold	96	9	19	13	148	4	16	19	179	1	15	18
Rem.												

3. Avoirdupois Weight.

	1. lb.	oz.	dr.	2. C.	qr.	lb.	3. T.	cwt.	qr.	lb.	4. T.	cwt.	qr.	lb.	oz.	dr.
Bought	7	9	12	8	2	13	5	13	1	12	9	11	3	17	5	12
Sold	3	12	19	4	1	15	1	12	2	17	3	12	1	19	10	9
Rem.																

4. Apothecaries' Weight.

1. ℔	℥	ʒ	℈	gr.	2. ℔	℥	ʒ	℈	gr.	3. ℔	℥	ʒ	℈	gr.
71	9	3	1	13	65	10	6	2	10	84	1	1	1	1
37	8	4	1	16	31	8	4	2	9	65	9	3	1	17

5.

5. Cloth Measure.

1.

Yds.	qr.	n.
35	1	2
19	1	3

2.

E. E.	qr.	n.
467	3	1
291	3	2

3.

E. Fl.	qr.	n.
765	1	3
149	2	1

4.

E. Fr.	qr.	n.
549	4	2
197	4	3

6. Long Measure.

1.

Yds.	ft.	in.
28	2	10
17	2	11

2.

Pol.	ft.	in.
21	11	9
9	13	8

3.

Mil.	fur.	pol.
76	3	11
27	3	21

4.

Deg.	m.	fur.	p.	yds.	ft.	in.	bar.
38	41	3	29	2	1	7	2
19	35	5	31	3	1	9	1

7. Time.

1.

Mo.	d.	h.	m.	s.
6	17	13	27	19
1	21	16	41	35

2.

Mo.	w.	d.	h.
9	2	5	15
4	3	5	15

3.

Y.	mo.	d.
7	3	13
4	2	19

4.

Y.	m.	w.	d.	h.	m.	s.
48	9	2	5	19	27	31
19	9	3	4	20	19	49

8. Motion.

1.

79°	21′	31″
41	41	52

2.

6s	11°	12′	48″
3	8	39	29

3.

4s	19°	41′	22″
1	22	19	45

9. Land, or Square Measure.

1.

A.	R.	Pol.
29	1	10
24	1	25

2.

A.	R.	Pol.
29	2	17
17	1	36

3.

A.	R.	Pol.	ft.	in.
56	3	19	27	110
29	-	21	210	129

10. Solid Measure.

1.

Tons.	ft.	in.
49	19	1100
38	36	1296

2.

Yds.	ft.	in.
79	11	017
17	25	1095

3.

Cords.	ft.	in.
349	97	1250
192	127	1349

11. Wine Measure.

1. Tun.	gal.	qt.	pt.	2. Tier.	gal.	qt.	3. Hhd.	gal.	qt.	4. Tun.	hhd.	gal.
79	1	2	1	19	17	1	375	41	2	532	1	19
38	1	3	1	12	29	2	197	36	3	197	1	47

12. Ale, and Beer Measure.

1. A.B.	fir.	gal.	qt.	2. B.B.	fir.	gal.	qts.	pts.	3. Hhds.	gal.	qts.
39	1	2	1	21	3	5	2	-	769	17	1
24	3	6	2	19	1	7	2	1	391	42	3

13. Dry Measure.

1. Qu.	bu.	pk.	qt.	2. Bu.	pk.	qts.	pts.	3. Chal.	bu.	pk.	qts.
56	2	2	1	91	1	3	2	39	12	2	1
39	3	1	2	29	2	1	1	24	25	3	2

PROBLEMS

Resulting from a Comparison *of the preceding Rules.*

PROB. 1. Having the sum of two numbers, and one of them given, to find the other.

Rule. Subtract the given number from the given sum, and the remainder will be the number required.

LET 288 be the sum of two numbers; one of which is 115, the other is required?

From 288 the Sum,
Take 115 the given number.
Remains 173 the other.

PROB. 2. Having the greater of two numbers, and the difference between that and the less given, to find the less.

Rule. Subtract the one from the other.

LET the greater number be 325 and the difference between that and the other, 198: What is the other?

From 325 the greater.
Take 198 the difference.
Rem. 127 the less.

PROB. 3. Having the least of two numbers given, and the difference between that and a greater, to find the greater.

Rule. Add them together.

Given { 127 the less number.
198 the difference.

Sum 325 the greater number required.

PROB. 4. Having the sum and difference of two numbers given, to find those numbers.

Rule.

Rule. To half the ſum add half the difference, and the ſum is the greater, and from half the ſum take half the difference, and the remainder is the leſs.—Or, from the ſum take the difference, and half the remainder is the leaſt: To the leaſt add the given difference and the ſum is the greateſt.

WHAT are thoſe two numbers, whoſe ſum is 48, and difference 14?

$2)\underline{48}$ $\quad 2)\underline{14}$ $\quad 24+7=31$ the greater, & $24-7=17$ the leſs.

$\frac{1}{2}$ Sum $=24$ $\quad\frac{1}{2}$ diff. $=7$ $\quad$ Or $\overline{48-14}\div2=17$, & $17+14=31$.

PROB. 5. Having the ſum of two numbers and the difference of their ſquares given, to find thoſe numbers.

Rule. Divide the difference of their ſquares by the ſum of the numbers, and the quotient will be their difference: you will then have their ſum and difference to find the numbers by Prob. 4.

WHAT two numbers are thoſe, whoſe ſum is 32, and the difference of whoſe ſquares is 256?

32)256(8 difference.
256

Half ſum	16
Half diff.	4
Greater	20
Leſs	12

PROB. 6. Having the difference of two numbers and the difference of their ſquares given, to find thoſe numbers.

Rule. Divide the difference of their ſquares by the difference of the numbers, and the quotient will be their ſum, then proceed by Prob. 4.

WHAT are thoſe two numbers, whoſe difference is 20, and the difference of whoſe ſquares is 2000?

20)2000(100 Sum. $50+10=60$, the greater, & $50-10=40$, the leſs.

FOR more Queſtions of this nature, ſee Miſcell. Queſ.—Problems 46, 47, 48 and 49; but, as the extraction of the ſquare root is there concerned, they could not be admitted here.

PROB. 7. Having the product of two numbers, and one of them given, to find the other.

Rule. Divide the product by the given number, and the quotient will be the number required.

LET the product of two numbers be 288, and one of them 8; I demand the other? $\quad$ 8)288 *Anſw.* 36

PROB. 8. Having the dividend and quotient, to find the diviſor.

Rule. Divide the dividend by the quotient.

COR. Hence we get another method of proving *Diviſion.*

Given $\begin{cases}288 \text{ the Dividend.}\\ 36 \text{ the Quotient.}\end{cases}$ $\quad$ 36)288(8 Diviſor.
288

Required the Diviſor.

PROB. 9. Having the Diviſor and Quotient given, to find the Dividend.

Rule. Multiply them together.

Given { 8 the Divisor. 36 the Quotient. }

$$\begin{array}{r} 36 \\ 8 \\ \hline 288 \end{array}$$ the Dividend.

Required the Dividend

By a due consideration and application of these Problems only, many questions (of which kind are some of the following) may be resolved in a short and elegant manner, although some of them are generally supposed to belong to higher rules.

Application *of the preceding Rules.*

1. The least of two numbers is 19418, and the difference between them is 2384; What is the greater, and sum of both?

19418+2384=21802 greater, and 19418+21802=41220 sum.

2. Suppose a man born in the year 1743; when will he be 57 years of age? 1743+57= 1800 *Ans.*

3. What number is that, which being added to 19418, will make 21802? 21802—19418=2384 *Ans.*

4. General Washington was born in 1732; what is his age in 1787? 1787—1732= 55 *Ans.*

5. America was discovered by Columbus in 1492, and its Independence declared in 1776; How many years have elapsed between those two Æras? 1776—1492= 284 *Ans.*

6. The Massacre at Boston, by the British Troops, happened, March 5th, 1770, and the Battle at Lexington, April 19th, 1775; How long between?

April 19th, 1775—March 5th, 1770= 5y. 1m. 14d. *Ans.*

7. General Burgoyne and his Army were captured October 17th, 1777, and Earl Cornwallis and his Army, October 19th, 1781; What space of time between?

October 19th, 1781—October 17th, 1777 = 4 years & 2 days, *Ans.*

8. The war between America and England commenced April 19th, 1775, and a general peace took place January 20th, 1783; How long did the war continue?

January 20th, 1783—April 19th, 1775 = 7y. 9m. 1d. *Ans.*

9. A, B, C & D purchased a quantity of Goods in partnership; A paid £.12 10*s*, a dollar and a crown piece; B, 35/ C 29/10, and D, 79*d*.: What did the Goods cost? *Ans.* £.16 14 1.

10. A man borrowed, at different times, these several sums, viz. £.29 5*s*, £.18 17*s*. 6*d*, £.45 12*s*, £.98, 3 dollars, one crown piece and an half; pray how much was he in debt? *Ans.* £.193 2 6.

11. There are 4 numbers; the first 317, the second 912, the third 1229, and the fourth as much as the other three, abating 97: What is the sum of them all? *Ans.* 4819.

12. Bought a quantity of Goods for £.125 10*s*. paid for truckage 45/ for freight 79/6, for duties 35/10, and my expences were 53/9: what did the goods stand me in? *Ans.* £.136 4*s*. 1*d*.

13. A Gentleman left his son £.1725 more than his daughter, whose

whose fortune was 15 thousand, 15 hundred and 15 pounds: What was the son's portion, and what did the whole estate amount to?

Ans. The son's fortune, £.18240, & the whole estate £.34755.

14. A merchant had 6 debtors, who together owed him £.2917 10*s.* 6*d.* A, B, C, D & E, owed him £.1675 13*s.* 9*d* of it: What was F's debt? *Ans.* 1241 16 9.

15. What is the difference between £.1309 7*s.* 1*d.* and the amount of £.345 13*s.* 4*d.* and £.571 4*s.* 8*d.*? *Ans.* £.392 9*s.* 1*d.*

16. A Merchant, at his first engaging in trade, owed £.937 15*s.* he had in cash £.1755 3*s.* 6*d.* in goods £.459 12*s.* 3*d.* in good debts £.197 16*s.* and he cleared the first year £.249 19*s*, 10*d.* What was the neat balance at the year's end? *Ans.* £.1724 16*s* 7*d.*

17. What sum of money must be divided between 12 men, so as that each may receive £.155? £.155×12=£.1860, *Ans.*

18. What number must I multiply by 9, that the product may be 675? $675\div9=75$ *Ans*,

19. A Privateer of 175 men took a prize which amounted to £.59 per man, beside the owner's half: What was the value of the prize?

$175\times59\times2=$ £.20650 *Ans.*

20. What is the difference between thrice five, and thirty; and thrice thirty-five? $35\times3-\overline{5\times3\times30}=60$ *Ans.*

21. The sum of two numbers is 750; the less 248; What is their difference, product and the square of their difference? ¶

$750-248=502$ the greater number, $502-248=254$ difference, $502\times248=124496$ product, and $254\times254=64516$ square of the difference.

22. What is the difference between six dozen dozen, and half a dozen dozen; and what is their product, and the quotient of the greater by the less? *Ans.* $6\times12\times12-6\times12=792$ diff. $6\times12\times12\times6\times12=62208$ prod. and $6\times12\times12\div6\times12=12$ Quotient.

23. There are two numbers; the greater of them is 25 times 78, and their difference is 9 times 15; their sum and product are required.

Ans. $78\times25=1950$ the greater, $1950-\overline{15\times9}=1815$ the less. $1950+1815=3765$ the sum, and $1950\times1815=3539250$ the product.

24. A Merchant began trade with £.25327—for 6 years together, he cleared £.1253 *per annum*; the next 5 years, he cleared £.1729 *per annum*; but, the last 4 years, had the misfortune to lose £.3019 *per annum*: What was he worth at the 15 years' end?

Ans. £.29414.

25. If a man spends £.192 in a year: What is that per Calendar-month? $192\div12=$ £.16 *Ans.*

26. If the Federal Debt, which is 42 million dollars, be equally divided between the 13 States: What will be the share of each?

Ans. $3230769\frac{3}{13}$ dollars.

27.

¶ A Number is said to be squared, when it is multiplied into itself.

27. If 9000 men march in a column of 750 deep: How many march abreaſt? $9000 \div 750 = 12$ *Ans.*

28. What number, deducted from the 32d part of 3072, will leave the 96th part of the ſame? $\overline{3072 \div 32} - 32 = 64$ *Ans.*

29. What number is that, which, multiplied by 3589, will produce 92050672? $92050672 \div 3589 = 25648$ *Ans.*

30. Suppose the quotient ariſing from the diviſion of two numbers to be 5379, the diviſor 37625; What is the dividend, if the remainder came out 9357?

$37625 \times 5379 + 9357 = 202394232$ *Ans.*

31. There is a certain number, which being divided by 7, the quotient reſulting, multiplied by 3, that product divided by 5, from the quotient 20 being ſubtracted; and 30 added to the remainder, the half ſum ſhall make 35: Can you tell me the number?

$\overline{35 \times 2 - 30 + 20} \times 5 \times 7 \div 3 = 700$ *Ans.*

32. A Sheepfold was robbed three nights ſucceſſively; the firſt night, half the ſheep were ſtolen, and half a ſheep more; the ſecond, half the remainder were loſt, and half a ſheep more; the laſt night they took half what were left and half a ſheep more; by which time they were reduced to 30: How many were there at firſt?

Begin with 30, and, reckoning back from the laſt night to the firſt, you will find that 31 were ſtolen the 3d night, 62 the 2d, and 124 the firſt. *Ans.* 247.

33. Two Boys, A and B, had 850 Cheſnuts between them; but A had 150 more than B: How many had each?

$850 \div 2 = 425$ half ſum, and $150 \div 2 = 75$ half diff. then $425 + 75 = 500$ A's, and $425 - 75 = 350$ B's.

34. A and B played at marbles, having 14 a-piece at the firſt; but after playing ſeveral games, B, having loſt ſome of his, would play no longer, and it was found that the difference of the ſquares of the numbers, which each then had, was 336: Pray, how many did B loſe?

$14 + 14 = 28$ ſum, $336 \div 28 = 12$ diff. $28 \div 2 = 14$ half ſum, and $12 \div 2 = 6$ half diff. then $14 + 6 = 20$ A retired with, and $14 - 6 = 8$ B had left, therefore B loſt $14 - 8 = 6$.

35. Said Harry to Charles, my father gave me 12 more apples than he gave my brother Jack, and the difference of the ſquares of our ſeparate parcels was 288: Now, if you are Arithmetician enough to tell how many he gave us, each, you ſhall have half of mine?

$288 \div 12 = 24$ the whole: $24 \div 2 = 12$ and $12 \div 2 = 6$, then $12 + 6 = 18 =$ Harry's ſhare, and $12 - 6 = 6 =$ Jack's ſhare.

36. What number added to the 27th part of 6615, will make 570? $570 - \overline{6615 \div 27} = 325$ *Ans.*

REDUC.

REDUCTION

Teacheth to bring, or exchange, numbers of one denomination to others of different denominations, retaining the ſame value.

It is of *two* ſorts, viz. Deſcending and Aſcending; the former of which is performed by multiplication, and the latter, by diviſion.

Reduction descending.

Rule.*

Multiply the higheſt denomination, given, by ſo many of the next leſs, as make one of that greater, and thus continue 'till you have brought it down as low as your queſtion requires.

Proof. Change the order of the queſtion, and divide your laſt product by the laſt multiplier, and ſo on.

Examples.

1. In £.27 15*s.* 9*d.* 2*qrs.* how many farthings?

	£.	s.	d.	qr.	
	27	15	9	2	
multiplied by	20	= *Shillings in a pound.*			
	555	= *Shillings.*			
—— *by*	12	= *Pence in a ſhilling.*			
	6669	= *Pence.*			
—— *by*	4	= *farthings in a penny.*			
Anſ.	= 26678	*farthings.*			

Note. In multiplying by 20, I added in the 15*ſ.*—by 12, the 9*d.* and by 4, the 2*qrs.* which muſt always be done in like caſes.

To prove the above queſtion, change the order of it, and it will ſtand thus: In 26678 farthings how many pounds?

4)26678

12)6669 2*qrs.*

2|0)55|5 9*d.*

Anſwer, £.27 15 9 2

2. In £.36 12*s.* 10*d.* 1*qr.* how many farthings? *Anſ.* 35177.
3. In £.95 11*s.* 5*d.* 3*qr.* how many farthings? *Anſ.* 91751.
4. In £.719 9*s.* 11*d.* how many half-pence? *Anſ.* 345358.
5. In 29 Guineas, at 28*ſ.* how many pence? *Anſ.* 9744.

6. In

* The Reaſon of this Rule is exceedingly obvious; for pounds are brought into ſhillings by multiplying them by 20; ſhillings into pence by multiplying them by 12; and pence into farthings by multiplying them by 4; and the contrary, by diviſion: and this will be true in the reduction of numbers conſiſting of any denomination whatever.

6. In 37 Pistoles, at 22*s.* how many shillings, pence and farthings?

Ans. 814*s.* 9768*d.* 39072 *farthings.*

7. In 49 half Johannes, at 48*s.* how many sixpences?

Ans. 4704.

8. In 473 French Crowns, at 6/8, how many three-pences?

Ans. $12613\frac{1}{3}$

9. In 53 Moidores, at 36*s.* how many shillings, pence, and farthings?

Ans. 1908*s.* 22896*d.* 91584*qrs.*

10. In £.29 how many groats, three-pences, pence and farthings?

Ans. 1740 *groats,* 2320 *three-pences,* 6960*d.* 27840*qrs.*

11. Reduce 47 guineas and one fourth of a guinea into shillings, six-pences, groats, three-pences, two-pences, pence and farthings?

Ans. 1323 *shill.* 2646 *six-pences,* 3969 *groats,* 5292 *three-pences,* 7938 *two-pences,* 15876 *pence, and* 63504*qrs.*

Reduction Ascending.

Rule.

Divide the lowest denomination given, by so many of that name, as make one of the next higher, and thus continue till you have brought it into that denomination which your question requires.

Examples.

1. In 547325 farthings how many pence, shillings and pounds?

Farthings in a penny = 4)547325

Pence in a shilling = 12)136831 1 *qr.*

Shillings in a pound = 2|0)1140|2 7*d.*

£.570 2*s.* 7*d.* 1*qr.*

Ans. 136831*d.* 11402*s.* and £.570.

Note. The remainder is always of the same name as the dividend.

2. Bring 35177 farthings into pounds.
3. Bring 91751 farthings into pence, &c.
4. Bring 345358 half pence into pence, shillings and pounds.
5. Reduce 9744 pence to guineas, at 28*s.* per guinea.
6. In 39072 farthings, how many pistoles, at 22*s*?
7. In 4704 sixpences how many half Johannes?
8. In $12613\frac{1}{3}$ three-pences how many French Crowns, at 6/8?
9. In 91584 farthings how many Moidores, at 36*s*?
10. In 27840 farthings how many pence, three-pences, groats, shillings and pounds?
11. In 63504 farthings how many pence, two-pences, three-pences, groats, sixpences, shillings and guineas?

Note. The preceding questions may serve as proofs to those in Reduction descending.

REDUCTION DESCENDING and ASCENDING.

I. MONEY.

1. In £.97 how many pence and English or French Crowns, at 6/8?
Ans 23280*d.* *and* 291 *crowns.*

2. In 947 English crowns, at 6/8, how many shillings, and English Guineas? *Ans.* 6313*s.* 4*d.* *and* 225 *guin.* 13*s.* 4*d.*

3. In 519 English half crowns how many pence and pounds?
Ans. 20760*d.* *and* £.86 10*s.*

4. In 1259 groats how many farthings, pence, shillings & Guineas?
Ans. 20144*qrs* 5036*d.* 419*s.* 8*d.* *and* 14 *Guineas* 27*s.* 8*d.*

5. In 75 Pistoles how many pounds? *Ans.* £.82 10*s.*

6. In 735 French Crowns how many shillings and French Guineas, at 26/8? *Ans.* 4900*s.* *and* 183 *Guin.* 20*s*

7. In 5793 pence how many farthings, pounds and pistoles?
Ans. 23172*qrs.* £.24 2*s.* 9*d.* *and* 21 *Pistoles,* 20/9.

8. In £.99 how many shillings, and half Johannes, at 48/?
Ans. 1980*s.* *and* 41 *half-Joes,* 12*s.*

9. In £.179 how many Guineas? *Ans.* 127 *Guin.* 24*s.*

10. In £.345 how many Moidores? *Ans.* 191 *Moid.* 24*s.*

11. In 59 half joes, 37 moidores, 45 guineas, 63 pistoles, 24 English crowns, & 19 dollars; how many pounds, half-joes, moidores, guineas, pistoles, Eng. crowns, dollars, shillings, pence & farthings?
Ans. £.354 4*s.* 147 *half-joes,* 28*s.* 196 *moidores,* 28*s.* 253 *Guineas,* 322 *pistoles,* 1062 *Eng. crowns,* 4*s.* 1180 *dollars,* 4 *shillings,* 7084 *shill.* 85008*d.* *and* 340032*qrs.*

When it is required to know how many sorts of Coin, of different values, and of equal number, are contained in any number of another kind; reduce the several sorts of coin into the lowest denomination mentioned, and add them together for a divisor; then, reduce the money given, into the same denomination for a dividend, & the quotient, arising from the division, will be the number required.

Note. Observe the same direction in weights and measures.

1. In 275 half-Johannes how many moidores, guineas, pistoles, dollars, shillings and sixpences, of each, the like number?

A Moidore is 36*s.* that is	72 *sixpences.*		275 *half-joes.*
			48 *shill. in a johan.*
A Guinea is 28*s.* that is	56 *ditto.*		2200
			1100
A Pistole is 22*s.* that is	44 *ditto.*		13200 *shillings.*
			2 *sixp. in a shill.*
A Dollar is 6*s.* that is	12 *ditto.*	*Dividend* =	26400 *sixpences.*
One shilling has	2 *do.*		187)26400(141 *of each,* & 33 *sixp. or* 16/6. *over, the answer.*
	1 *do.*		
Divisor =	187 *sixp.*		

2. A Gentleman distributed £.37 10*s*. between 4 poor persons, in the following manner, viz. that as often as the first had 20*s*. the second should have 15*s*. the third, 10*s*. and the fourth, 5*s*. What did each person receive? *Ans. The first man* £.15.

2. TROY WEIGHT.

1. How many grs. in a silver Bowl, that weighs 3 lb. 10*oz*. 12 *pwts*.?

lb.	*oz.*	*pwt.*
3	10	12

12 *Ounces in a pound.*

46 *Ounces.*
20 *Pennyweights in an Ounce.*

932 *Pennyweights.*
24 *Grains in one pwt.*

3728
1864

Proof. 24)22368 *grains, answer.*

2|0)93|2
12)46—12 *pwt.*

lb. 3—10 *oz.*

2. In 487 *ozs*. how many *pwts*. & *grs*? *Ans*. 9740 *pwt*. & 233760 *gr*.

3. In 13 Ingots of Gold, each weighing 9*oz*. 5*pwt*. how many Grains? *Ans*. 57720 *gr*.

4. In 97397 *grs*. how many pounds? *Ans*. 16 lb. 10*oz*. 18*pwt*. 5*gr*.

5. How many Rings, each weighing 5*pwt*. 7*gr*. may be made of 3 lb. 5*oz*. 16*pwt*. 2*gr*. of gold? *Ans*. 158.

3. AVOIRDUPOIS.

	Cwt.	*qrs.*	*lb.*	*oz.*	
1. In	91	3	17	14	how many ounces?

4

367 *quarters.*
28

2943
735

10293 *pounds.*
16

61762
10294

164702 *Ounces.*

Proof.
16)164702

28)10293 14*oz*.

367 17 lb.

Cwt. 91 3*qrs*.

2. In

2. In 12 tons, 15*cwt.* 1*qr.* 19℔. 6*oz.* 12*dr.* how many drams?

Ans. 7323500 *dr.*

3. In 24℔. 11*oz.* 9*dr.* how many drams? *Ans.* 6329*dr.*

4. In 44800 pounds, how many Drams and Tons?

Ans. 11468800 *dr.* and 20 *Tons.*

5. In 28℔. Avoirdupois, how many pounds Troy?

```
              28
            7000 grains in 1℔. Avoird.
grs. in  }
1lb tr.  }=576|0)19600|0(34℔.
                 1728
                 ----
                 2320
                 2304
                 ----
                  160
                   12
                 ----
         576|0)192|0(0 oz.
                 20
                 ----
         576|0)3840|0(6 pwt.
               3456
               ----
               3840
                 24
               ----
               1536
               768
               ----
         576|0)9216|0(16 gr.
               576
               ----
               3456
               3456
               ----
```

6. In 47 ℔. 9 *oz.* 13 *pwt.* 17 *gr.* Troy, how many pounds Avoirdupois?

```
             ℔.  oz. pwt. gr.
             47   9   13   17
             12
             ---
             573
              20
             ----
            11473
               24
            -----
            45899
           22947
           ------
7|000)275|369(39℔.
      21
      --
       65
       63
       --
       2369
         16
       -----
       14214
       2369
       -----
7|000)37|904(5 oz.
      35
      --
       2904
         16
       -----
       17424
       2904
       -----
7|000)46|464(6 4464/7000 dr.
      42
      --
       4464
```

4. Apothecaries

4. Apothecaries Weight.

1. How many grains are there in 37℔. 6℥ ?

℔. ℥.	*Proof.*
37 6	2|0)21600|0
12	
450 *Ounces.*	3)10800
8	
3600 *Drams.*	8)3600
3	
10800 *Scruples.*	12)450
20	
216000 *Grains.*	37℔. 6℥.

2. In 9℔. 8℥. 1ʒ. 2℈. 19*gr.* how many grains ?
Ans. 55799*gr.*

3. In 55799 grains how many pounds, &c.
Ans. 9℔. 8℥. 1ʒ. 2℈. 19*gr*

5. Cloth Measure.

1. In 127 yards how many quarters and nails ?

4	*Proof.*
508 *qrs.*	4)2032
4	4)508
Ans. 2032 *nails.*	127 *Yards.*

2. In 9173 nails how many yards? *Ans.* 573*yds.* 1*qr.* 1*n.*

3. In 75 Ells English how many quarters and nails ?
Ans. 375*qrs.* 1500*n.*

4. In 56 Ells-Flemish how many quarters and nails ?
Ans. 168*qrs.* 672*n.*

5. In 39 Ells-French how many quarters and nails ?
Ans. 234*qrs.* 936*n.*

6. In 7248 nails how many yards, ells-flemish, ells-English and ells-French ?
Ans. 453*yds.* 604 *Ells-flem.* 362 *Ells-Eng.* 2*qrs.* 302 *Ells-French.*

7. In 19 pieces of Cloth, each 15 yards 2 quarters, how many yards, quarters and nails?
Ans. 294*yds.* 2 *qrs.* 1178 *qrs.* & 4712*n.*

6. Long Measure.

1. How many barley-corns will reach from Newbury-port to Boston, it being 43 miles ?

43 *Miles.*
8

344 *Furlongs.*
40

13760 *Rods.*
$5\frac{1}{2}$

68800
6880

75680 *Yards.*
3

227040 *Feet.*
12

2724480 *Inches.*
3

8173440 *Answer.*

Proof.
3)8173440
12)2724480
3)227040
11)75680
6880
2
4,0)1376,0
8)344
43

HERE I divide by 11, and multiply the quotient by 2, because twice $5\frac{1}{2}$ is 11,—or I might first have multiplied by 2, and, then, have divided the Product by 11.

2. How many barley-corns will reach round the globe, it being 360 degrees? *Ans.* 4755801600.

3. How many inches from Newbury-port to London, it being 2700 miles? *Ans.* 171072000.

4. How often will a wheel, of 16 feet and 6 inches circumference, turn round in the distance from Newbury-port to Cambridge, it being 42 miles? *Ans.* 13440 *times.*

5. IN 190080 inches, how many yards and leagues? *Ans.* 5280 *yds.* and 1 *League.*

7. TIME.

1. IN 20 years how many seconds?

d. *h.*
365 6 *in a year.*
24

1466
730

8766 *hours in* 1 *year.*
20

175320 *hours in* 20 *years.*
60

10519200 *minutes in ditto.*
60

631152000 *seconds in ditto.*

Proof.
6|0)63115200|0
6|0)1051920|0
2|0)17532|0
4×6)8766
4)1461
365*d.* 6*h.*

2. SUPPOSE your age to be 15*y.* 19*d.* 11*h.* 37*m.* 45*s.* how many seconds are there in it, allowing 365 days and 6 hours to the year? *Ans.* 475047465.

3. In 31538937 seconds how many solar years? *Ans.* 1 *year.*

4. How many minutes from the first day of January to the 14th day of August, inclusively? *Ans.* 325440.

5. How many days since the commencement of the christian Æra? ———

6. How many minutes since the commencement of the American war, which happened on the 19th day of April 1775? ———

7. How many seconds between the commencement of the war, April 19th, 1775, and the independence of the United States of America, which took place the 4th day of July 1776?

Ans. 38102400.

8. Motion.

1. In 9 signs, 13° $25'$, how many seconds?

9s 13° $25'$		*Proof.*
30		6|0)102030|0
283 *degrees.*		6|0)1700|5
60		3|0)28|3—25
17005 *minutes.*		9s 13° $25'$
60		
1020300 *seconds.*		

9. Land or Square Measure.

1. In 29 acres, 3 roods, 19 Poles how many roods and perches?

29 3 19	4|0)477|9
4	4)119—19*p.*
119 *Roods.*	29*Ac.*—3*r.*
40	
Answer 4779 *Perches.*	

2. In 1997 Poles how many Acres? *Ans.* 12*A.* 1*r.* 37*p.*

3. In 89763 square yards how many acres, &c.?

Ans. 18*A.* 2*r.* 7*p.* 101*ft.* 36*in.*

4. How many square feet, square yards and square poles in a square mile?

Ans. 27878400 *feet*—3097600 *yards, and* 102400 *poles.*

10. Solid Measure.

1. In 15 Tons of hewn Timber how many solid Inches?

15

```
 15 Tons.                    Proof.
 50                                  5|0
 ---                  1728)1296000(75|0
 750 Feet.                 12096     ---
1728                       -----     15 Tons.
-----                       8640
 6000                       8640
 1500                       ----
5250
750
--------
```

Ans. 1296000 *Inches.*

2. In 9 Tons of round timber how many Inches? *Ans.* 622080.

3. In 25 cords of Wood how many Inches? *Ans.* 5529600.

11. Wine Measure.

1. In 9*hhds*, 15*gal.* 3*qts.* of Wine how many quarts?

```
hhds. gal. qts.          Proof.
  9   15   3            4)2331
 63                     ------
 --                  63)582--3qts.
 32                     ---
 55                     9hhds.--15gal.
 --
582 gallons.
  4
-----
```

Ans. 2331 *quarts.*

2. In 12 Pipes of Wine how many Pints? *Ans.* 12096.

3. In 9758 pints of Brandy how many Pipes? *Ans.* 9*p.* 1*hhd.* 22*gal.* 3*qts.*

5. In 1008 Gallons of Cyder how many Tons? *Ans.* 1 *Ton.*

12. Ale *or* Beer Measure.

1. In 29*hhds.* of Beer how many pints?

```
hhds.                    Proof.
  29                   2)12528
  54                    ------
  --                   4)6264
 116                    -----
 145                  54)1566
-----                   -----
1566 Gallons.            29hhds.
   4
-----
6264 Quarts.
   2
-----
```

Ans. 12528 *Pints.*

2. In 47 *bar.* 18 *gal.* of Ale how many pints? *Anf.* 13680.
3. In 36 puncheons of Beer how many Butts? *Anf.* 24.

13. Dry Measure.

1. In 42 Chaldrons of Coals how many Pecks?

```
  42                    Proof.
  32                   4)5376
 ---                 32)1344(42
  84                    128
 126                    ---
 ---                     64
1344 Bushels.            64
   4                     --
 ---
Anf. 5376 Pecks.
```

2. In 75 bushels of Corn how many pints? *Anf.* 4800.
3. In 9376 quarts how many bushels? *Anf.* 293.

VULGAR FRACTIONS.

Fractions, or broken numbers, are expreffions for any affignable parts of an unit, or whole number; and are reprefented by two numbers, placed one above another, with a line drawn between them, thus, $\frac{5}{8}$, $\frac{4}{3}$, &c. fignifying five eighths, four thirds, that is, one and one third, &c.

The figure, above the line, is called the *numerator*, and that below it, the *denominator*.

The denominator (which is the divifor in divifion) fhews how many parts the Integer is divided into; and the numerator (which is the remainder after divifion) fhews how many of thofe parts are meant by the fraction.

Fractions are either proper, improper, fingle, compound, or mixed. Any whole number may be made an improper fraction by drawing a line under it, and putting unity, or 1 for a denominator, as 9 may be expreffed fractionwife, thus $\frac{9}{1}$, and 12 thus $\frac{12}{1}$, &c.

1. A *fingle*, *fimple*, or *proper fraction* is when the numerator is lefs than the denominator, as, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{19}{35}$, &c. and is a *fimple* expreffion for any number of parts of the integer.

2. An improper fraction is when the numerator exceeds the denominator, as, $\frac{5}{3}$, $\frac{8}{5}$, $\frac{12}{4}$, &c.

3. A *compound fraction* is the fraction of a fraction, coupled by the word *of*, thus, $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{8}$, &c. which are read thus, two thirds of three fourths; one half of three fifths of feven eighths.

4. A *mixed number* is compofed of a whole number and a fraction, as $7\frac{3}{5}$, $35\frac{4}{13}$, &c. that is, feven and three fifths, &c.

5. A

5. A fraction is said to be in its least, or lowest terms, when it is expressed by the least numbers possible.

6. The *common measure* of two, or more numbers, is that number which will divide each of them without a remainder: thus, 5 is the common measure of 10, 20 and 30; and the *greatest* number, which will do this, is called the *greatest common measure.*

7. A number, which can be measured by two, or more, numbers, is called their *common multiple:* and, if it be the least number, which can be so measured, it is called the *least common multiple*; thus, 40, 60, 80, 100 are multiples of 4 and 5; but their least common multiple is 20.

8. A *prime number* is that, which can only be measured by itself, or an unit.

9. That number, which is produced by multiplying several numbers together, is called a *composite number.*

10. A *perfect number* is equal to the sum of all its aliquot parts.*

Problem I.†

To find the greatest common measure of two, or more, numbers.

Rule.

1. If there be two numbers only, divide the greater by the less, and this divisor by the remainder, and so on, always dividing the last divisor by the last remainder, till nothing remain, then will the last divisor be the greatest common measure required.

2. When there are more than two numbers, find the greatest common measure of two of them, as before; then, of *that* common measure and one of the other numbers, and so on, through all the numbers, to the last; then will the greatest common measure, last found, be the answer.

3. If 1 happens to be the common measure, the given numbers are prime to each other, and found to be incommensurable, or in their lowest terms.

Examples.

* The Following perfect numbers are all which are, at present, known.

6	8589869056
28	137438691328
496	2305843008139952128
8128	2417851639228158837784576
33550336	9903520314282971830448816128

† This and the following problem will be found very useful in the doctrine of fractions, and several other parts of Arithmetic.

The truth of the rule may be shewn from the first example: for, since 108 measures 216, it also measures $216+108$, or 324.

Again, since 108 measures 216 and 324, it also measures $\overline{5\times324+216}$, or 1836. In the same manner it will be found to measure $\overline{2\times1836+324}$, or 3996, and so on.

It is also the greatest common measure; for, suppose there be a greater; then, since the greater measures 1836 and 3996, it also measures the remainder 324; and since it measures 324 and 1836, it also measures the remainder 216; in the same manner it will be found to measure the remainder 108; that is, the greater measures the less, which is absurd; therefore, 108 is the greatest common measure.

In the same manner, the demonstration may be applied to one or more numbers.

EXAMPLES.

1. WHAT is the greatest common measure of 1836, 3996, and 1044?

```
1836)3996(2
     3672
     ----
      324)1836(5
          1620
          ----
           216)324(1
               216
               ---
Common meas. = 108)216(2
                   216
```

So 108 is the greatest common measure of 3996 and 1836.

```
Hence 108)1044(9
           972
           ---
            72)108(1
                72
                --
Last greatest com. meas. = 36)72(2
                              72
```

THEREFORE, 36 is the answer required.

2. WHAT is the greatest common measure of 1224 and 1080? *Ans.* 72.

3. WHAT is the greatest common measure of 1440, 672 and 3472? *Ans.* 16.

PROBLEM 2. ‡

To find the least common multiple of two, or more, numbers.

RULE.

1. DIVIDE by any number that will divide two, or more, of the given numbers without a remainder, and set the quotients, together with the undivided numbers, in a line beneath.

2. DIVIDE the second line, as before, and so on, till there are no two numbers, that can be divided; then, the continued product of the divisors and quotients will give the multiple required.

EXAMPLES.

1. WHAT is the least common multiple of 6, 10, 16 and 20?

```
*5)6  10  16  20
   -------------
*2)6   2  16   4
   -------------
*2)3   1   8   2
   -------------
  *3   1  *4   1
   -------------
```

$\overset{*}{5}\times\overset{*}{2}\times\overset{*}{2}\times\overset{*}{3}\times\overset{*}{4}=240$ *Ans.*

I survey my given numbers and find that five will divide two of them, *viz.* 10 and 20, which I divide by 5, bringing into a line with the quotients the numbers, which 5 will not measure: Again, I view the numbers in the second line, and find 2 will measure them all, and I get 3, 1, 8, 2 in the third line, and find that two will measure 8 and 2, and in the fourth line get 3, 1, 4, 1, all prime, I then multiply the prime numbers and the divisors continually into each other, for the number sought, and find it to be 240.

2. WHAT

‡ THE reason of this Rule may also be shewn from the first example; thus, it is evident that 6x10x16x20 (=19200) may be divided by 6, 10, 16 and 20, without a remainder; but 20 is a multiple of 5, therefore 6x10x16x4, or 3840, is also divisible by 6, 10, 16 and 20.——Also, 16 is a multiple of 4; therefore, 6x10x4x4=960 is also divisible by 6, 10, 16 and 20.——Also 10 is a multiple of 2; therefore 6x5x4x4=480 is also divisible by 6, 10, 16 and 20.—Also 6 is a multiple of 2; therefore 3x5x4x4=240 is also divisible by 6, 10, 16 and 20; and is evidently the least number that can be so divided.

2. WHAT is the leaſt common multiple of 6 and 8? *Anſ.* 24.

3. WHAT is the leaſt number that 3, 5, 8 and 10 will meaſure? *Anſw.* 120.

4. WHAT is the leaſt number which can be divided by the 9 digits, ſeparately, without a remainder? *Anſw.* 2520.

REDUCTION *of* VULGAR FRACTIONS

Is the bringing of them out of one form into another, in order to prepare them for the operations of Addition, Subtraction, &c.

CASE I.*

To abbreviate, or reduce fractions to their loweſt terms.

RULE.

DIVIDE the terms of the given fraction by any number, which will divide them without a remainder, and the quotients, again, in the ſame manner; and ſo on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its loweſt terms. *Or,*

DIVIDE both the terms of the fraction by their greateſt common meaſure, and the quotients will be the terms of the fraction required.

EXAMPLES.

1. REDUCE $\frac{288}{480}$ to its loweſt terms.

$$8\left\{\frac{288}{480} \overset{(4)}{=} \frac{36}{60} \overset{(3)}{=} \frac{9}{15} = \frac{3}{5}\right. \text{ the anſwer.}$$

* THAT dividing both the terms, that is, both numerator and denominator of the fraction equally by any number whatever, will give another fraction, equal to the former, is evident: and if thoſe diviſions be performed as often as can be done, or the common diviſor be the greateſt poſſible, the terms of the reſulting fraction muſt be the leaſt poſſible.

Note 1. Any number, ending with an even number or cypher, is diviſible by 2.

2. ANY number, ending with 5 or 0, is diviſible by 5.

3. IF the right hand place of any number be 0, the whole is diviſible by 10.

4. IF the two right hand figures of any number be diviſible by 4, the whole is diviſible by 4.

5. IF the three right hand figures of any number be diviſible by 8, the whole is diviſible by 8.

6. IF the ſum of the digits, conſtituting any number, be diviſible by 3 or 9, the whole is diviſible by 3 or 9.

7. IF a number cannot be divided by ſome number leſs than the ſquare root thereof, that number is a *prime*.

8. ALL *prime* numbers, except 2 and 5, have 1, 3, 7, or 9 in the place of units: and all other numbers are *compoſite*.

9. WHEN numbers, with the ſign of Addition or Subtraction between them, are to be divided by any number, each of the numbers muſt be divided: thus, $\frac{6+9+12}{3} = 2+3+4 = 9$.

10. BUT if the numbers have the ſign of Multiplication between them; then only one of them muſt be divided: thus, $\frac{4 \times 6 \times 10}{2 \times 5} = \frac{2 \times 6 \times 10}{1 \times 5} = \frac{2 \times 6 \times 2}{1 \times 1} = \frac{24}{1} = 24.$

Or thus :

288)480(1
288
——
192)288(1
192
——
Com. meas. 96)192(2
192

Therefore 96 *is the greatest common measure.*

and 96 $\left\{ \frac{288}{480} = \frac{3}{5} \right.$ *the same as before.*

2. REDUCE $\frac{96}{544}$ to its lowest terms. *Answ.* $\frac{3}{17}$.
3. REDUCE $\frac{384}{1152}$ to its lowest terms. *Answ.* $\frac{1}{3}$.
4. REDUCE $\frac{57}{456}$ to its lowest terms. *Answ.* $\frac{1}{8}$.
5. REDUCE $\frac{46}{184}$ to its lowest terms. *Answ.* $\frac{1}{4}$.
6. REDUCE $\frac{1429}{2858}$ to its lowest terms. *Answ.* $\frac{1}{2}$.

CASE 2.

To reduce a mixed number to its equivalent improper fraction.

RULE.†

MULTIPLY the whole number by the denominator of the fraction, and add the numerator of the fraction to the product; under which subjoin the denominator, and it will form the fraction required.

EXAMPLES.

1. REDUCE $36\frac{5}{8}$ to its equivalent improper fraction.

36
8+5
——
Answ. $\frac{293}{8}$

I multiply 36 by 8, and adding the numerator 5 to the product, as I multiply, the sum 293 is the numerator of the fraction sought, and 8 the denominator: so that $\frac{293}{8}$ is the improper fraction, equal to $36\frac{5}{8}$.

Or, $\frac{\overline{36 \times 8 + 5}}{8} = \frac{293}{8}$ *Answ. as before.*

2. REDUCE $127\frac{4}{17}$ to its equivalent improper fraction. *Answ.* $\frac{2163}{17}$

3. REDUCE $653\frac{3}{19}$ to its equivalent improper fraction. *Answ.* $\frac{12410}{19}$

CASE 3.¶

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.

† ALL fractions represent a division of the numerator by the denominator, and are taken altogether as proper and adequate expressions of the quotient. Thus the quotient of 3 divided by 4 is $\frac{3}{4}$; from whence the rule is manifest; for if any number is multiplied and divided by the same number, it is evident the quotient must be the same as the quantity first given.

¶ MULTIPLICATION and Division are here equally used, and consequently the result is the same as the quantity first proposed.

RULE.

MULTIPLY the whole number by the given denominator:—place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. REDUCE 6 to a fraction, whose denominator shall be 8.

$6 \times 8 = 48$, *and* $\frac{48}{8}$ *the answer.—Proof* $\frac{48}{8} = 48 \div 8 = 6$.

2. REDUCE 15 to a fraction, whose denominator shall be 12.

Answ. $\frac{180}{12}$.

3. REDUCE 100 to a fraction, whose denominator shall be 70.

Answ. $\frac{7000}{70} = \frac{100}{1} = 100$.

CASE 4. ‡

To reduce an improper fraction to its equivalent whole, or mixed number.

RULE.

DIVIDE the numerator by the denominator; the quotient will be the whole number, and the remainder, if any, will be the numerator to the given denominator.

EXAMPLES.

1. REDUCE $\frac{293}{8}$ to its equivalent whole, or mixed number.

8)293(36$\frac{5}{8}$ *Answer.*
24
——
53
48
——
5

Or $\frac{293}{8} = 293 \div 8 = 36\frac{5}{8}$ *as before.*

2. REDUCE $\frac{2163}{17}$ to its equivalent whole, or mixed number.

Ans. $127\frac{4}{17}$.

3. REDUCE $\frac{12410}{19}$ to its equivalent whole, or mixed number.

Answ. $653\frac{3}{19}$.

4. REDUCE $\frac{45}{5}$ to its equivalent whole number. *Answ.* 9.

CASE 5. *

To reduce a compound fraction to an equivalent simple one.

RULE.

MULTIPLY all the numerators continually together for a new numerator,

‡ THIS rule is evidently the reverse of case 2d, and has its reason in the nature of common division.

* THAT a compound fraction may be represented by a simple one is very evident; since a part of a part must be equal to some part of the whole. The truth of the rule for this reduction may be shewn as follows.

LET the compound fraction, to be reduced, be $\frac{3}{4}$ of $\frac{6}{10}$. Then $\frac{1}{4}$ of $\frac{6}{10} = \frac{6}{10} \div 4 = \frac{6}{40}$, and consequently $\frac{3}{4}$ of $\frac{6}{10} = \frac{6}{40} \times 3 = \frac{18}{40}$ the same as by the rule.

IF the compound fraction consists of more numbers than two, the two first may be reduced to one, and that one and the third will be the same as a fraction of two numbers, and so on.

numerator, and all the denominators, for a new denominator, and they will form the simple fraction required.

If part of the compound fraction be a whole or mixed number, it must be reduced to an improper fraction, by case 2d, or 3d.

If the denominator of any member of a compound fraction be equal to the numerator of another member thereof, these equal numerators and denominators may be expunged and the other members continually multiplied, (as by the rule) will produce the fraction required in lower terms.

Examples.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

$$\frac{1\times2\times3\times4}{2\times3\times4\times5}=\frac{24}{120}=\frac{1}{5}\ \textit{the answer.}$$

Or, by expunging the equal numerators and denominators, it will give $\frac{1}{5}$ as before.

2. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{11}{12}$ to a simple fraction.

$\frac{3\times4\times5\times11}{4\times5\times6\times12}=\frac{660}{1440}=\frac{11}{24}$ *Answ.* Or, by expunging the equal numerators and denominators it will be $\frac{3\times11}{6\times12}=\frac{33}{72}=\frac{11}{24}$ *as before.*

3. Reduce $\frac{5}{8}$ of $\frac{6}{7}$ of $\frac{15}{19}$ to a simple fraction. *Answ.* $\frac{225}{532}$.

4. Reduce $\frac{3}{12}$ of $\frac{13}{15}$ of $\frac{8}{17}$ of 20 to a simple (or improper) fraction. *Answ.* $\frac{624}{306}=2\frac{2}{61}$.

5. Reduce $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $12\frac{1}{2}$ to a simple (or improper) fraction. *Answ.* $\frac{75}{64}=1\frac{11}{64}$.

Case 6.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

Rule. 1 *

Multiply each numerator into all the denominators, except its own, for a new numerator, and all the denominators into each other, continually, for a common denominator.

Examples.

1. Reduce $\frac{1}{4}$, $\frac{2}{5}$ and $\frac{5}{8}$ to equivalent fractions, having a common denominator.

$1\times5\times8=40$ *the new numerator for* $\frac{1}{4}$.

2

* By placing the numbers multiplied properly under one another, it will be seen that the numerator and denominator of every fraction are multiplied by the very same number, and consequently their values are not altered. Thus, in the first example.

$$\frac{1}{4}\left|\frac{\times5\times8}{\times5\times8}\right.\quad \frac{2}{5}\left|\frac{\times4\times8}{\times4\times8}\right.\quad \frac{5}{8}\left|\frac{\times4\times5}{\times4\times5}\right.$$

In the second rule, the common denominator is a multiple of all the denominators, and consequently will divide by any of them: therefore proper parts may be taken for all the numerators as required.

$2 \times 4 \times 8 = 64$ *ditto for* $\frac{2}{5}$.

$5 \times 4 \times 5 = 100$ *ditto for* $\frac{5}{8}$.

$4 \times 5 \times 8 = 160$ *the common denominator.*

THEREFORE the new equivalent fractions are $\frac{40}{160}$, $\frac{64}{160}$, & $\frac{100}{160}$, *the answer.*

2. REDUCE $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to fractions having a common denominator. *Answ.* $\frac{576}{1152}$, $\frac{768}{1152}$, $\frac{864}{1152}$, $\frac{960}{1152}$, $\frac{1008}{1152}$.

3. REDUCE $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{6}$, $7\frac{3}{4}$, and $\frac{3}{13}$ to a common denominator.

Answ. $\frac{936}{1872}$, $\frac{1040}{1872}$, $\frac{14508}{1872}$, $\frac{432}{1872}$.

4. REDUCE $\frac{11}{15}$, $\frac{3}{4}$ of 2^{1}, $\frac{7}{12}$, and $\frac{5}{8}$ to a common denominator.

Answ. $\frac{8448}{11520}$, $\frac{21600}{11520}$, $\frac{6720}{11520}$, $\frac{7200}{11520}$.

RULE 2.

To reduce any given fractions to others, which shall have the least common denominator.

1. BY Prob. 2, Page 72, find the least common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. DIVIDE the common denominator by the denominator of each fraction, and multiply the quotient by the numerator and the product will be the numerator of the fraction required.

EXAMPLES.

1. REDUCE $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{7}{8}$ to fractions, having the least common denominator possible.

$$\begin{array}{r|ccc} 4) & 3 & 4 & 8 \\ \hline & 3 & 1 & 2 \end{array} \qquad 4 \times 3 \times 2 = 24 = \textit{least common denominator.}$$

$\overline{24 \div 3} \times 1 = 8$ *the 1st numerator*; $\overline{24 \div 4} \times 3 = 18$ *the 2d numerator*; $24 \div 8 \times 7 = 21$ *the 3d numerator.*

Whence, the required fractions are $\frac{8}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.

2. REDUCE $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to fractions having the least common denominator. *Answ.* $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$ *and* $\frac{48}{60}$.

CASE 7.

To reduce a fraction of one denomination to the fraction of another, but greater, retaining the same value.

RULE. ¶

REDUCE the given fraction to a compound one, by comparing it with all the denominations between *it* and *that* denomination you would reduce it to; lastly, reduce this compound fraction to a single one, by case 5th, and you will have a fraction of the required denomination, equal in value to the given fraction.

EXAMPLES.

¶ THE reason of this and the next rule is explained in the rule for reducing compound fractions to simple ones.

EXAMPLES.

1. REDUCE $\frac{3}{5}$ of a penny to the fraction of a pound.

BY comparing it, it becomes $\frac{3}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}$; which reduced by case 5th, will be $3 \times 1 \times 1 = 3$

And $5 \times 12 \times 20 = 1200 = \frac{1}{400}$.

2. REDUCE $\frac{3}{4}$ of a farthing to the fraction of a pound.
Answ. £. $\frac{1}{1280}$.

3. REDUCE $\frac{5}{8}$ of a penny to the fraction of a guinea.
Answ. $\frac{5}{2688}$ *Guinea.*

4. REDUCE $\frac{12}{19}$ of a shilling to the fraction of a moidore.
Answ. $\frac{1}{57}$ *Moidore.*

5. REDUCE $\frac{4}{7}$ of an ounce to the fraction of a ℔. Avoirdupois.
Answ. $\frac{1}{28}$ ℔.

†6. REDUCE 3/6 to the fraction of a pound. *Answ.* $\frac{7}{40}$£.

§7. REDUCE 13/6 to the fraction of a pistole.
Answ. $\frac{27}{44}$ *Pistole.*

‡8. REDUCE $\frac{4}{5}$ of a pound to the fraction of a guinea.
Answ. $\frac{4}{7}$ *Guinea.*

9. REDUCE $\frac{7}{8}$ of a *pwt.* to the fraction of a pound Troy.
Answ. $\frac{7}{1920}$ ℔.

10. REDUCE $\frac{8}{9}$ of a ℔. Avoirdupois to the fraction of 1 *Cwt.*
Answ. $\frac{1}{126}$ *Cwt.*

11. EXPRESS $5\frac{1}{2}$ furlongs in the fraction of a mile.
Answ. $\frac{11}{16}$ *mile.*

CASE 8.

To reduce a fraction of one denomination to the fraction of another, but less, retaining the same value.

RULE.

MULTIPLY the given numerator by the parts of the denominations between *it* and *that* denomination you would reduce it to, for a new numerator, which place over the given denominator; *Or*, only invert the parts contained in the integer, and make of them a compound fraction as before, then, reduce it to a simple one.

EXAMPLES.

1. REDUCE $\frac{1}{400}$ of a pound to the fraction of a penny.

BY comparing it, the fraction will be $\frac{1}{400}$ of $\frac{20}{1}$ of $\frac{12}{1}$, then $\frac{1 \times 20 \times 12}{400 \times 1 \times 1} = \frac{240}{400} = \frac{3}{5}$ *Answer.*

2. REDUCE

† 3s. 6*d.* = 42*d.* and 1£. = 240*d.* therefore, $\frac{42}{240} = \frac{7}{40}$£.

§ $\frac{4}{5}$£. $= \frac{4}{5}$ of $\frac{20}{1} = \frac{4 \times 20}{5 \times 1} = \frac{80}{5}$ *s.* and $\frac{80}{5}$ of $\frac{1}{28} = \frac{80 \times 1}{5 \times 28} = \frac{112}{140}$ $= \frac{4}{7}$ *Guineas.*

‡ $\frac{4}{7}$ *Guin.* $= \frac{4}{7}$ of $^{28} = \frac{4 \times 28}{7 \times 1} = \frac{112}{7}$ *s.* & $\frac{112}{7}$ of $\frac{1}{20} = \frac{112}{140} = \frac{4}{5}$£.

2. Reduce $\frac{1}{1280}$ of a pound to the fraction of a farthing. *Answ.* $\frac{3}{4}$ *qr.*

3. Reduce $\frac{5}{2688}$ of a guinea to the fraction of a penny. *Answ.* $\frac{5}{8}$ *d.*

4. Reduce $\frac{1}{37}$ of a moidore to the fraction of a shilling. *Answ.* $\frac{12}{19}$ *s.*

5. Reduce $\frac{1}{28}$ of a ℔. Avoirdupois to the fraction of an ounce. *Answ.* $\frac{4}{7}$ *oz.*

6. Reduce $\frac{4}{7}$ of a guinea to the fraction of a pound. *Answ.* $\frac{4}{5}$ £.

7. Reduce $\frac{7}{1920}$ of a ℔. Troy to the fraction of a *pwt.* *Answ.* $\frac{7}{8}$ *pwt.*

8. Reduce $\frac{1}{126}$ of *Cwt.* to the fraction of a ℔. Avoirdupois. *Answ.* $\frac{8}{9}$ ℔.

Case 9.

To find the value of a fraction in the known parts of the integer, as of coin, weight, measure, &c.

Rule.||

Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator; and if any thing remain, multiply it by the next inferior denomination, and divide by the denominator as before, and so on, as far as necessary; and the quotients placed after one another, in their order, will be the answer required.

Examples.

1. What is the value of $\frac{5}{7}$ of a pound?

```
     5
    20
   ---
 7)100
   ---
 14s.—2
     12
     --
   7)24
     --
   3d.—3.
        4
        --
      7)12
        --
      1 5/7qr.
```

I multiply 5 by 20, the number of shillings in £.1, and the product 100 I divide by the denominator 7, and get the quotient 14*s.* and 2 remaining, I multiply it by 12, and again dividing the product by 7, find the quotient 3*d.* and 3 remains, which I multiply by 4, and, dividing as before, the quotient is 1*qr.* and 5 remaining, I place it over the numerator, and find the *answer* 14*s.* 3*d.* $1\frac{5}{7}$*qr.*

2. What

|| As the numerator of a fraction may be considered as a remainder, and the denominator as a divisor: this rule therefore has its reason in the nature of division.

2. WHAT is the value of $\frac{9}{24}$ of a shilling? *Answ.* $4\frac{1}{2}d.$

3. WHAT is the value of $\frac{17}{29}$ of a *Cwt.*?
Answ. 2qr. 9℔. 10oz. $7\frac{21}{29}dr.$

4. WHAT is the value of $\frac{4}{5}$ of a ℔. Avoirdupois?
Answ. 12oz. $12\frac{4}{5}dr.$

5. WHAT is the value of $\frac{3}{5}$ of a ℔. Troy?
Answ. 7oz. 4pwt.

6. WHAT is the value of $\frac{3}{13}$ of a Ton?
Answ. 4Cwt. 2qrs. 12℔. 14oz. $12\frac{4}{13}dr.$

7. WHAT is the value of $\frac{6}{9}$ of a yard? *Answ.* 2qrs. $2\frac{2}{3}n.$

8. WHAT is the value of $\frac{7}{8}$ of an Ell-English?
Answ. 4qrs. $1\frac{1}{2}n.$

9. WHAT is the value of $\frac{5}{8}$ of a mile? *Answ.* 6fur. 26p. 11ft.

10. WHAT is the value of $\frac{9}{13}$ of a day? *Answ.* 16h. 36m. $55\frac{5}{13}s.$

11. THE value of $\frac{12}{17}$ of a Julian year is required?
Answ. 257d. 19h. 45m. $52\frac{16}{17}s.$

12. THE value of $\frac{9}{14}$ of a guinea is demanded? *Answ.* 18s.

13. WHAT is the value of $\frac{15}{16}$ of a dollar? *Answ.* 5s. $7\frac{1}{2}d.$

14. WHAT is the value of $\frac{3}{5}$ of a moidore? *Answ.* 21s. $7\frac{1}{5}d.$

15. WHAT is the value of $\frac{6}{7}$ of an acre? *Answ.* 3R. $17\frac{1}{7}P.$

CASE 10.

To reduce any given quantity to the fraction of any greater denomination of the same kind.

RULE.§

REDUCE the given quantity to the lowest term mentioned, for a numerator; then reduce the integral part to the same term, for a denominator; which will be the fraction required.

EXAMPLES.

1. REDUCE 14s. $3\frac{1}{4}d.$ $\frac{5}{7}$ to the fraction of a pound.

20 *Integral part.*	s. d.
12	14 $3\frac{1}{4}$, $\frac{5}{7}$
	12
240	
4	171
	4
960	
7	685
	7+5
6720 *Denominator.*	
	4800 *Num.*

Answ. $\frac{4800}{6720} = \frac{5}{7}$£.

2. REDUCE $4\frac{1}{2}d.$ to the fraction of a shilling. *Answ.* $\frac{3}{8}s.$

3. REDUCE

§ THIS case is the reverse of the former, therefore proves it.
Note. If there be a fraction given with the said quantity, it must be further reduced to the denominative parts thereof, adding thereto the numerator.

3. REDUCE *2qrs.* 9℔. *10oz.* $11\frac{5}{29}$*dr.* to the fraction of a *Cwt.*
Answ. $\frac{17}{29}$ *Cwt.*

4. REDUCE *12oz.* $12\frac{4}{5}$*dr.* to the fraction of a ℔. Avoirdupois.
Answ. $\frac{4}{5}$ ℔.

5. REDUCE *7oz.* *4pwt.* to the fraction of a ℔. Troy.
Answ. $\frac{3}{5}$ ℔.

6. REDUCE 4*Cwt.* *2qrs.* 12 ℔. *14oz.* $12\frac{4}{13}$*dr.* to the fraction of a Ton. *Answ.* $\frac{3}{13}$ *T.*

7. REDUCE *2qrs.* $2\frac{2}{3}$*n.* to the fraction of a yard. *Answ.* $\frac{2}{3}$ *yd.*

8. REDUCE *4qrs.* $1\frac{1}{2}$*n.* to the fraction of an Ell-English.
Answ. $\frac{7}{8}$ *E. E.*

9. REDUCE 6 *furl.* 26 *po.* 11 *ft.* to the fraction of a mile.
Answ. $\frac{5}{6}$ *M.*

10. REDUCE *16h.* *36m.* $55\frac{5}{13}$*s.* to the fraction of a day.
Answ. $\frac{9}{13}$ *Day.*

11. REDUCE *257d.* *19h.* *45m.* $52\frac{16}{17}$*s.* to the fraction of a Julian year. *Answ.* $\frac{12}{17}$ *J. Y.*

12. REDUCE *18s.* to the fraction of a guinea. *Answ.* $\frac{9}{14}$ *G.*

13. REDUCE *5s.* $7\frac{1}{2}$*d.* to the fraction of a dollar. *Answ.* $\frac{15}{16}$ *dol.*

14. REDUCE *21s.* $7\frac{1}{5}$*d.* to the fraction of a Moidore.
Answ. $\frac{3}{5}$ *Moid.*

15. REDUCE 3 *Roods,* $17\frac{1}{7}$*P.* to the fraction of an Acre.
Answ. $\frac{6}{7}$ *Acr.*

ADDITION *of* VULGAR FRACTIONS.

RULE.‡

REDUCE compound fractions to single ones; mixed numbers to improper fractions; fractions of different integers to those of the same; and all of them to a common denominator; then the sum of the numerators written over the common denominator will be the sum of the fractions required.

EXAMPLES.

1. ADD $7\frac{4}{5}$, $\frac{5}{7}$ of $\frac{3}{8}$ and 7 together.

First. $7\frac{4}{5} = \frac{39}{5}$, $\frac{5}{7}$ of $\frac{3}{8} = \frac{15}{56}$, and $7 = \frac{7}{1}$.

Then the fractions are $\frac{39}{5}$, $\frac{15}{56}$, and $\frac{7}{1}$; *therefore*

$$\begin{array}{l} 39 \times 56 \times 1 = 2184 \\ 15 \times 5 \times 1 = 75 \\ \underline{7 \times 5 \times 56 = 1960} \end{array}$$

$$\frac{4219}{5 \times 56 \times 1 = 280} = 15\tfrac{19}{280}$$

Or thus, $\frac{2184+75+1960}{280} = 15\frac{19}{280}$.

L 2. ADD

‡ FRACTIONS, before they are reduced to a common denominator, are entirely dissimilar, and therefore cannot be incorporated with one another; but when they are reduced to a common denominator, and made parts of the same thing; their sum, or difference, may then be as properly expressed by the sum or difference of the numerators, as the sum or difference of any two quantities whatever, by the sum of difference of their individuals; whence the reason of the rules, both for Addition and Subtraction is manifest.

2. ADD $\frac{3}{7}$, $9\frac{1}{5}$, and $\frac{2}{3}$ of $\frac{1}{2}$ together. *Answ.* $9\frac{101}{105}$.

3. WHAT is the ſum of $\frac{4}{5}$, $\frac{5}{6}$ of $\frac{3}{8}$ of $\frac{1}{4}$, and $8\frac{4}{13}$? *Answ.* $9\frac{773}{4160}$.

4. WHAT is the ſum of $\frac{7}{10}$ of $4\frac{5}{8}$, $\frac{3}{4}$ of $\frac{1}{3}$ and $9\frac{1}{4}$? *Answ.* $12\frac{59}{80}$.

5. ADD $\frac{1}{9}$£. $\frac{3}{7}$*s.* and $\frac{4}{5}$*d.* together. *Answ.* 2/$8\frac{64}{105}$.

6. WHAT is the ſum of $\frac{2}{5}$ of 17£. $9\frac{5}{8}$£. and $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{4}{7}$£.? *Answ.* £.16 12*s.* $3\frac{5}{7}$*d.*

7. ADD $\frac{3}{4}$ of a yard, $\frac{1}{3}$ of a foot, and $\frac{5}{8}$ of a mile together. 1100*yds.* 2*ft.* $6\frac{517938}{557569}$ *Inch.*

8. ADD $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute together.. *Answ.* 2 *days,* 2 *hours,* 30 *minutes,* 45 *ſeconds.*

SUBTRACTION *of* VULGAR FRACTIONS.

RULE.*

PREPARE the fractions as in Addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required.—*Note,* a fraction is ſubtracted from a whole number, by taking the numerator of the fraction from its denominator, and placing the remainder over the denominator, then taking one from the whole number.

EXAMPLES.

1. FROM $\frac{3}{4}$ take $\frac{2}{7}$ of $\frac{5}{8}$.

$\frac{2}{7}$ of $\frac{5}{8} = \frac{10}{56} = \frac{5}{28}$. *Then the fractions are* $\frac{3}{4}$ *and* $\frac{5}{28}$.

$$\left.\begin{array}{l} 3\times 28 = 84 \\ 5\times 4 = 20 \\ 4\times 28 = 112 \textit{ com. den.} \end{array}\right\} \begin{array}{l} \frac{3}{4} = \frac{84}{112} \textit{ and } \frac{5}{28} = \frac{20}{112}, \textit{ therefore,} \\ \frac{84}{112} - \frac{20}{112} = \frac{64}{112} = \frac{4}{7} \textit{ Remainder.} \end{array}$$

2. FROM $\frac{49}{50}$ take $\frac{5}{9}$. *Answ.* $\frac{191}{450}$.
3. FROM $37\frac{1}{4}$ take $19\frac{4}{7}$. *Answ.* $17\frac{19}{28}$.
4. FROM $13\frac{1}{3}$ take $\frac{3}{4}$ of 15. *Answ.* $2\frac{1}{12}$.
5. FROM $\frac{1}{4}$£. take $\frac{9}{10}$*s.* *Answ.* 4/$1\frac{1}{5}$.
6. FROM $\frac{5}{7}$*oz.* take $\frac{3}{4}$*pwt.* *Answ.* 13*pwt.* $12\frac{6}{7}$*gr.*
7. FROM $\frac{1}{2}$ of a league take $\frac{3}{8}$ of a mile. *Answ.* 1*mi.* 1*fur.*
8. FROM 5 weeks take $19\frac{4}{5}$ days. *Answ.* 15*da.* 4*ho.* 48*min.*

MULTI-

* IN ſubtracting mixed numbers, when the lower fraction (the ſubtrahend) is greater than the upper one, (the minuend) you may, without reducing them to improper fractions, ſubtract the numerator of the ſubtrahend from the common denominator, and to that difference add the numerator of the minuend, and carry one to the integer of the ſubtrahend.

Example. From $19\frac{3}{8}$ *take* $12\frac{7}{8}$?———$19\frac{3}{8} - 12\frac{7}{8} = 6\frac{1}{2}$.

MULTIPLICATION of VULGAR FRACTIONS.

RULE.‖

REDUCE compound fractions to simple ones, and mixed numbers to improper fractions; then the product of the numerators will be the numerator, and the product of the denominators, the denominator of the product required.—*Note,* where several fractions are to be multiplied, if the numerator of one fraction be equal to the denominator of another, their equal numerators and denominators may be omitted.

EXAMPLES.

1. WHAT is the continued product of $4\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{4}$ of $\frac{7}{8}$ and 6.

$$4\tfrac{1}{3} = \tfrac{13}{3},\ \tfrac{1}{4}\ \textit{of}\ \tfrac{7}{8} = \frac{1 \times 7}{4 \times 8} = \tfrac{7}{32},\ \textit{and}\ 6 = \tfrac{6}{1}.$$

$$\textit{Then}\ \tfrac{13}{3} \times \tfrac{1}{5} \times \tfrac{7}{32} \times \tfrac{6}{1} = \frac{13 \times 1 \times 7 \times 6}{3 \times 5 \times 32 \times 1} = \tfrac{546}{480} = 1\tfrac{11}{80}\ \textit{the answer.}$$

2. MULTIPLY $\frac{4}{17}$ by $\frac{5}{27}$. *Answ.* $\frac{20}{459}$.
3. MULTIPLY $9\frac{1}{4}$ by $\frac{1}{8}$. *Answ.* $\frac{7}{8}$.
4. MULTIPLY $\frac{1}{3}$ of 5 by $\frac{3}{4}$ of $\frac{2}{7}$. *Answ.* $\frac{5}{14}$.
5. MULTIPLY $\frac{3}{7}$ of $\frac{5}{6}$ by $\frac{4}{5}$ of $\frac{1}{5}$ of $11\frac{3}{7}$. *Answ.* $\frac{64}{147}$.
6. MULTIPLY $9\frac{3}{4}$, $\frac{1}{2}$ of $\frac{2}{3}$, and $12\frac{4}{7}$ continually together. *Answ.* $24\frac{18}{35}$.
7. WHAT is the continual product of $\frac{3}{4}$ of $\frac{2}{3}$, $5\frac{1}{2}$, 7 and $\frac{1}{3}$ of $\frac{5}{8}$? *Answ.* $4\frac{1}{96}$.
8. WHAT is the continual product of 7, $\frac{1}{2}$, $\frac{5}{7}$ of $\frac{3}{8}$, and $3\frac{1}{9}$? *Answ.* $2\frac{11}{12}$.

Another method for the Multiplication of mixed Quantities.

Case 1. To multiply a *whole number* by a *fraction,* or a *fraction* by a *whole number.*

Rule. Multiply the whole number by the numerator of the fraction, and divide the product by the denominator: but if the numerator be 1, divide by the denominator only.

	1.	2.	3.	4.	5.	6.	7.
Mult.	8	15	28	36	48	325	259
By	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{7}{12}$
Prod.	2	$7\frac{1}{2}$	$9\frac{1}{3}$	3)72	4)144	8)1625	12)1813
			Prod.	24	38	$203\frac{1}{8}$	$151\frac{1}{12}$

Case 2. To multiply a *whole number* by a *mixed one.*

Rule. Multiply by the fraction as in case 1st; then multiply by the whole number, and add the two products, as in the examples—or,

‖ MULTIPLICATION of a fraction implies the taking some part or parts of the multiplicand, and therefore may truly be expressed by a compound fraction. Thus $\frac{4}{5}$ multiplied by $\frac{3}{8}$ is the same as $\frac{4}{5}$ of $\frac{3}{8}$; and as the directions of the rule agree with the method already given, to reduce these fractions to simple ones, it is shewn to be right.

or, to multiply a mixed number by a whole one, change the place of the factors, and proceed as the rule directs.—*See Examp.* 6.

	1.	2.	3.	4.	5.	6.
Mult.	15	35	68	42	129	$\frac{7}{15}$
By	$3\frac{1}{2}$	$5\frac{1}{3}$	$7\frac{11}{12}$	$9\frac{3}{7}$	$8\frac{5}{8}$	24
	$7\frac{1}{2}$	$11\frac{2}{3}$	748	126	645	*Mult.* 24
	45	175	$62\frac{4}{12}$	18	$80\frac{5}{8}$	*by* $\frac{7}{15}$
	$52\frac{1}{2}$	$186\frac{2}{3}$	476	378	1032	168
			$538\frac{4}{12}$	396	$1112\frac{5}{8}$	$11\frac{3}{15}$

Case 3. To multiply a *mixed number* by a *mixed number.*

Rule. Multiply the integral part of the multiplicand by the denominator of its fractional part, and add thereto its numerator: then multiply by the mixed multiplier, by Case 2d, and divide the product by the denominator of the fractional part of the multiplicand, as in the following example.

Mult. $42\frac{3}{5}$ } *1st* $42\frac{3}{5} = 213$
By $8\frac{2}{3}$ } *which mult. by* $8\frac{2}{3}$

$$3)426$$
$$142$$
$$1704$$
$$5)1846$$
Product $= 369\frac{1}{5}$

After this manner may feet and inches be multiplied, calling 1 inch $\frac{1}{12}$ of a foot, 2 inches $\frac{1}{6}$, 3 inches $\frac{1}{4}$, inches $\frac{1}{3}$, 5 inches $\frac{5}{12}$, 6 inches $\frac{1}{2}$, 7 inches $\frac{7}{12}$, 8 inches $\frac{2}{3}$, 9 inches $\frac{3}{4}$, 10 inches $\frac{5}{6}$, and 11 inches $\frac{11}{12}$ of a foot.

Division *of* Vulgar Fractions.

Rule.†

Prepare the fractions as before: then, invert the divisor and proceed exactly as in Multiplication:—The products will be the quotient required.

Examples.

1. Divide $\frac{1}{3}$ of 17 by $\frac{2}{3}$ of $\frac{6}{8}$.

$\frac{1}{3}$ *of* $17 = \frac{1}{3}$ *of* $\frac{17}{1} = \frac{1 \times 17}{3 \times 1} = \frac{17}{3}$, & $\frac{2}{3}$ *of* $\frac{6}{8} = \frac{12}{24} = \frac{1}{2}$; *therefore,* $\frac{17}{3} \div \frac{1}{2} = \frac{17}{3} \times \frac{2}{1} = \frac{34}{3} = 11\frac{1}{3}$ *the quotient required.*

2. Divide $\frac{5}{7}$ by $\frac{3}{5}$. *Answ.* $1\frac{4}{21}$.
3. Divide $12\frac{1}{2}$ by $\frac{1}{3}$ of 7. *Answ.* $5\frac{8}{35}$.

† The reason of the Rule may be shewn thus. Suppose it were required to divide $\frac{4}{5}$ by $\frac{2}{7}$. Now $\frac{4}{5} \div 2$ is manifestly $\frac{1}{2}$ of $\frac{4}{5}$, or $\frac{4}{2 \times 5}$; but $\frac{2}{7} = \frac{1}{7}$ of 2; therefore $\frac{1}{7}$ of 2, or $\frac{2}{7}$ must be contained 7 times as often in $\frac{4}{5}$ as 2, that is $\frac{4 \times 7}{5 \times 2} =$ the answer, which is according to the rule.

Note. To multiply a fraction by an integer, divide the denominator, or multiply the numerator by it; and to divide by an integer, divide the numerator, or multiply the denominator by it.

4. Divide $5\frac{1}{8}$ by $7\frac{3}{4}$. *Answ.* $\frac{41}{62}$.
5. Divide $\frac{3}{7}$ by 9. *Answ.* $\frac{1}{21}$.
6. Divide $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{8}$ of $\frac{3}{4}$. *Answ.* $\frac{8}{9}$.
7. Divide 7 by $\frac{3}{8}$. *Answ.* $18\frac{2}{3}$.
8. Divide $4204\frac{1}{6}$ by $\frac{7}{8}$ of 112. *Answ.* $42\frac{520}{588}$.

DECIMAL FRACTIONS.

Decimal Fractions are of ſuch a nature, that they vary in the ſame proportion, and are managed by the ſame method of Operation, as whole numbers are.

On this account, every proper Fraction is ſuppoſed to be reducible to another, whoſe denominator ſhall be 10, 100, 1000, &c. *viz.* Unity, with a number of cyphers annexed; and Fractions with ſuch denominators are called *Decimal Fractions*: ſuch are $\frac{5}{10}$, $\frac{55}{100}$, $\frac{675}{1000}$, &c.

As the denominator of a decimal fraction is always 10, or 100, or 1000, &c. the denominators need not be expreſſed: for the numerator only may be made to expreſs the true value: for this purpoſe it is only required to write the numerator with a point before it at the left hand, to diſtinguiſh it from a whole number, when it conſiſts of ſo many figures as the denominator hath cyphers annexed to unity, or 1; ſo $\frac{5}{10}$ is written ,5; $\frac{33}{100}$,33; $\frac{735}{1000}$,735, &c.

Note. The point prefixed is called a Separatrix.

But if the numerator has not ſo many places as the denominator has cyphers, put ſo many cyphers before it, *viz.* at the left hand, as will make up the defect; ſo write $\frac{5}{100}$ thus ,05; and $\frac{6}{1000}$ thus ,006, &c. And thus do theſe fractions receive the form of whole numbers.

The 1ſt, 2d. 3d. 4th. &c. places of decimals, counting from the left hand toward the right, are called primes, ſeconds, thirds, fourths, &c.

We may conſider unity as a fixed point, from whence whole numbers proceed infinitely increaſing toward the left hand, and decimals infinitely decreaſing toward the right hand to 0, as in the following

TABLE.

C Millions	X Millions	Millions	C Thouſands	X Thouſands	Thouſands	Hundreds	Tens	Units	Tenth Parts	Hundredth Parts	Thouſandth Parts	X Thouſandth Parts	C Thouſandth Parts	Millionth Parts	X Millionth Parts	C Millionth Parts
9	8	7	6	5	4	3	2	1,	2	3	4	5	6	7	8	9

From this table it is evident, that, in decimals, as well as in whole numbers, each figure takes its value by its diſtance from unit's place;

place: If it be in the firſt place after units (or the ſeparating point) it ſignifies tenths; if in the ſecond, hundredths, &c. decreaſing in each place in a ten-fold proportion.

Consequently, every ſingle figure expreſſing a decimal has for its denominator an unit or 1, with ſo many cyphers as its place is diſtant from unit's place: Thus ,2 in the decimal part of the table $=\frac{2}{10}$; $3=\frac{3}{100}$; $4=\frac{4}{1000}$, &c. And if a decimal be expreſſed by ſeveral figures, the denominator is 1, with ſo many cyphers as the loweſt figure is diſtant from unit's place. So ,357 ſignifies $\frac{357}{1000}$, and $,0053=\frac{53}{10000}$, &c.

Cyphers, placed at the right-hand of a decimal fraction, do not alter its value, ſince every ſignificant figure continues to poſſeſs the ſame place: So ,5, ,50 and ,500 are all of the ſame value, and each equal to $\frac{1}{2}$.

But cyphers, placed at the left hand of a decimal, do alter its value, every cypher depreſſing it to $\frac{1}{10}$ of the value it had before, by removing every ſignificant figure one place further from the place of units. So ,5, ,05, ,005, all expreſs different decimals, *viz.* ,5, $\frac{5}{10}$; ,05, $\frac{5}{100}$; ,005, $\frac{5}{1000}$.

Hence may be obſerved the contrary effect of cyphers being annexed to whole numbers, and decimals.

It is likewiſe evident from the table, that, ſince the places of decimals decreaſe in a ten-fold proportion from units downwards, ſo they conſequently increaſe in a ten-fold proportion from the right-hand toward the left, as the places of whole numbers do: for, ten hundredth parts make one tenth, ten tenths make 1; ten units, ten; ten tens, one hundred, &c. viz. $\frac{10}{100}=\frac{1}{10}$, $\frac{10}{10}=1$, and $1\times10=10$, which proves that decimals are ſubject to the ſame law of Notation, and conſequently of operation, as whole numbers are.

Decimal Fractions of unequal denominators are reduced to one common denominator, when there are annexed to the right hand of thoſe, which have fewer places, ſo many cyphers, as make them equal in places with that which has the moſt. So theſe decimals, ,5, ,06, ,455 may be reduced to the decimals ,500, ,060, and ,455, which have, all, 1000 for their denominator.

Of Decimals, that is the greateſt, whoſe higheſt figure is greateſt, whether they conſiſt of an equal or unequal number of places: Thus, ,5 is greater than ,459, for if it be reduced to the ſame denominator with ,459, it will be ,500.

A mixt number, *viz.* a whole number, with a decimal annexed, is equal to an improper fraction, whoſe numerator is all the figures of the mixed number, taken as one whole number, and the denominator, that of the decimal part. So 45,309 is equal to $\frac{45309}{1000}$, as is evident from the method given to reduce a mixt number to an improper fraction:

Thus, $45\times1000+,309=\frac{45309}{1000}$ *as above.*

Addition

Addition of Decimals.

Rule.

1. Place the numbers, whether mixed, or pure decimals, under each other, according to the value of their places.

2. Find their sum as in whole numbers, and point off so many places for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

Examples.

1. Find the sum of 19,073+2,3597+223+,0197581+3478,1 +12,358.

```
  19,073
   2,3597
 223
    ,0197581
3478,1
  12,358
------------
3734,9104581 the sum.
```

2. Required the sum of 429+21,37+355,003+1,07+1,7 ? *Answ.* 808,143.

3. Required the sum of 5,3+11,973+49+,9+1,7314+34,3 ? *Answ.* 103,2044.

4. Required the sum of 973+19+1,75+93,7164+,9501 ? *Answ.* 1088,4165.

Subtraction of Decimals.

Rule.

Place the numbers according to their value ; then subtract as in whole numbers, and point off the decimals as in Addition.

Examples.

1. Find the difference of 1793,13 and 817,05693 ?

```
     From 1793,13
     Take  817,05693
Remainder  976,07307
```

2. From 171,195 take 125,9176. *Answ.* 45,2774.

3. From 219,1384 take 195,91. *Answ.* 23,2284.

4. From 480 take 245,0075. *Answ.* 234,9925.

Multiplication

MULTIPLICATION *of* DECIMALS.

CASE I.

RULE.

1. WHETHER they be mixt numbers, or pure decimals, place the factors and multiply them as in whole numbers.

2. POINT off so many figures from the product as there are decimal places in both the factors; and if there be not so many places in the product, supply the defect by prefixing cyphers.

EXAMPLES.

1. MULTIPLY ,02345
by ,00163

$$\begin{array}{r} 7035 \\ 14070 \\ 2345 \\ \hline ,0000382235 \end{array}$$

the product.

2. MULTIPLY 25,238 by 12,17. *Answ.* 307,14646.
3. MULTIPLY ,3759 by ,945. *Answ.* ,3552255.
4. MULTIPLY ,84179 by ,0385. *Answ.* ,032,08915.

To multiply by 10, 100, 1000, &c. remove the separating point so many places to the right hand, as the multiplier has cyphers.

$$\textit{So } ,345 \left\{ \textit{Multip. by} \right\} \left\{ \begin{array}{l} 10 \\ 100 \\ 1000 \end{array} \right\} \textit{makes} \left\{ \begin{array}{l} 3,45 \\ 34,5 \\ 345 \end{array} \right.$$

FOR ,345 × 10 is 3,450, &c.

CASE 2.

To contract the operation, so as to retain so many decimal places in the Product as may be thought necessary.

RULE.

1. WRITE the unit's place of the multiplier under that figure of the multiplicand, whose place you would reserve in the product; and dispose of the rest of the figures in a contrary order to what they are usually placed in.

2. IN multiplying, reject all the figures which are to the right hand of the multiplying digit, and set down the products, so that their right hand figures may fall in a straight line below each other; observing to increase the first figure of every line with what would arise by carrying 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. from the preceding figures when you begin to multiply, and the sum will be the product required.

EXAM-

EXAMPLE.

1. It is required to multiply 56,7534916 by 5,376928 and to retain only five places of decimals in the product.

Contracted way.	*Common way.*
56,7534916	56,7534916
829673,5	5,376928
28376746 . .	45 | 40279328
1702605 . . .	113 | 5069832
397274	5107 | 184244
34052	34052 | 09496
5108	397274 | 4412
113	1702604 | 748
45	28376745 | 80
305,15943	305,15943 | 80818048

By the operation in the *common way*, it is evident that all the figures, which are cut off at the right hand by the perpendicular line, are wholly omitted in the *contracted way*, and the last product here is the first there; consequently the reason of placing the multiplier in a reverse order, must appear very plainly.

DIVISION *of* DECIMALS.

RULE.*

1. The places of decimal parts in the divisor and quotient counted together must always be equal to those in the dividend, therefore divide as in whole numbers, and, from the right hand of the quotient, point off so many places for decimals, as the decimal places in the dividend exceed those in the divisor.

2. If the places of the quotient be not so many as the rule requires, supply the defect by prefixing cyphers to the left hand.

3. If at any time there be a Remainder, or the decimal places in the divisor be more than those in the dividend, cyphers may be annexed to the dividend, or to the remainder, and the quotient carried on to any degree of exactness.

EXAMPLES.

* The reason of pointing off so many decimal places in the quotient, as those in the dividend exceed those in the divisor, will easily appear; for since the number of decimal places in the dividend is equal to those in the divisor and quotient taken together, by the nature of Multiplication: it therefore follows that the quotient contains so many as the dividend exceeds the divisor.

EXAMPLES.

1.		2.
219),117841075(,000538087, &c.		,3719)38,0000(102,178, &c.
1095	*In Example 1st. the divisor having no decimals, the quotient must have so many as there are in the dividend. In Example 2, the dividend being an integer must have at least so many cyphers annexed, as there are decimals in the divisor, and so far the quotient will be whole numbers, then annexing more cyphers, the remaining figures in the quotient will be decimals, according to the Rule.*	3719
834		8100
657		7438
1771		6620
1752		3719
1907		29010
1752		26033
1555		29770
1533		29752
22		18

3d. 133)5737(43,1353+ (4th) 23,7)65321(2756,16+
5th. ‡ 72)918,217(12753+ (6th) 25,17)315,6293(1253+
7th. ,317)29,417(92+ (8th) 37,9),0059374(156+
9th. ,375),173948375(463862+

Having a multiplier, to find a divisor which shall give a quotient equal to the product by that multiplier.

RULE.

DIVIDE unity by the given multiplier, and the quotient will be the divisor sought.

WHAT divisor is that, by which dividing 5357, shall give a quotient equal to the product of the same number multiplied by 250?

250)1,000(,004 *the answer.* *And* ,004)5357,000(1339250.
Proof. 5357×250=1339250.

Having a divisor, to find a multiplier which shall give a product equal to the quotient by that divisor.

RULE.

DIVIDE unity by the given divisor, and the quotient will be the multiplier sought.

WHAT multiplier is that, by which multiplying 5357, shall give a product equal to the quotient of the same number divided by ,004?
,004)1,000(250 *the answer: Therefore* 5357×250=5357÷,004 =1339250.

CASE 2.

To contract Division, when there are many decimals in the dividend, and the divisor is large.

RULE.

‡ THE following questions are left unpointed in the quotient to exercise the learner.

Rule.

1. Whatever place of the dividend correſponds with the unit's place of the diviſor, at the firſt ſtep of the Diviſion, the ſame place muſt the firſt figure of the quotient have.

2. In dividing, reject the laſt right hand figure of the diviſor, at every ſtep (inſtead of bringing down a figure, as in common) and make the laſt remainder the dividend for the new diviſor at every ſtep: Thus continue the Diviſion 'till the diviſor ſhall be exhauſted.

```
 . . . . .
99,5678)4,6789837568(,0469931 Quotient.
          3 982712
          --------
   99,567)696271
          597402
          ------
    99,56) 98869
           89604
           -----
     99,5) 9265
           8955
           ----
       99) 310
           297
           ---
        9) 13
            9
           --
Remainder   4
```

Here, the unit's place of the diviſor in the firſt ſtep falls under 7 in the place of hundredths in the dividend, therefore I put 4, the firſt quotient-figure, in the place of hundredths, *by prefixing a cypher.*

I have ſet down every diviſor to explain the work; but you need only put a daſh over every figure rejected, as you proceed, to ſhew it is omitted.

When decimals or whole numbers are to be divided by 10, 100, 1000, &c. (*viz.* unity with cyphers,) it is performed by removing the ſeparatrix, in the dividend, ſo many places toward the left hand as there are cyphers in the diviſor.

Examples.

Dividing		The Quot. is
10	7654	765,4
100	7654	76,54
1000	7654	7,654
10000	7654	,7654

Reduction *of* Decimals.

Case I.

To reduce a Vulgar Fraction to its equivalent Decimal.

Rule. *

Divide the numerator by the denominator, as in diviſion of decimals, and the quotient will be the decimal required:—Or, ſo many

* Let the given vulgar fraction, whoſe decimal expreſſion is required, be $\frac{9}{15}$. Now, ſince every decimal fraction has 10, 100, 1000, &c. for its denominator; and if two fractions be equal, it will be, as the denominator of one is to its numerator; ſo is the denominator of the other to its numerator, therefore, As 15 : 9 :: 10, &c. : $\frac{9\times10}{15} = \frac{90}{15} =$,6 the numerator of the decimal required; and is the ſame as by the rule,

ny cyphers as you annex to the given numerator, ſo many places muſt be pointed off in the quotient, and if there be not ſo many places of figures in the quotient, the deficiency muſt be ſupplied by prefixing ſo many cyphers before the quotient figures.

Examples.

1. Reduce $\frac{1}{8}$ to a decimal. $8)\underline{1,000}$
$\underline{,125}$ *Anſwer*.

2. Reduce $\frac{3}{8}$, $\frac{5}{8}$ and $\frac{2}{3}$ to Decimals. *Anſwers.* ,375. ,625. ,666+
3. Reduce $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{7}{8}$ to Decimals.
Anſwers. ,25. ,5. ,75. ,333+. ,8. ,833+. ,875.
4. Reduce $\frac{5}{19}$, $\frac{27}{39}$, $\frac{12}{480}$ and $\frac{9}{36}$ to Decimals.
Anſwers. ,263+. ,692+. ,025. ,25.
5. Reduce $\frac{7}{375}$, $\frac{9}{1129}$, and $\frac{5}{1875}$ to Decimals.
Anſwers. ,0186+. ,00797+. ,00266+.

Case 2.

To reduce numbers of different denominations, as of Money, Weight and Meaſure, to their equivalent decimal values.

Rule.*

1. Write the given numbers perpendicularly under each other for dividends, proceeding orderly from the leaſt to the greateſt.

2. Opposite to each dividend, on the left hand, place ſuch a number for a diviſor as will bring it to the next ſuperior denomination, and draw a line perpendicularly between them.

3. Begin with the higheſt, and write the quotient of each diviſion, as decimal parts, on the right hand of the dividend next below it, and ſo on, till they are all uſed, and the laſt quotient will be the decimal ſought.

1. Reduce 17*s.* 8*d.*$\frac{3}{4}$ to the decimal of a pound.

4	3,
12	8,75
20	17,729166 &c.
	,886458 &c. the decimal required.

Here, in dividing 3 by 4, I ſuppoſe 2 cyphers to be annexed to the 3, which makes it 3,00, and ,75 is the quotient, which I write againſt 8 in the next line; this quotient, viz. 8,75 being pence and decimal parts of a penny, I divide them by 12, which brings them to ſhillings and decimal parts, I therefore divide by 20, and (there being no whole number) the quotient is decimal parts of a pound.

2. Reduce

* The reaſon of the rule may be explained from the firſt example; thus, three farthings are $\frac{3}{4}$ of a penny, which, reduced to a decimal, is ,75; conſequently, 8$\frac{3}{4}$*d.* may be expreſſed, 8,75*d.* but 8,75 is $\frac{875}{100}$ of a penny $=\frac{875}{1200}$ of a ſhilling, which, reduced to a decimal, is ,729166*s.*+ In like manner, 17,729166*s.*+ are $\frac{17729166}{1000000}$ $=$,886458+ as by the rule.

2. REDUCE 1, 2, 3, 4, and ſo on to 19 ſhillings, to decimals.

Shillings.	1	2	3	4	5	6	7	8	9	10
Anſwers.	,05.	,1.	,15.	,2.	,25.	,3.	,35.	,4.	,45.	,5.
Shil.	11	12	13	14	15	16	17	18	19	
Anſ.	,55.	,6.	,65.	,7.	,75.	,8.	,85.	,9.	,95.	

Here, when the ſhillings are even, half the number, with a point prefixed, is their decimal expreſſion; but if the number be odd, annex a cypher to the ſhillings, and then halving them, you will have their decimal expreſſion.

† 3. REDUCE 1, 2, 3, and ſo on to 11 pence, to the decimals of a ſhilling.

Pence.	1	2	3	4	5	6
Anſwers.	,083+.	,166.	,25.	,333+.	,416+.	,5.
Pence.	7	8	9	10	11	
Anſ.	,583+.	,666+.	,75.	,833+.	,916+.	

4. REDUCE 1, 2, 3, &c. to 11 pence, to the decimals of a pound.

Pence.		1	2	3	4	5
Anſwers.		,00416+.	,0083+.	,0125.	,01666+.	,0208+.
Pence.	6	7	8	9	10	11
Anſ.	,025.	,02916+.	,0333+.	,0375.	,0466+.	,04583+.

5. REDUCE 1, 2 and 3 farthings to the decimals of a penny.
1qr.=,25*d.* *2qrs.*=,5*d.* *and 3qrs.*=,75*d.* *Anſwers.*

6. REDUCE 1, 2 and 3 farthings to the decimals of a ſhilling.
Anſw. *1qr.* =,02083+*s.* *2qrs.*=,04166+*s.* *3qrs.*=,0625*s.*

7. REDUCE 1, 2 and 3 farthings to the decimals of a pound.
Anſw. *1qr.*=,0010416.+£. *2qrs.*=,002083+£. *3qrs.*=,003125£.

8. REDUCE 13*s.* 5½*d.* to the decimal of a pound.
Anſw. ,6729+.

9 REDUCE 7*Cwt.* 3*qr.* 17℔. 10*oz.* 12*dr.* to the decimal of a Tun.
Anſw. ,39538+.

10. REDUCE 10*oz.* 13*pwt.* 9*gr.* to the decimal of a pound Troy?
Anſw. ,8890625.

11. REDUCE 3*qrs.* 3*n.* to the decimal of a yard.
Anſw. ,9375.

12. REDUCE 5*fur.* 12*po.* to the decimal of a mile.
Anſw. ,6625.

13. REDUCE 55*m.* 37*ſec.* to the decimal of a day.
Anſw. ,03862+.

CASE 3.

To find the decimal of any number of ſhillings, pence and farthings by Inſpection.

RULE.

† THE anſwers to this queſtion are the ſame as the decimal parts of a foot.

RULE. ‖

1. WRITE *half* the greatest even number of shillings for the first decimal figure.

2. LET the farthings in the given pence and farthings possess the second and third places; observing to increase the *second* place, or place of *hundredths*, by 5, if the shillings be odd, and the third place by 1, when the farthings exceed 12, and by 2, when they exceed 36.

3. IF more than three places be needful, divide half the number of farthings in the given pence and farthings (rejecting 24*qrs.* first, if there be so many) by 12, and the quotient, written after the three places before found, will give the decimal required.

EXAMPLES.

1. FIND the decimal of 13*s.* $9\frac{3}{4}$*d.* by Inspection.

	,6 . .	$=\frac{1}{2}$ *of* 12*s.*
	5	*for the odd shilling.*
	39	*= the farthings in* $9\frac{3}{4}$*d.*
Add	2	*for the excess of* 36.
	,691	= *decimal required.*

2. FIND, by Inspection, the decimal expressions of 18*s.* $3\frac{1}{4}$*d.* and 17*s.* $8\frac{1}{2}$*d.* *Answ.* £. ,914 *and* £. ,885.

3. VALUE the following sums, by Inspection, and find their total, *viz.* 15*s.* 3*d.* + 8*s.* $11\frac{1}{2}$*d.* + 10*s.* $6\frac{1}{4}$*d.* + 1*s.* $8\frac{1}{2}$*d.* + $\frac{1}{2}$*d.* + $2\frac{3}{4}$*d.* *Answ.* £. 1,834 *the total.*

CASE 4.

To find the value of any given decimal in the terms of the Integer.

RULE.

1. MULTIPLY the decimal by the number of parts in the next less denomination, and cut off so many places for a remainder, to the right hand, as there are places in the given decimal.

2. MULTIPLY the remainder by the next inferior denomination, and cut off a remainder as before.

3. PROCEED

‖ THE invention of the rule is as follows: As Shillings are so many 20ths of a pound, half of them must be so many tenths, and consequently take the place of tenths in the decimal; but when they are odd, their half will always consist of two figures, the first of which will be half the even number, next less, and the second a 5: Again, farthings are so many 960ths of a pound, and had it happened that 1000, instead of 960, had made a pound, it is plain any number of farthings would have made so many thousandths, and might have taken their place in the decimal accordingly. But 960 increased by $\frac{1}{24}$ part of itself, is = 1000, consequently, any number of farthings, increased by their $\frac{1}{24}$ part, will be an exact decimal expression for them: whence, if the number of farthings be more than 12, $\frac{1}{24}$ part is greater than $\frac{1}{2}$*d.* and, therefore, 1 must be added; and when the number of farthings is more than 36, $\frac{1}{24}$ part is greater than $1\frac{1}{2}$*d.* for which 2 must be added.

3. PROCEED in this manner through all the parts of the integer, and the ſeveral denominations, ſtanding on the left hand, make the anſwer.

EXAMPLES.

1. FIND the value of ,73968 of a pound.

```
 ,73968
     20
 ------
14,79360
     12
 ------
 9,52320
      4
 ------
 2,09280
```

2,09280 *Anſw.* 14*s.* $9\frac{1}{2}$*d.*

2. WHAT is the value of ,679 of a ſhilling? *Anſw.* $8\frac{1}{7}$*d.*

3. WHAT is the value of ,9999£.? *Anſw.* 19*s.* $11\frac{3}{4}$*d.* $\frac{9}{10}$—*or* £1.

4. WHAT is the value of ,617 of a *Cwt.*? *Anſw.* 2*qrs.* 13℔. 1*oz.* $10\frac{6}{10}$*dr.*

5. WHAT is the value of ,8593 of a ℔. Troy? *Anſw.* 10*oz.* 6*pwt.* 5*gr.*

6. WHAT is the value of ,397 of a yard? *Anſw.* 1*qr.* 2*n.*

7. WHAT is the value of ,8469 of a degree? *Anſw.* 58*m.* 6*fur.* 35*po.* 0*ft.* 11*in.*

8. WHAT is the value of ,569 of a year? *Anſw.* 207*da.* 16*h.* 26*m.* 24*ſec.*

9. WHAT is the value of ,713 of a day? *Anſw.* 17*h.* 6*m.* 43*ſec.*

CASE 5.

To find the value of any decimal of a pound by inſpection.

RULE.

DOUBLE the firſt figure, or place of tenths, for ſhillings, and if the ſecond figure be 5, or more than 5, reckon another ſhilling; then, after the 5 is deducted, call the figures in the ſecond and third places ſo many farthings, abating 1 when they are above 12, and 2 when above 36, and the reſult will be the anſwer.

Note. WHEN the Decimal has but 2 figures, if any thing remain after the ſhillings are taken out, a cypher muſt be annexed to the right hand, or ſuppoſed to be ſo.

EXAMPLES.

1. FIND the value of ,876£. by inſpection.

16*s.* = *double of* 8.
1*s.* *for the* 5 *in the ſecond place, which is to be taken out of* 7.
and $6\frac{1}{2}$*d.* = 26 *farthings remain, to be added.*
deduct $\frac{1}{4}$*d.* *for the exceſs of* 12.

17*s.* $6\frac{1}{4}$*d.* *the anſwer.*

2. FIND

2. Find, by inspection, the value of ,49£.

8*s.* - = *double of* 4.

1*s.* - *for the* 5 *in the place of hundredths.*

10*d.* = 40 *farthings, a* 0 *being annexed to the remaining* 4.

Deduct ½*d. for the excess of* 36.

9*s.* 9½*d. the answer.*

3. Find the value of ,097£. by inspection. *Answ.* 1*s.* 11¼*d.*

4. Value the following decimals by inspection, and find their sum, *viz.* ,785£. + ,537£. + ,916£. + ,74£. + ,5£. + ,25£. + ,09£. + ,008£. *Answ.* £.3 16*s.* 4¾*d.*

FEDERAL MONEY.

The Pupil being well acquainted with Decimals, it will be proper to introduce here an account of the Federal Money, as settled by Congress the 8th. of August, 1786, when it was "*Resolved,*

"That the Standard of the United States of America, for Gold and Silver, shall be eleven parts fine and one part alloy.

"That the *Money-Unit* of the United States (being by the Resolve of Congress of the 6th. of July 1785, a Dollar) shall contain, of fine silver, $375\frac{64}{100}$ grains.

"That the money of account, to correspond with the division of Coins, agreeably to the above Resolve, proceed in a decimal Ratio, agreeably to the forms and manner following, *viz.*

"Mill, the lowest money of account, of which 1000 shall be equal to the federal dollar, or money-unit, - - - 0,001.

"Cent, the highest copper-piece, of which 100 shall be equal to the federal dollar, - - - - - - 0,010.

"Dime, the lowest Silver Coin, of which 10 shall be equal to the dollar, - - - - - - 0,100.

"Dollar, the highest Silver Coin, - - - 1,000.

"That betwixt the Dollar and the lowest Copper Coin, as fixed by the Resolve of Congress of the 6th. of July, 1785, there shall be *three* silver Coins, and *one* copper Coin.

"That the silver Coins shall be as follow: One Coin containing $187\frac{82}{100}$ grains of fine silver, to be called a *Half-dollar*: One Coin containing $75\frac{128}{1000}$ grains of fine silver, to be called a *double-Dime*: and one Coin containing $37\frac{564}{1000}$ grains of fine silver, to be called a *Dime.*

"That the two Copper Coins shall be as follow: One equal to the one hundredth part of the federal Dollar, to be called *A Cent*; and one equal to the two hundredth part of the federal Dollar, to be called *A Half Cent.*

"That 2¼ ℔. Avoirdupois weight of Copper, shall constitute 100 Cents.

"That

"THAT there ſhall be two gold Coins: One containing $246\frac{268}{1000}$ grains of fine gold, equal to 10 Dollars, to be ſtamped with the impreſſion of the American Eagle, and to be called an *Eagle:* One containing $123\frac{134}{1000}$ grains of fine gold, equal to 5 dollars, to be ſtamped in like manner, and to be called a *Half-Eagle.*

"THAT the Mint-price of a pound troy-weight, of uncoined ſilver, eleven parts fine and one part alloy, ſhall be 9 dollars, 9 dimes and 2 Cents.

"THAT the Mint-price of one pound troy-weight of uncoined Gold, eleven parts fine and one part alloy, ſhall be 209 dollars, 7 dimes and 7 Cents."

As the Money of Account proceeds in a decuple, or ten-fold proportion, ſo any number of Dollars, Dimes, Cents and Mills, is ſimply the expreſſion of Dollars and Decimal parts of a Dollar:—Thus 9 Dollars and 8 Dimes are expreſſed $9,8=9\frac{8}{10}$ *doll.*—12 Dollars, 4 Dimes and 7 Cents thus, $12,47=12\frac{47}{100}$ *dol.* 20 Dollars, 3 Dimes, 4 Cents and 5 Mills, thus $20,345=20\frac{345}{1000}$ *dol.*—100 Dollars and 9 Mills, thus $100,009=100\frac{9}{1000}$ *dol.* and 50 dollars, 5 Cents, thus, $50,05=50\frac{5}{100}$ *dol.* wherefore, it is, in all reſpects, added, ſubtracted, multiplied and divided, the ſame as Decimals; and, of all Coins, it is the moſt ſimple.

			marked.		*Mills.*	*Cent.*	*Dime.*	*Dol.*	*Eag.*
Note.	10 Mills	*make one*	Cent.	*m.* *c.*	10 =	1			
	10 Cents		Dime.	*d.*	100 =	10 =	1		
	10 Dimes		Dollar.	*D.*	1000 =	100 =	10 =	1	
	10 Dollars		Eagle.	*E.*	10000 =	1000 =	100 =	10 =	1.

ADDITION *of the* FEDERAL MONEY.

ADD $25\frac{1}{2}$ Eagles; 7 Dollars, 8 Dimes, 3 Cents, 4 Mills; 125 Dollars, 8 Cents; 5 Eagles, 9 Mills; 18 Dollars, 7 Cents and 4 Mills together.†

	Eagles. Dollars. Dimes. Cents. Mills.
1ſt.	255
2d.	7,834
3d.	125,080
4th.	50,009
5th.	18,074
Sum	455,997

Note, That Dollars occupy the firſt place at the left hand of the comma; and Eagles, all the places at the left of Dollars: But Eagles and Dollars, reckoned together, expreſs the number of Dollars contained in the ſum, as 349 is 34 Eagles and 9 Dollars; equal to 349 Dollars, &c.

N SUBTRACTION.

† IT may be obſerved that the ſum exhibits the particular number of each different piece of money contained in it, viz. 455997 Mills $=45599\frac{7}{10}$ Cents $=4559\frac{97}{100}$ Dimes $=455\frac{997}{1000}$ Dollars $=45\frac{5997}{10000}$ Eagles = 4 5 5,9 9 7.
E.D.d.c.m.

ALSO,

SUBTRACTION.

	E.D.d.c.m.	D. d.c.m.	D.d.c.m.
From	13479,8 1 5	495641,001	798,
Take	8985,946	218796,795	459,379
Rem.	4493,8 6 9	276844,206	338,621
Proof.	13479,8 1 5		

MULTIPLICATION.

WHEN you have pointed off the decimals in the product, according to the rule in Multiplication of Decimals, all beyond the Mills, or third place of decimals, are decimal parts of a Mill.

BOUGHT 37 Horses for shipping, at 48, 5 7 3 (D. d. c. m.) per head: What came they to?

```
      D.d. c. m.
      48,5 7 3
           3 7
      --------
      340011
     145719
     ---------
Answ. 1797,201
      D.d.c.m.
```

```
          D.d. c.m.
   Mult.  45,8 4 6
   by     23,0 9 4
          --------
          183384
         412614
       1375380
       91692
      -----------
Answ. 1058,767 524/1000
```

DIVISION.

1. IF 1000 Oranges cost 10 dollars, what is that a-piece?

```
        D.d.c. D.d.c.
1000)10,0 0(0,0 1 Answ.‡
     10 0 0
     ------
```

2. IF 3730 bushels of Corn cost 2025,39 dollars; what is that per Bushel?

```
                 D.c. m.
3730)2025,39(0,5 4 3 Answ.
     1865 0
     ------
      160 39
      149 20
      ------
       11 190
       11 190
```

3. DIVIDE 12976 dollars between 7 men.

```
     7)12976
     --------
1853,714 2/7 Answ. in dol. dim. &c.
```

DECIMAL

ALSO, the names of the Coins, less than a dollar, are significant of their values. FOR the *Mill*, which stands in the 3d place at the right hand of the comma, or place of thousandths, is contracted from *Mille*, the Latin for *Thousand*:—*Cent*, which occupies the second place, or place of Hundredths, is an abbreviation of *Centum*, the Latin for *Hundred*:—and *Dime*, which is in the first place, or place of tenths, is derived from *Disme*, the French for *Tenths*.

‡ AND here I would remind the learner, that, when he has brought down all his whole numbers, or dollars, in the dividend, he must place a comma in the quotient; and if, when he has brought down the next figure, he cannot have the divisor once, he must place a cypher at the right hand of the comma, in the place of dimes.

DECIMAL TABLES OF COIN, WEIGHT *and* MEASURE.

TABLE I. COIN.

£1. the Integer.

Shil.	dec.	Shil.	dec.
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,8	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		

Pence	Decimals
11	,045833
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Farthings	Decimals
3	,003125
2	,0020833
1	,0010416

TABLE II.

COIN & Long Meas. 1 Shill. & 1 Foot the Integer.

Pence & Inches.	Decimals.
11	,916666
10	,833333
9	,75
8	,666666
7	,583333
6	,5
5	,416666
4	,333333
3	,25
2	,166666
1	,083333

Farthings	Decimals
3	,0625
2	,041666
1	,020833

TABLE III.

TROY WEIGHT. 1lb. the Integer. Ounces the same as TABLE II.

Penny-weights.	Decimals.
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Grains.	Decimals.
12	,002083
11	,00191
10	,001736
9	,001562
8	,001389
7	,001215
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173

1 Oz. the Integer. Pennyweights the same as Shillings in the first Table.

Grains.	Decimals.
12	,025
11	,022916
10	,020833
9	,01875
8	,016666
7	,014583
6	,0125
5	,010416
4	,008333
3	,00625
2	,004166
1	,002083

TABLE IV.

AVOIRDUPOIS WT. 112lb. the Integer.

Qrs.	Decimals.
3	,75
2	,5
1	,25

Pounds.	Decimals.
27	,241071
26	,232143
25	,223214
24	,214286
23	,205357
22	,196428
21	,1875
20	,178571
19	,169643
18	,160714
17	,151786
16	,142857
15	,133928
14	,125
13	,116071
12	,107143
11	,098214
10	,089286
9	,080357
8	,071428
7	,0625
6	,053571
5	,044643
4	,035714
3	,026786
2	,017857
1	,008928

Ounces.	Decimals.
15	,008370
14	,007812
13	,007254
12	,006696
11	,006138
10	,00558
9	,005022
8	,004464
7	,003906
6	,003348
5	,00279
4	,002232
3	,001674
2	,001116
1	,000558

qrs. of ozs.	Decim.
3	,000418
2	,000279
1	,000139

TABLE V.

AVOIRD. WEIGHT. 1lb. the Integer.

Ounces.	Decimals.
15	,9375
14	,875
13	,8125
12	,75
11	,6875
10	,625
9	,5625
8	,5
7	,4375
6	,375
5	,3125
4	,25
3	,1875
2	,125
1	,0625

Drams.	Decimals.
15	,059493
14	,055587
13	,051681
12	,047775
11	,043868
10	,039962
9	,036056
8	,03215
7	,027343
6	,023437
5	,019531
4	,015625
3	,011718
2	,007812
1	,003906

TABLE VI.

CLOTH MEASURE. 1 Yard the Integer.

Quarters.	Decimals.
3	,75
2	,5
1	,25

Nails.	Decimals.
3	,1875
2	,125
1	,0625

TABLE VII.

LONG MEASURE. 1 Mile the Integer.

Yards.	Decimals.
1000	,568182
900	,511364
800	,454545
700	,397727
600	,34
500	,284091
400	,227272
300	,170454
200	,113636
100	,056818
90	,051136
80	,045454
70	,039773
60	,034091
50	,028409
40	,022727
30	,017045
20	,011364
10	,005682
9	,005114
8	,004545
7	,003977
6	,003409
5	,002841
4	,002273
3	,001704
2	,001136
1	,000568

Feet.	Decimals.
2	,0003787
1	,0001892

Inches.	Decimals.
6	,0000947
5	,000079
4	,0000632
3	,0000474
2	,0000316
1	,0000158

TABLE VIII.
LIQUID MEAS'R.
1 Gallon the Integer.
Quarts the same as qrs. in TABLE VI.

1 Pint.	,125
3 Gills.	,09375
2	,0625
1	,03125

TABLE IX.
TIME.
1 Year the Integer.
Months the same as Pence in TABLE II.

Days.	Decimals.
365	1,000000
300	,821928
200	,547945
100	,273973
90	,246575
80	,219178
70	,191781
60	,164383
50	,136986
40	,109589
30	,082192
20	,054794
10	,027397
9	,024657
8	,021918
7	,019178
6	,016438
5	,013698
4	,010959
3	,008219
2	,005479
1	,002739

1 Day the Integer.

Hours.	Decimals.
23	,958333
22	,916666
21	,875
20	,833333
19	,791666
18	,75
17	,708333
16	,666666
15	,625
14	,583333
13	,541666
12	,5
11	,458333
10	,416666
9	,375
8	,333333
7	,291666
6	,25
5	,208333
4	,166666
3	,125
2	,083333
1	,041666

Minutes.	Decimals.
30	,020833
20	,013888
10	,006944
9	,00625
8	,005555
7	,004861
6	,004166
5	,003472
4	,002777
3	,002083
2	,001388
1	,000694

COMPOUND MULTIPLICATION‡

Is extremely useful in finding the value of Goods, &c.—And as in Compound Addition we carry from the lowest denomination to the next higher, so we begin and carry in compound multiplication; one general rule being to multiply the price by the quantity.

CASE I.

When the quantity does not exceed 12 *yards, pounds, &c.* set down the price of 1, and place the quantity underneath the least denomination, for the multiplier, and, in multiplying by it, observe the same rules for carrying from one denomination to another, as in Compound Addition.

INTRODUCTORY EXAMPLES.

	1. £.	s.	d.	2. £.	s.	d.	3. £.	s.	d.	4. £.	s.	d.
Multiply	15	17	1	25	12	8	45	9	10	67	18	$6\frac{1}{2}$
by			2			3			4			5

5.			6.			7.			8.		
19	6	$8\frac{1}{4}$	13	12	11	31	16	$8\frac{1}{4}$	12	17	$10\frac{1}{2}$
		6			7			8			9

9.

‡ THE product of a number, consisting of several parts or denominations, by any simple number whatever, will be expressed by taking the product of that simple number, and each part by itself, as so many distinct questions;—Thus £.33 15*s.* 9*d.* multiplied by 5, will be £.165 77*s.* 45*d.* = (by taking the shillings from the pence, and the pounds from the shillings, and placing them in the shillings and pounds respectively,) £.168 18*s.* 9*d.* and this will be true when the multiplicand is any compound number whatever.

9.			10.			11.			12.		
£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
4	13	$4\frac{3}{4}$	8	15	$11\frac{3}{4}$	6	19	$4\frac{1}{2}$	14	17	$8\frac{1}{4}$
		10			11			12			9
									133	19	$2\frac{1}{4}$

In the last example, I say, 9 times 1 is 9 farthings = $2\frac{1}{4}d.$ I set down $\frac{1}{4}$ and carry 2, saying, 9 times 8 is 72, and 2 I carry, makes 74 pence = 6*s.* 2*d.* I set down 2 in the pence, and carry 6; then, 9 times 7 (the unit-figure in the shillings) is 63, and 6 I carry is 69, I set down 9 under the unit-figure of the shillings, and carry 6, saying, 9 times 1 is 9, and 6 I carry is 15, then I halve 15, which is 7 and 1 over, which I set in the tens' place in shillings, and that, with the 9, makes 19 shillings; then I carry the 7 as pounds: lastly, 9 times 4 is 36, and 7 I carry, are 43 pounds; I set down 3 and carry 4, saying, 9 times 1 is 9, and 4 I carry makes 13, which I set down, and the product is £.133 19*s.* $2\frac{1}{4}d.$

Practical Questions.

1. What will 9 yards of Cloth, at 5*s.* 4*d.* *per* yard, come to?

	£.	s.	d.	
	0	5	4	*price of one yard.*
Multiplied by			9	*yards.*
Answ.	£.2	8	0	*price of 9 yards.*

Questions.			s.	d.	
2d.	3	yards, at	15	4	*per yard.*
3d.	6	—— at	9	10	
4th.	5	—— at	29	6	
5th.	9	—— at	13	$7\frac{1}{2}$	
6th.	7	—— at	39	$10\frac{3}{4}$	

Case 2.

When the multiplier, that is, the quantity, is above 12, you must multiply by two such numbers, as, when multiplied together, will produce the given quantity.

Examples.

1. What will 144 yards of Cloth cost, at 3*s.* $5\frac{1}{2}d.$ *per* yard?

	£.	s.	d.	
	0	3	$5\frac{1}{2}$	*price of one yard.*
Multiplied by			12	
Produces	2	1	6	*price of 12 yards.*
Multiplied by			12	
Answer	£.24	18	0	*price of 144 yards*

Questions.

Questions.							*Answers.*		
			s.	d.			£.	s.	d.
2d.	24 yards	at	6	$3\frac{3}{4}$	per yard.	=	7	11	6
3d.	27 ——	at	9	10	——	=	13	5	6
4th.	44 ——	at	12	$4\frac{1}{2}$	——	=	27	4	6
5th.	55 ——	at	8	$3\frac{1}{4}$	——	=	22	14	$10\frac{3}{4}$
6th.	72 ——	at	19	11	——	=	71	14	—

CASE 3.

When the quantity is such a number, as that no two numbers in the table will produce it, exactly; Then multiply by two such numbers as come the nearest to it; and for the number wanting, multiply the given price of one yard by the said number of yards wanting, and add the products together for the answer; but if the product of the two numbers exceed the given quantity, then find the value of the overplus, which subtract from the last product, and the remainder will be the answer.

EXAMPLES.

1. WHAT will 47 yards of Cloth, at 17*s.* 9*d.* *per* yard, come to?

	£.	s.	d.	
	0	17	9	*price of one yard.*
Multiplied by			5	
Produces	4	8	9	*price of 5 yards.*
Multiplied by			9	
Produces	39	18	9	*price of 45 yards.*
Add	1	15	6	*price of two yards.*
Answ. £.	41	14	3	*price of 47 yards.*

Note, This may be performed by first finding the value of 48 yards, from which if you subtract the price of 1, the remainder will be the answer, as above.

Questions.							*Answers.*		
	Yds.		s.	d.			£.	s	d.
2d.	75	at	5	$7\frac{1}{2}$	*per* yard.	=	21	1	$10\frac{1}{2}$
3d.	$67\frac{1}{2}$	—	16	$3\frac{1}{4}$	——	=	54	18	$3\frac{1}{4}$
4th.	59	—	9	7	——	=	28	5	5
5th.	$135\frac{1}{4}$	—	43	4	——	=	293	—	10
6th.	$112\frac{3}{4}$	—	15	$11\frac{3}{4}$	——	=	90	1	$7\frac{3}{4}$

CASE 4.

When the quantity is any number above the Multiplication Table; Mul- the price of 1 yard by 10, which will produce the price of 10 yards: *this* product, multiplied by 10, will give the price of 100 yards; then if the quantity do not exceed hundreds, you must multiply the price of one hundred by the number of hundreds in your question; the

the price of ten by the number of tens; and the price of unity, or 1, by the number of units: lastly, add these several products together, and the sum will be the answer.

EXAMPLES.

1. WHAT will 359 yards of Cloth, at 4s. 7½d. *per* yard amount to

	£.	s.	d.	
	—	4	7½	*price of one yard.*
			10	
	2	6	3	*price of* 10 *yards.*
			10	
	23	2	6	*price of* 100 *yards.*
			3	
	69	7	6	*price of* 300 *yards.*
5 *times the price of* 10 *yds.* =	11	11	3	*price of* 50 *yards.*
9 *times the price of* 1 *yd.* =	2	1	7½	*price of* 9 *yards.*
Answer	£.83	—	4½	*price of* 359 yards.

Questions.		s.	d.		£.	s.	d.
2d.	297 yards, at	17	3½	*per* yard. =	204	8	1½
3d.	473 ———	9	11¼	——— =	235	5	5¾
4th.	512 ———	15	10	——— =	405	6	8
5th.	624 ———	12	8	——— =	395	4	-
6th.	765 ———	19	9½	——— =	757	-	7½

CASE 5.

WHEN the quantity does not exceed 200, nor the price 12 pence, then by the pence-table, find what it comes to, at one penny per yard, &c. and multiplying this sum by the number of pence in the price, the product will be the answer.

EXAMPLES.

1. WHAT will 129 yards cost, at 9¾d. *per* yard?

	£.	s.	d.	
129 *pence* =	—	10	9	*the price at* 1d. *per yard.*
			9	
	4	16	9	*the price at* 9d. *per yard.*
Half of 10s. 9d. =		5	4½	*the price at* ½d. *per yard.*
One fourth of 10s. 9d. =		2	8¼	*the price at* ¼d. *per yard.*
Answer.	£.5	4	9¾	*the price at* 9¾d. *per yard.*

Questions.

Questions.	*Yds.*					*Answers.* £.	*s.*	*d.*
2d.	$148\frac{1}{2}$	at	$11\frac{1}{4}d.$	*per* yard.	=	6	19	$3\frac{1}{2}$
3d.	75	—	7	———	=	2	3	9
4th.	$63\frac{1}{2}$	—	$3\frac{3}{4}$	———	=	-	19	10
5th.	$123\frac{3}{4}$	—	$2\frac{1}{2}$	———	=	1	5	$9\frac{1}{4}$
6th.	137	—	$4\frac{1}{4}$	———	=	2	8	$6\frac{1}{4}$

C A S E 6.

To find the value of one hundred weight:—As 112 is the groſs hundred, ſo 112 farthings are = 2*s.* 4*d.* and 112 pence = 9*s.* 4*d.* therefore if the price be farthings, or not more than 3*d.* multiply 2*s.* 4*d.* by the farthings in the price of 1 ℔. or, if the price be pence, multiply 9*s.* 4*d.* by the pence in the price of 1 ℔. and in either caſe the product will be the anſwer.

E X A M P L E S.

1. WHAT will 1 *Cwt.* of Chalk come to, at $1\frac{1}{2}d.$ *per* ℔. ?

		£.	*s.*	*d.*	
112 *farthings*	=	-	2	4	*price of* 1 *Cwt. at* $\frac{1}{4}d.$ *per* ℔.
$1\frac{1}{2}d.$	=			6	*farthings in the price.*
Anſwer.		£. -	14	-	*price of* 1 *Cwt.* at $1\frac{1}{2}d.$ *per* ℔.

	£.	*s.*	*d.*	
2. 1 *Cwt.* of Tin at $2\frac{1}{4}d.$ per ℔. ?		2	4	*price of* 1 *Cwt. at* $\frac{1}{4}d.$ *per* ℔.
			9	*farthings in the price of* 1 ℔.
Anſwer.	£. 1	1	-	*price of* 1 *Cwt. at* $2\frac{1}{4}d.$ *per* ℔.

	£.	*s.*	*d.*	
3. 1 *Cwt.* of Lead at 7*d.* *per* ℔. ?		9	4	*price of* 1 *Cwt. at* 1*d. per* ℔.
			7	*pence in the price of* 1 ℔.
	£. 3	5	4	*price of* 1 *Cwt. at* 7*d. per* ℔.

Questions. *Cwt.*						*Answers.* £.	*s.*	*d.*
4th.	1	of Copper at	- $\frac{3}{4}$	*per* ℔.	=	-	7	-
5th.	1	ditto ———	$2\frac{1}{2}d.\frac{1}{2}$	———	=	1	4	6
6th.	1	ditto ———	$4\frac{1}{2}$		=	2	2	-

C A S E 7.

To find the value of two, or more, hundreds, by having the price of one pound:—Firſt, find the price of 1 *Cwt.* by the laſt Caſe, and then

then proceed to find the value of the whole by Case 1st. or 2d. as the question may require.

EXAMPLES.

1. WHAT is the value of $5\frac{1}{4}$ *Cwt.* of Sugar, at 6*d. per* ℔.?

	£.	s.	d.	
	-	9	4	*price of* 1 *Cwt. at* 1*d. per* ℔.
			6	
	2	16	-	*price of ditto at* 6*d. per* ℔.
			5	
	14	-	-	*price of* 5 *Cwt.*
1 *quarter of a Cwt.* =	—	14		*price of* $\frac{1}{4}$ *Cwt.*
Answer.	£.14	14		*price of* $5\frac{1}{4}$ *Cwt.*

Questions.					*Answers.*		
	Cwt.				£.	s.	d.
2d.	4	of Sugar at	$2\frac{1}{2}$*d.*	*per* ℔. =	4	13	4
3d.	$8\frac{1}{2}$	———	5	——— =	19	16	8
4th.	7	———	$4\frac{3}{4}$	——— —	15	10	4
5th.	$4\frac{3}{4}$	———	$3\frac{1}{2}$	——— =	7	15	2
6th.	$9\frac{1}{2}$	———	8	——— =	35	9	4

CASE 8.

To find the value of a hundred-weight, when the price of 1 ℔. *is any number of pounds and shillings; or shillings, pence and farthings:*— Multiply the price of 1 ℔. by 7, its product by 8, and *this* product by 2; which last product will be the answer required.

EXAMPLES.

1. WHAT will 1 *Cwt.* of Tobacco cost, at 5*s.* $7\frac{1}{2}$*d.* per ℔.?

	£.	s.	d.	
	-	5	$7\frac{1}{2}$	*price of* 1 ℔.
			7	
	1	19	$4\frac{1}{2}$	*price of* 7 ℔.
			8	
	15	15	-	*price of* 56 ℔. *or* $\frac{1}{2}$ *Cwt.*
			2	
Answer.	£.31	10	-	*price of* 112 ℔. or 1 *Cwt.*

O *Questions.*

Questions.					*Answers.*		
		s.	*d.*		£.	*s.*	*d.*
2d.	1 *Cwt.* at	3	$10\frac{1}{2}$	per ℔. =	21	14	-
3d.	1 ditto —	9	6	—— =	53	4	-
4th.	1 ditto —	16	$11\frac{1}{2}$	—— =	94	19	4
5th.	1 ditto —	19	$8\frac{1}{4}$	—— =	110	5	-
6th.	1 ditto — £.1	7	10	—— =	155	17	4

PRACTICAL QUESTIONS *in* WEIGHTS *and* MEASURES.

1. WHAT is the weight of 4 hogſheads of Sugar, each weighing 7*Cwt.* 3*qrs.* 19℔.? *Anſw.* 31*Cwt.* 2*qrs.* 20℔.

2. WHAT is the weight of 6 cheſts of Tea, each weighing 3*Cwt.* 2*qrs.* 9℔.? *Anſw.* 21*Cwt.* 1*qr.* 26℔.

3. IF I am poſſeſſed of $1\frac{1}{2}$ dozen of Silver Spoons, each weighing 3*oz.* 5*pwt.*—2 dozen of Tea-ſpoons, each weighing 15*pwt.* 14*gr.*—3 Silver Cans, each 9*oz.* 7*pwt.*—2 Silver Tankards, each 21*oz.* 15*pwt.* and 6 Silver Porrengers, each 11*oz.* 18*pwt.* Pray, what is the weight of the whole? *Anſw.* 18℔. 4*oz.* 3*pwt.*

4. IN 35 pieces of Cloth, each meaſuring $27\frac{3}{4}$ yards, how many yards? *Anſw.* $971\frac{1}{4}$ *yds.*

5. How much Brandy in 9 Caſks each, containing 45*gal.* 3*qts.* 1*pt.*? *Anſw.* 412*gal.* 3*qts.* 1*pt.*

6. IF I have 9 fields, each of which contains 12 acres, 2 roods and 25 poles; how many acres are there in the whole? *Anſw.* 113*A.* 3*r.* 25*p.*

COMPOUND DIVISION*

Is the dividing of numbers of different denominations: in doing which, always begin at the higheſt, and when you have divided that, if any thing remain, reduce it to the next lower denomination, and ſo on, till you have divided the whole, taking care to ſet down your quotient-figures under their reſpective denominations.

INTRODUCTORY EXAMPLES.

	1. £.	*s.*	*d.*		2. £.	*s.*	*d.*	3. £.	*s.*	*d.*
Divide	549	17	9	*by* 5	3)197	13	$7\frac{1}{2}$	4)731	5	$10\frac{1}{4}$
Quotient.	£109	19	$6\frac{1}{2}$							

4.

* To divide a number conſiſting of ſeveral denominations by any ſimple number whatever, is the ſame as dividing all the parts or members of which that number is compoſed by the ſame number. And this will be true when any of the parts are not an exact multiple of the diviſor; for by conceiving the number, by which it exceeds that multiple, to have its proper value by being placed in the next lower denomination, the dividend will ſtill be divided into parts, and the true quotient found as before: thus, £.41 17*s.* 6*d.* divided by 6, will be the ſame as £.36 117*s.* 42*d.* divided by 6, which is equal to £.6 19*s.* 7*d.* as by the rule.

4. £. s. d. 2)97 19 $10\frac{1}{2}$

5. £. s. d. 6)37 11 $4\frac{3}{4}$

6. £. s. d. 7)193 15 $9\frac{1}{2}$

7. £. s. d. 8)739 12 $1\frac{1}{2}$

8. £. s. d. 9)471 18 $10\frac{1}{2}$

9. £. s. d. 10)79 13 $9\frac{1}{2}$

10. £. s. d. 11)58 19 $11\frac{1}{2}$

11. £. s. d. 12)13 17 $9\frac{1}{2}$

In the firſt example, having divided the pounds, the 4, which remains, is 4 pounds, which are equal to 80 ſhillings, and 17 in the ſhillings make 97; I then find 5 is contained 19 times in 97, and 2 over: I ſet down 19 under the ſhillings, and reduce the 2 ſhillings, which remain, into pence, and they make 24, and the 9 pence, in the queſtion, added, make 33: then, how often 5 in 33; I find it 6 times, and 3 over: I ſet down 6 under the pence, and reduce the 3 pence, which remain, to farthings, and they make 12; then, how often 5 in 12; I find it to be twice: I therefore ſet down $\frac{1}{2}$*d.* and the 2, which remains, is $\frac{2}{5}$ of a farthing, which I make no account of.

12. *T. cwt. qr. lb. oz. dr.* 3)29 13 2 25 12 13

13. *T. cwt. qr. lb.* 4)6 11 3 19

14. *Cwt. qr. lb.* 5)14 1 12

15. *lb. oz. dr.* 6)10 13 9

16. *lb. oz.* 7)20 13

17. *lb. oz. pwt. gr.* 8)7 10 15 2

18. *lb. oz. pwt. gr.* 9)56 9 13 19

19. *lb. oz. pwt. gr.* 10)849 11 12 14

20. ℔. *oz. pwt. gr.* 12)529 10 7 14

21. ℔ ℥ ʒ ℈ *gr.* 2)37 10 5 1 12

22. ℔ ℥ ʒ ℈ *gr.* 5)37 11 6 2 17

23. ℔ ℥ ʒ ℈ *gr.* 9)348 9 4 0 19

24. *Yds. qr. n.* 4)76 3 2

25. *E.E. qr. n.* 5)97 4 1

26. *E Fl. qr. n.* 7)58 2 2

27. *E. Fr. qr. n.* 12)17 5 1

28. *Deg. m. f. p. ft. in. br.* 6)97 55 7 35 4 2 1

29. *M. f. p.* 7)5 5 19

30. *P. ft. in.* 8)9 13 10

31.	32.	33.	34.
Yds. ft. in.	Ac. r. p.	Yr. m. w. d. h. m. s.	Y. m. d.
9)8 2 11	3)76 3 37	4)25 10 3 5 21 39 45	5)5 9 25

35.	36.	37.
M. w. d. h. m.	M. d. h. m.	
6)6 3 5 10 29	7)9 21 12 45	8)3s 25° 55′ 25″

38.	39.
9)19° 45′ 38″	12)189° 37′ 29″

40. Suppose that two places lie east and west of each other, and it is found by observation that it is noon at the former 2 hours, 6′ 30″ sooner than at the latter; how many degrees are they asunder?

4′)2h 6′ 30″ *Reduce the hours and minutes to minutes, then, minutes divided by 4′, give degrees in the quotient.*

31° 37′ 30″ *Answ.*

41. The longitude of Cambridge is 4^h 44′ 17″, and that of Philadelphia, 5^h 0′ 43″; how many degrees difference?

5^h 0′ 43″
4 44 17

4′)0 16 26

4° 6′ 30″ *Answer.*

Practical Questions.

Case I.

Having the price of any number of yards, &c. within the pence-table, to find the price of unity, or 1 yard:—If the quantity do not exceed 12, proceed by setting down the price and dividing it by the quantity; which quotient will be the price of one yard, required; but if the quantity exceed 12, then divide by 2 such numbers, as, when multiplied together, will produce the quantity, and the last quotient will be the value of 1 yard, required.

Note. This case proves the first and second cases in Compound Multiplication.

1. If 9 yards of Cloth cost £.4 3s. 7½d. what is it *per* yard?

£. s. d.
9)4 3 7½

- 9 3½ *Answer.*

2.

2. If 7 Ells cost £.5 17*s.* 5*d.* what cost 1 Ell?
3. If 11 Sheep cost £.6 5*s.* 9*d.* what did each cost?
4. If 12 gallons of Rum cost £.8 11*s.* 9½*d.* what is it *per* gallon?
5. If 84 Cows cost £.253 13*s.* what is the price of each?

Answ. £.3 0*s.* 4⅔*d.*

6. If 132 bushels of Corn cost £.20 12*s.* 6*d.* what is that *per* bushel? *Answ.* 3*s.* 1½*d.*

Note. When the given quantity (or divisor) is large, and not a composite number, the operation may be performed by long Division.

Case 2.

Having the price of a hundred weight, to find the price of 1 ℔.—— Divide the given price by 8, that quotient by 7, and this quotient by 2, and the last quotient will be the price of 1 ℔ required.

1. If 1 *Cwt.* of Flax cost £.2 7*s.* 8*d.* what is that *per* ℔?

	£.	s.	d.	
8)	2	7	8	
7)	—	5	11½	
2)	—	-	10*d.* $0\frac{6}{7}$ *qr.*	
	—	-	$5\frac{3}{28}$*d.*	*price of one pound.*

2. At £.3 10*s.* *per Cwt.* what cost 1 ℔? *Ans.* 7½*d.*
3. At £.6 6*s.* *per Cwt.* what cost 1 ℔? *Ans.* 1*s.* 1½.
4. At £.42 11*s.* 8*d.* *per Cwt.* what cost 1 ℔? *Ans.* 7*s.* 7¼*d.*
5. At £.19 5*s.* *per Cwt.* what cost 1 ℔? *Ans.* 3*s.* 5¼*d.*

Case 3.

Having the price of several hundred weight, to find the price per ℔:— Divide the whole price by the number of hundreds, which will give the price *per Cwt.* and then proceed as in the last Case.

1. If 5 *Cwt.* of Sugar cost £.13 8*s.* 4*d.* what is that *per* ℔?

	£.	s.	d.	
5)	13	8	4	
8)	2	13	8	*price of* 1 *Cwt.*
7)	-	6	8½*d.*	*price of* 14*lb.* or ⅛ *Cwt.*
2)	-	-	11½*d.*	*price of* 2 *lb.* or $\frac{1}{56}$ *Cwt.*
	-	-	5¾	*price of* 1 *lb.*

2. If 8 *Cwt.* of Cocoa cost £.15 17*s.* 4*d.* what is that *per lb.*?

Answ. $4\frac{3}{14}$*d.*

3. If 3¼ *Cwt.* Sugar cost £.9 17*s.* 2*d.* what is that *per lb.*?

Answ. 6½*d.*

4.

4. If $1\frac{3}{4}$ *Cwt.* of Cotton-wool cost £.6 10*s.* 8*d.* what is that *per lb.*? *Answ.* 8*d.*

Note. This case proves the 7th, in Compound Multiplication.

Case 4.

Having the price of any number of yards, &c. to find the price of 1 *yard:*——Divide the price by the quantity, beginning at the highest denomination, and, if any thing remain, reduce it into the next, and every inferior denomination, and, at each reduction, divide as before, remembering, each time, to add the odd shillings, pence, &c. if there be any, and you will have the value of unity required.

Note. If there be $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ of a yard, pound, &c. multiply both the price and quantity by 4, and then proceed as above directed.

1. If $95\frac{1}{2}$ *lb.* of Figs cost £.16 13*s.* $6\frac{3}{4}$*d.* what are they *per lb.*?

	lb.			£.	*s.*	*d.*	
Quantity =	$95\frac{1}{2}$		*Price* =	16	13	$6\frac{3}{4}$	
Mult. by	4					4	
Produces	382	*for a divisor.*	*Product.*	£.66	14	3	*for a dividend.*

```
       £.   s.  d.  £.  s.  d.
382)  66   14   3 (0   3   5¾ 250/382 per lb.
      20
      ----
382)1334(3
    1146
    ----
     188
      12
    ----
382)2259(5
    1910
    ----
     349
       4
    ----
382)1396(3
    1146
    ----
     250
```

2. If 147 bushels of Rye cost £.47 12*s.* 6*d.* what is it *per* bushel? *Answ.* 6*s.* $5\frac{3}{4}$*d.*

3. If $33\frac{1}{4}$ yards of Baize cost £.25 13*s.* $9\frac{1}{2}$*d.* what is it *per* yard? *Answ.* 15*s.* $5\frac{1}{4}$*d.* $\frac{95}{133}$.

4. If $172\frac{3}{4}$ gallons of Wine cost £.43 5*s.* 2*d.* what is it *per* gallon? *Answ.* $5\frac{3}{691}$*s.*

5. If

5. If a dozen Sheep cost £.7 2*s.* 9*d.* what are they a-piece?
Answ. 11*s.* 10$\frac{1}{4}$*d.*

Note. This proves the 3d and 4th Cases in Multiplication.

Practical Questions *in* Money.

1. Divide £.273 9*s.* 4*d.* among 5 men and 4 women, and give the men twice as much as the women.

men. *women.*	£. s. d.	£. s. d.
5 *and* 4	*Divide by* 14)273 9 4(	19 10 8 = 1 *woman's share.*
Mult. by 2	14	4 *women.*
10 *shares.*	133	78 2 8 = *women's shares.*
Add 4 *women's shares.*	126	
		£.19 10 8
14 *the number of equal*	7	2
shares in the whole=Divisor.	20	
		£.39 1 4 = 1 *man's share.*
	14)149(10	5 *men.*
	14	
		£.195 6 8 = *men's share.*
	9	78 2 8 = *women's share.*
	12	
		£.273 9 4 *Proof.*
	14)112(8	
	112	

2. Divide £.120 17*s.* 4*d.* among 7 men and 7 women, and give the women 3 times so much as the men.

	£.	s.	d.	
Answer.	4	6	4	= *a man's share.*
	12	19	-	= *a woman's share.*

3. Divide £.39 12*s.* 5*d.* among 4 men, 6 women, and 9 boys: give each man double to a woman, and each woman double to a boy.

	£.	s.	d.	
Ans.	1	1	5	= *a boy's share.*
	2	2	10	= *a woman's ditto.*
	4	5	8	= *a man's ditto.*

4. Divide 5 guineas among 8 men:—give *A* 8*d.* more than *B*, and *B* 8*d.* more than *C*, &c. *Ans. H's share* = 15*s.*

RULES

For reducing the Federal Coin, and the Currencies of the several United States; also English, Irish, Canada, Nova-Scotia, Livres Tournois and Spanish milled Dollars, each to the *par* of all the others.

I. *To reduce New Hampshire, Massachusetts, Rhode-Island, Connecticut, and Virginia currency.*

1. *To New-York and North-Carolina currency.*

Rule.—Add one third to the New-Hampshire, &c. sum, and the sum total will be the New-York, &c. currency.

Reduce £.100 New-Hampshire, &c. to New-York, &c.

	£.		
3)	100		
+	33	6	8
£	133	6	8 *Ans.*

2. *To*

2. *To Pennsylvania, New-Jersey, Delaware & Maryland currency.*

Rule.—Add one fourth to the New-Hampshire, &c. sum.

REDUCE £.100 New-Hampshire, &c. to Pennsylvania, &c.

4)100
+ 25
£.125 *Ans.*

3. *To South-Carolina and Georgia currency.*

Rule.—Multiply the New-Hampshire, &c. sum by 7, and divide the product by 9, and the quotient is the answer.

REDUCE £.100 New-Hampshire, &c. to South-Carolina, &c.

100
7
9)700
£.77 15 $6\frac{2}{3}$ *Ans.*

4. *To English Money.*

Rule.—Deduct one fourth from the New-Hampshire, &c. sum.

REDUCE £.100 New-Hampshire, &c. to English Money.

4)100
— 25
£.75 *Ans.*

5. *To Irish Money.*

Rule.—Multiply the New-Hampshire, &c. sum by 13, and divide the product by 16.

REDUCE £.100 New-Hampshire, &c. to Irish Money.

100
4x3+ the given Sum.
400
3
1200
+100
16 = 4x4)1300
4)325
£.81 5 *Ans.*

6. *To Canada and Nova-Scotia currency.*

Rule.—Multiply the New-Hampshire, &c. sum by 5, and divide the product by 6.

REDUCE £.100 New-Hampshire &c. to Canada, &c.

100
5
6)500
£.83 6 8 *Ans.*

7. *To Livres Tournois.*

Rule.—Multiply the New-Hampshire &c. pounds by $17\frac{1}{2}$ & the product will be Livres :—Or, multiply the sum in shillings by 7 : divide the product by 8, and the quotient will be livres, sous &c.

REDUCE £.100 New-Hampshire, &c. to Livres Tournois.

100	*Or,*	100
$17\frac{1}{2}$		20
700		2000
100		7
50		8)14000
Ans. 1750 *Liv.*		
		Ans. 1750 *Livres.*

1*d.* = 1*sou.* $5\frac{1}{2}$*den.* 1*s.* = $17\frac{1}{2}$ *sous.*
1£. = $17\frac{1}{2}$ *livres.*

8. *To Spanish milled Dollars.*

Rule 1.—When the sum consists of pounds only : annex a cypher to the pounds, and divide the whole by 3 : the quotient is dollars.

REDUCE £.100 New-Hampshire, &c. to dollars.

3)1000
Dol. $333\frac{1}{3}$ *Ans.*

Rule 2.—When the sum consists of pounds and shillings :—

Divide

Divide the pounds by 3, and the shillings by 6, not separating them in the quotient, and the quotient will be dollars.

REDUCE £.152 15/6 to dollars.

£'s. by 3 & *S. by* 6. } 152 15/6

509 *dol.* & 1/6 *Ans.*

Note. This article may be applied to the federal dollar, it being of the same value with a Spanish dollar.

II. *To reduce New-Jersey, Pennsylvania, Delaware and Maryland Currency.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule.—Deduct one fifth from the New-Jersey, &c. sum, and the remainder will be New-Hampshire, &c. currency.

REDUCE £.100 New-Jersey, &c. to New-Hampshire, &c.

5)100
—20

£.80 *Answer.*

2. *To New-York and North-Carolina currency.*

Rule. Add one fifteenth to the New-Jersey, &c. sum.

REDUCE £.100 New-Jersey, &c. to New-York, &c.

15=3×5)100

3)20

+6 13 4+ *giv. sum.*

£.106 13 4 *Answer.*

3. *To South-Carolina and Georgia currency.*

Rule. Multiply the New-Jersey, &c. sum by 28, and divide the product by 45, and the quotient is South-Carolina, &c.

REDUCE £.100 New-Jersey, &c. to South-Carolina, &c.

100

4×7=28

400
7

45=5×9)2800

5)311 2 $2\frac{2}{3}$

£.62 4 $5\frac{1}{3}$ *Answer.*

4. *To English Money.*

Rule. Multiply the New-Jersey, &c. by 3, and divide the product by 5.

REDUCE £.100 New-Jersey, &c. to English money.

100
3

5)300

£.60 *Answer.*

5. *To Irish Money.*

Rule. Multiply the New-Jersey, &c. by 13, and divide the product by 20.

REDUCE £.100 Irish to New-Jersey, &c.

100

4×3+ *the given sum.*

400
3

1200
+100

20=4×5)1300

4)260

£.65 *Answer.*

6. *To*

6. *To Canada and Nova-Scotia currency.*

Rule. Deduct one third from the New-Jersey, &c.

REDUCE £.100 New-Jersey, &c. to Canada, &c.

3)100
— 33 6 8

£.66 13 4 *Answer.*

7. *To Livres Tournois.*

Rule. Multiply the New-Jersey, &c. pounds by 14, and the product will be Livres Tournois—or multiply the sum in shillings by 7; divide the product by 10, and the quotient will be livres, sous, &c.

REDUCE £.100 New-Jersey, &c. to Livres Tournois.

100 Or 100 { 1*d.*=$1\frac{1}{6}$ *Sous.*
14 20 { 1*s.*=14 *Sous.*
{ 1£.=14 *Liv.*
400 2000
100 7

Ans. 1400 *Liv.* 10)14000

1400 *as before.*

8. *To Spanish milled dollars.*

Rule. Multiply the New-Jersey, &c. pounds by $2\frac{2}{3}$ and the product will be dollars.—-Or multiply them by 8: divide the product by 3, and the quotient will be dollars.—If there be shillings in the given sum, for every 7/6 add 1 dollar to the quotient.

REDUCE £.100 10*s.* New-Jersey, &c. to dollars.

100 Or 100
8 2

3)800 200
$266\frac{2}{3}$ 100 x $\frac{2}{3}$= $66\frac{2}{3}$
10*s*= $1\frac{1}{3}$ 10*s*= $1\frac{1}{3}$

Ans. 268 *dol.* 268 *as before.*

III. *To reduce New-York and North-Carolina currency.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule. Deduct one fourth from the New-York, &c.

REDUCE £.100 New-York, &c. to New-Hampshire, &c.

4)100

—25

£.75 *Answer.*

2. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule. Deduct one sixteenth from the New-York, &c. sum.

REDUCE £.100 New-York, &c. to New-Jersey, &c.

16=4×4)100

4)25

—£.6 5

£.93 15 *Answer.*

3. *To South-Carolina and Georgia currency.*

Rule. Multiply the New-York, &c. sum by 7, and divide the product by 12: the quotient is South-Carolina, &c.

REDUCE £.100 New-York, &c. to South-Carolina, &c.

100
7

12)700

£.58 6 8 *Answer.*

4. *To English Money.*

Rule. Multiply the New-York, &c. sum by 9: divide the product by 16, and the quotient is English.

REDUCE £.100 New-York, &c. to English money.

100

100
9
$16=4\times4)900$
$4)225$
£.56 5 *Answer.*

5. *To Irish Money.*

Rule. Multiply the New-York, &c. sum by 39: divide the product by 64, and the quotient is Irish.

REDUCE £.100 New-York, &c. to Irish money.

100
6 x 6 + *thrice the given sum.*
600
6
3600
+ 300 = 100 x 3
$64=8\times8)3900$
$8)487\ \ 10$
£.60 18 9 *Answer.*

6. *To Canada and Nova-Scotia currency.*

Rule. Multiply the New-York, &c. sum by 5, and divide the product by 8.

REDUCE £.100 New-York, &c. to Canada, &c.

100
5
$8)500$
£.62 10 *Answer.*

7. *To Livres Tournois.*

Rule. Multiply the New-York, &c. sum in shillings by 21: divide the product by 32, and the quotient will be livres, sous, &c.

REDUCE £.100 New-York, &c. to Livres Tournois.

100
20
2000
21
2000
4000
$32=4\times8)42000$
$4)5250$
Answ. $1312\frac{1}{2}$ *Livres.*

Note. 1*d.*=$1\frac{3}{32}$ *Sous.*
1*s.*=$13\frac{1}{8}$ *Sous.*
1£.=$13\frac{1}{8}$ *Liv.*

8. *To Spanish milled Dollars.*

Rule. If the New-York sum be pounds only, annex a cypher to them, then divide by 4, and the quotient is dollars: but if it be pounds and shillings; annex half the shillings to the pounds and divide as before, and the quotient is dollars.

REDUCE £.100 New-York, &c. to Dollars.

$4)1000$
250 *Doll. Answer.*

REDUCE £.100 8*s.* to Dollars.

$4)1004$
251 *Dol. Answer.*

IV. *To reduce South-Carolina and Georgia currency.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule. Multiply the South-Carolina, &c. sum by 9, and divide the product by 7.

REDUCE £.100 South-Carolina, &c. to New-Hampshire, &c.

100
9
$7)900$
£.128 11 $5\frac{1}{7}$ *Answer.*

2. *To*

2. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule. Multiply the South-Carolina, &c. sum by 45, and divide the product by 28.

REDUCE £.100 South-Carolina, &c. to New-Jersey, &c.

$$\begin{array}{r} 100 \\ 9\times5=45 \\ \hline 900 \\ 5 \\ \hline 28=4\times7)4500 \\ \hline 4)642\ \ 17\ \ 1\frac{5}{7} \\ \hline £.160\ \ 14\ \ 3\frac{3}{7}\ \textit{Answer.} \end{array}$$

3. *To New-York and North-Carolina currency.*

Rule. Multiply the South-Carolina, &c. sum by 12, and divide the product by 7.

REDUCE £.100 South-Carolina, &c. to New-York, &c.

$$\begin{array}{r} 100 \\ 12 \\ \hline 7)1200 \\ \hline £.171\ \ 8\ \ 6\frac{6}{7}\ \textit{Answer.} \end{array}$$

4. *To English Money.*

Rule. From the South-Carolina, &c. sum, deduct one twenty-eighth.

REDUCE £.100 South-Carolina, &c. to English Money.

$$\begin{array}{r} 28=4\times7)100 \\ \hline 4)14\ \ 5\ \ 8\frac{4}{7} \\ -3\ \ 11\ \ 5\frac{1}{7}\ \textit{from the given sum.} \\ \hline £.96\ \ 8\ \ 6\frac{6}{7}\ \textit{Answer.} \end{array}$$

5. *To Irish Money.*

Rule. Multiply the South-Carolina, &c. sum by 117, and divide the product by 112.

REDUCE £.100 South-Carolina, &c. to Irish.

$$\begin{array}{r} 100 \\ 12\times9+9\ \textit{times the giv. sum.} \\ \hline 1200 \\ 9 \\ \hline 10800 \\ +100\times9=\ 900 \\ \hline 112=4\times4\times7)11700 \\ \hline 4)1671\ \ 8\ \ 6\frac{6}{7} \\ \hline 4)417\ \ 17\ \ 1\frac{5}{7} \\ \hline £.104\ \ 9\ \ 3\frac{3}{7}\ \textit{Answer.} \end{array}$$

6. *To Canada and Nova-Scotia currency.*

Rule. Multiply the South-Carolina, &c. sum by 15, and divide the product by 14.

REDUCE £.100 South-Carolina, &c. to Canada, &c.

$$\begin{array}{r} 100 \\ 5\times3 \\ \hline 500 \\ 3 \\ \hline 14=2\times7)1500 \\ \hline 2)214\ \ 5\ \ 8\frac{4}{7} \\ \hline £.107\ \ 2\ \ 10\frac{2}{7}\ \textit{Answer.} \end{array}$$

7. *To Livres Tournois.*

Rule. Multiply the South-Carolina, &c. pounds by $22\frac{1}{2}$ and the product will be Livres.

REDUCE £.100 South-Carolina, &c. to Livres.

Note. 1*d.*$=1\frac{7}{8}$ *sous.*
1*s.*$=1\frac{1}{8}$ *livre.*
1£$=22\frac{1}{2}$ *livres.*

$$\begin{array}{r} 100 \\ 22\frac{1}{2} \\ \hline 200 \\ 200 \\ 50 \\ \hline \end{array}$$

Ans. 2250 *livres.*

8. *To*

8. *To Spanish milled Dollars.*

Rule.—Multiply the South-Carolina, &c. pounds by 30, and divide the product by 7, and if there be shillings, turn them into dollars and add them.

REDUCE £.100 South-Carolina, &c. to Dollars.

100
10×3=30

1000
3

7)3000

Dollars 428$\frac{4}{7}$. *Note,* $\frac{1}{7}$ = 8*d,*

V. *To reduce English Money.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule.—To the English sum add one third.

REDUCE £.100 English to New-Hampshire, &c.

3)100
+ 33 6 8

£.133 6 8 *Ans.*

2. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Multiply the English money by 5, and divide the product by 3.

REDUCE £.100 English to New-Jersey, &c. 100
5

3)500

£.166 13 4 *Ans.*

3. *To New-York and North-Carolina currency.*

Rule.----Multiply the English money by 16, and divide the product by 9.

REDUCE £.100 English to New-York, &c.

100
4×4

400
4

9)1600

£.177 15 6$\frac{2}{3}$ *Ans*

4. *To South-Carolina and Georgia currency.*

Rule.---To the English money add one twenty-seventh.

REDUCE £.100 English to South-Carolina, &c.

27=3×9)100

3)11 2 2$\frac{2}{3}$

+ 3 14 0$\frac{8}{9}$

£.103 14 0$\frac{8}{9}$ *Ans.*

5. *To Irish Money.*

Rule.—To the English sum add one twelfth.

REDUCE £.100 English money to Irish money.

12)100
+ 8 6 8

£.108 6 8 *Ans.*

6. *To Canada and Nova-Scotia currency.*

Rule.—To the English sum add one ninth.

REDUCE £.100 English to Canada, &c.

9)100
+ 11 2 2$\frac{2}{3}$

£.111 2 2$\frac{2}{3}$ *Ans.*

7. *To Livres Tournois.*

Rule.—Multiply the English pounds by 23$\frac{1}{3}$, and the product will be Livres.

REDUCE

REDUCE £.100 English to Livres Tournois.

100 *Note,* 1*d.* = $1\frac{17}{18}$ *sous.*
$23\frac{1}{3}$ 1*s.* = $1\frac{1}{6}$ *livre.*
1£. = $23\frac{1}{3}$ *liv.*

300
200
$33\frac{1}{3}$

Liv. sou. den.
Ans. $2333\frac{1}{3}$ *Liv.* = 2333 6 8

VI. *To reduce Irish Money.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut, and Virginia currency.*

Rule.—Multiply the Irish sum by 16, and divide the product by 13.

REDUCE £.100 Irish to New-Hampshire, &c.

100
4×4

400
4

13)1600

£.123 1 $6\frac{6}{13}$ *Ans.*

2. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Multiply the Irish sum by 20, and divide the product by 13.

REDUCE £.100 Irish to New-Jersey, &c.

100
4×5 = 20

400
5

13)2000(153 16 $11\frac{1}{13}$ *Ans.*

13 11 12
— 20 12
70 — —
65 13)220(16 13)144(11
— 13 13
50 — —
39 90 14
— 78 13
11 — —

3. *To New-York and North-Carolina currency.*

Rule.—Multiply the Irish sum by 64 and divide the product by 39.

REDUCE £.100 Irish to New-York, &c.

100
8×8 = 64

800
8

£. s.
39)6400(164 $2\frac{2}{39}$ *Ans.*
39
—
250
234
—
160
156
—
4

4. *To South-Carolina and Georgia currency.*

Rule.—Multiply the Irish sum by 112, and divide the product by 117.

REDUCE £.100 Irish to South-Carolina, &c.

100
7×4×4 = 112

700
4

2800
4

£.
117)11200(95 14 $6\frac{42}{117}$ *Ans.*
1053
—
670
585
—
85

5. *To*

5. *To English Money.*

Rule.—From the Irish sum deduct one thirteenth.

REDUCE £.100 Irish to English money.

```
13)100(7
   91
   --
    9
   20
   --
13)180(13
   13
   --
   50
   39
   --
   11
   12
   --
13)132(10
   13
   --
    2
```

£.	s.	d.
100	0	0
— 7	13	$10\frac{2}{13}$
£.92	6	$1\frac{11}{13}$ *Ans.*

6. *To Canada and Nova-Scotia currency.*

Rule.—To the Irish sum add one thirty-ninth.

REDUCE £.100 Irish to Canada, &c.

```
39)100(2
   78
   --
   22
   20
   --
39)440(11
   39
   --
   50
   39
   --
   11
   12
   --
39)132(3
   117
   ---
```

$\frac{15}{39} = \frac{5}{13}$

100		
+ 2	11	$3\frac{5}{13}$
£.102	11	$3\frac{5}{13}$ *Ans.*

7. *To Livres Tournois.*

Rule.—Multiply the Irish sum, in pence, by 70; divide that product by 39, and the quotient will be sous, which, divided by 20, will be livres.

REDUCE £.100 Irish to Livres Tournois.

$100 \times 20 \times 12 = 24000d.$

```
          70
   -----------
          2,0
39)1680000(4307,6
         ------ sous.
```

Ans. Livres. $2153,,16\frac{12}{13}$

$1d. = 1\frac{31}{39}$ *sous.* $1s. = 21\frac{7}{13}$ *sous.*

$1£. = 21$ *liv.* $10\frac{10}{13}$ *sous.*

VII. *To reduce Canada and Nova-Scotia currency.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule.—To the Canada, &c. sum add one fifth.

REDUCE £.100 Canada, &c. to New-Hampshire, &c.

```
5)100
+  20
 ----
£.120 Ans.
```

2. *To New-York and North-Carolina currency.*

Rule.—Multiply the Canada, &c. sum by 8, and divide the product by 5.

REDUCE £.100 Canada, &c. to New-York, &c.

```
  100
    8
  ---
5)800
  ---
£.160 Ans.
```

3. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—To the Canada, &c. sum add one half.

REDUCE £.100 Canada, &c. to New-Jersey, &c.

```
2)100
+  50
 ----
£.150 Ans.
```

4. *To South-Carolina and Georgia currency.*

Rule.

Rule.—From the Canada, &c. sum deduct one fifteenth.

REDUCE £.100 Canada, &c. to South-Carolina, &c.

15 = 3×5)100
3)20
— 6 13 4
£.93 6 8 *Ans.*

5. *To English Money.*

Rule.—From the Canada, &c. deduct one tenth.

REDUCE £.100 Canada, &c. to English money.

10)100
— 10
£.90 *Ans.*

6. *To Irish Money.*

Rule.—From the Canada, &c. deduct one fortieth.

REDUCE £.100 Canada, &c. to Irish money.

40)100
— 2 10
£.97 10 *Ans.*

7. *To Livres Tournois.*

Rule.—Multiply the Canada, &c. pounds by 21, and the product will be Livres.

REDUCE £.100 Canada, &c. to Livres Tournois.

100
7×3=21
700
3
Ans. 2100

1*d.* = 1¾ *sous.*
1*s.* = 21 *sous.*
1£. = 21 *livres.*

8. *To Spanish Milled Dollars.*

Rule.—Reduce the Canada, &c. sum to shillings: divide them by 5, and the quotient is dollars.—*Or*, Multiply the pounds by 4, and the product is dollars: and if there be shillings, turn them into dollars, and add them to the product.

REDUCE £.100 Canada, &c. to Dollars.

100
20
5)2000
Dollars 400 *Ans.*

155 15
4
620
+ 3 = 15*s.*
Dol. 623 *Ans.*

VIII. *To reduce Livres Tournois.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule.—Multiply the Livres by 2: divide the product by 35, and the quotient will be pounds.—*Or*, Multiply the Livres by 8: divide the product by 7, and the product will be shillings.

REDUCE 1750 Livres to New-Hampshire, &c. currency.

1750
2
£.
35)3500(100 *Ans.*
35
00

Or, 1750
8
7)14000
2,0)200,0
£.100 *as before.*

2. *To New-York and North-Carolina currency.*

Rule.—Multiply the Livres by 32: divide the product by 21, and the quotient will be shillings.

REDUCE 1312½ Livres to New-York, &c. currency.

1312,5
32
26250
39375
2,0
21)42000(200,0
£.100 *Ans.*

3. *To*

3. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Divide the Livres by 14, and the quotient will be pounds: *Or,* Multiply the Livres by 10: divide the product by 7, and the quotient will be shillings.

REDUCE 1400 Livres to New-Jersey, &c. currency.

1400
10
—
7)14000
—
2,0)200,0
—
£.100 *Ans.*

Or,
14)1400(£100.
14
—
..00

4. *To South-Carolina and Georgia Currency.*

Rule.—Multiply the Livres by 2, divide the product by 45, and the quotient will be pounds.—Or deduct one ninth, and the remainder will be shillings.

REDUCE 2250 Livres to South-Carolina, &c. currency.

2250
2
— £.
45)4500(100 *Ans.*
45
—
00

Or,
9)2250
— 250
—
2,0)200,0
—
£100 *as before.*

5. *To English Money.*

Rule.—Multiply the Livres by 6: divide the product by 7, and the quotient is shillings:—Or deduct one seventh from the Livres, and the remainder will be shillings.

REDUCE $2333\frac{1}{3}$ Livres to English Money.

$2333\frac{1}{3}$
6
—
7)14000
—
2|0)200|0
—
Ans. £.100

Or
$7)2333\frac{1}{3}$
$-333\frac{1}{3}$
—
2|0)200|0
—
£.100 *as before.*

6. *To Irish Money.*

Rule.---REDUCE the livres to sous, then multiply them by 39: divide this product by 70, and the quotient will be pence.

REDUCE 2153*liv.* $16\frac{12}{13}$*so.* to Irish Money.

20
—
$43076\frac{12}{13}$
39
—
387720
129228
—
7,0)168000,0
—
12)24000
—
2,0)200,0
—
£. 100 *Ans.*

7. *To Spanish milled Dollars, or to Federal Dollars.*

Rule.—Multiply the Livres by 4: divide the product by 21, and the quotient will be Spanish or Federal Dollars.

REDUCE 1000 Livres to Dollars.

1000
4
—
21)4000(190 { *Span. Dol.*
21
—
190 = $190\frac{10}{21}$ *Dollars.*
189
—
10
6
—
21)60(2/10 1*qr.*

Or
1000
4
—
21)4000(190 { *Fed. Dol.*
21
—
190
189
—
10
10
— *d. c. m.*
21)100(4 7 $6\frac{4}{21}$

IX. *To reduce Spanish milled Dollars.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule.—-Multiply the Dollars by 3, and double the right-hand figure of the product, for shillings; the left hand figures are pounds.

REDUCE 529 Dollars to New-Hampshire, &c.

```
   529
     3
  ----
£.158  14 Ans.
```

2. *To New York and North-Carolina currency.*

Rule.—Multiply the number of Dollars by 4: double the right hand figure of the product for shillings, and the left hand figures are pounds.

REDUCE 529 Dollars to New-York, &c.

```
    529
      4
   ----
£.211  12 Ans.
```

3. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Multiply the number of Dollars by 3, and divide by 8.

REDUCE 529 Dollars to New-Jersey, &c.

```
   529
     3
  ---- £.
8)1587(198  7  6 Ans.
  8
  --           Or
  78        8)1587
  72        -------
  --        £.198 3/8 Ans.
   67
   64
   --
    3
```

4. *To South-Carolina and Georgia currency.*

Rule.—Multiply the number of Dollars by 7, and divide by 30.

REDUCE 529 Dollars to South-Carolina, &c.

```
     529
       7
    ----
3,0)370,3
    -----
£.123 13/30 Ans.
```

*5. *To English Money, at 4/6 per Dollar.*

Rule.—-Multiply the Dollars by 9, and divide by 40.

REDUCE 529 Dollars to English money.

```
     529
       9
    ----
4,0)476,1
    -----
£.119 1/40 Ans.
```

6. *To Canada and Nova-Scotia currency.*

Rule.—Divide the Dollars by 4.

REDUCE 529 Dollars to Canada, &c.

```
4)529
  ----
£.132 1/4 Ans.
```

7. *To Livres Tournois.*

Rule.—-Multiply the Dollars by $5\frac{1}{4}$, and the product will be Livres.—*Or*, Multiply them by 21: divide by 4, and the quotient will be Livres.

REDUCE 100 Spanish Dollars to Livres.

```
          100           Or
          5 1/4         100
          -----          21
          500           ----
100×1/4=25           4)2100
          -----         ----
Ans. 525 livres.    525 as before.
```

* Note, *That in England Dollars are Bullion, that is, they are bought and sold by weight, and their value varies as other articles of Merchandize.*

NOTE,

1 Cent	=	$1\frac{1}{20}$ Sous.
1 Dime	=	$10\frac{1}{2}$ Sous.
1 Dollar	=	$5\frac{1}{4}$ Livres.

The pound	contains	Grains of fine Silver.	Grains of Stand. silv.	
Sterling		$1718\frac{2}{3}$	1858	The proportion of alloy being $\frac{3}{37}$ of the fine Silver.
New-Hampshire &c.		1289	$1393\frac{1}{2}$	
New-York, &c.		$966\frac{3}{4}$	1045	
New-Jersey, &c.		$1031\frac{1}{4}$	$1114\frac{3}{4}$	
South-Carolina, &c.		1657,366	1791,819	

The Federal	contains	*Grains of fine*		
Dollar	375,64	Silver	409,78	*grs. of Stand. Silver.*
Eagle	246,268	Gold	268,659	*grs. of Stand. Gold.*

The alloy being $\frac{1}{11}$ of the fine *Silver.* / *Gold.* The Subdivisions are in the same proportion.

DUODECIMALS;

OR,

CROSS MULTIPLICATION

Is a Rule made use of by Workmen and Artificers in casting up the contents of their works.

DIMENSIONS are generally taken in feet, inches and parts.

INCHES and parts are sometimes called primes, seconds, thirds, &c. and are marked thus; inches or primes ($'$), seconds ($''$), thirds ($'''$), fourths ($''''$) &c.

THIS Method of multiplying is not confined to *twelves*; but may be greatly extended: for any number, whether its inferior denominations decrease from the integer in the same ratio, or not, may be multiplied cross-wise; and for the better understanding of it, the learner must observe, that if he multiplies any denomination by an integer, the value of an unit in the product will be equal to the value of an unit in the multiplicand; but if he multiplies by any number of an inferior denomination, the value of an unit in the product will be so much inferior to the value of an unit in the multiplicand as an unit of the multiplier is less than an integer.

THUS, Pounds multiplied by pounds are pounds; pounds multiplied by shillings are shillings, &c. shillings multiplied by shillings are twentieths of a shilling; shillings multiplied by pence are twentieths of a penny; pence multiplied by pence are 240ths of a penny, &c.

RULE.

1. UNDER the multiplicand write the corresponding denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning at the lowest, by the highest denomination in the multiplier, and write the result of each under its respective term, observing, in duodecimals, to carry an unit for every 12, from each lower denomination to its next superior, and for other numbers accordingly.

3. In the ſame manner multiply all the multiplicand by the primes or ſecond denomination in the multiplier, and ſet the reſult of each term one place removed to the right hand of thoſe in the multiplicand.

4. Do the ſame with the ſeconds in the multiplier, ſetting the reſult of each term two places to the right hand of thoſe in the multiplicand.

5. Proceed in like manner with all the reſt of the denominations, and their ſum will be the anſwer required.

EXAMPLES.

1. Multiply $2\frac{1}{2}$ feet by $2\frac{1}{2}$ feet.

F. ′	Or thus.	Or thus.
2 6	$2\frac{1}{2}$	2,5
2 6	$2\frac{1}{2}$	2,5
5 0	5	125
1 3 0	$1\frac{1}{4}$	50
Anſw. 6 3	*Anſw.* $6\frac{1}{4}$ *ſquare feet* = 6 *ft.* 36 *in.*	*Anſ.* 6,25 *Square Feet.*

So that the 3 is not 3 inches, but 36 inches, or $\frac{1}{4}$ of a ſquare foot.

2. Multiply 9*f.* 8′ 6″ by 7*f.* 9′ 3″

f.	′	″			
9	8	6			
7	9	3			
67	11	6			= *Product by the feet in the multiplier.*
7	3	4	6‴		= *ditto by the primes.*
	2	5	1	6⁗	= *ditto by the ſeconds.*
75	5	3	7	6	*Anſwer.*

3. How many ſquare feet in a board 17 feet 7 inches long, and 1 foot 5 inches wide? *Anſw.* 24*f.* 10′ 11″

4. How many cubic feet in a ſtick of timber 12 feet 10 inches long, 1 foot 7 inches wide, and 1 foot 9 inches thick? *Anſw.* 35*f.* 6′ 8″ 6‴

5. How many cubic feet of wood in a load 6 feet, 7 inches long, 3 feet, 5 inches high, and 3 feet, 8 inches wide? *Anſw.* 82*f.* 5′ 8″ 4‴

6. There is a houſe with 4 tiers of windows, and 4 windows in a tier; the height of the firſt tier is 6*f.* 8′; of the ſecond, 5*f.* 9′; of the third, 4*f.* 6′; and of the fourth, 3*f.* 10′, and the breadth of each is 3*f.* 5′; how many ſquare feet do they contain in the whole? *Anſ.* 283*f.* 7′.

The two following queſtions are Sexceſſimals.

7. If

7. If two places differ in longitude 2° 12′; what is their difference of time?

Mult. 2° 12′ 00″ 00‴

by 3′ 59″ 20‴ *the time in which the Sun passes through* 1 *degree.*

8′ 46″ 32‴ *Ans.*

8. Two places differ in longitude 31° 27′ 30″; what is the difference, in time, of the Sun's coming to the meridian of those places, the Sun passing through 15° in an hour?

31° 37′ 30″

4′ 00″ *In* 4′ *of a solar day, or day of* 24 *hours, the Sun passes* 1 *deg.*

2 6′ 30″ 00‴ *Ans.*

9. *Mult.* £.3 6 8 *by* £.2 5 7

	£.	s.	d.
	3	6	8
	2	5	7
£3×£2=£6 =	6	—	—
6s.×£2=12s. =	—	12	—
8d.×£2=16d. =	—	1	4
£3×5s.=15s. =	—	15	—
6s.×5s.=$\frac{30}{20}$s. =	—	1	6
8d.×5s.=$\frac{40}{20}$d. =	—	—	2
£3×7d.=21d. =	—	1	9
6s.×7d.=$\frac{42}{20}$d. =	—	—	2$\frac{1}{10}$
8d.×7d.=$\frac{56}{240}$d.=	—	—	0$\frac{7}{30}$
Ans.	£.7	11	11$\frac{1}{3}$

10. A, B and C bought a drove of sheep in company; A paid £14 5*s.* B, £13 10*s.* and C, £11 5*s.* They agreed to dispose of them at the market;—that each man should take 18*s.* as pay for his time, &c. and that the remainder should be divided in proportion to their several stocks: At the close of the sale, they found themselves possessed of £46 5*s.* what was each man's gain, exclusive of the pay for his time &c?

£14 5+£13 10+£11 5 =£39. and £46 5—£39=£7 5, and £7 5—18*s.*×3 =£4 11*s.* whole gain, and £4 11÷39=2/4 gain in the pound.

A	B	C	
£14 5 0	£13 10 0	£11 5 0	
×2 4	×2 4	×2 4	£. s. d.
1 8 6	1 7 0	1 2 6	*Proof* 1 13 3
4 9	4 6	3 9	1 11 6
			1 6 3
A. £1 13 3	B. £1 11 6	C. £1 6 3	£4 11 0

SINGLE RULE OF THREE DIRECT.

The Rule of Three direct teacheth, by having three numbers given, to find a fourth, that shall have the same proportion to the third, as the second hath to the first.

If

If *more* require *more*, or *less* require *less*, the question belongs to the Rule of Three direct.

But if *more* require *less*, or *less* require *more*, it belongs to the Rule of Three Inverse. †

Rule.*

1. State the question by making that number, which asks ‡ the question, the third term, or putting it in the third place: that, which is of the same name or quality as the demand, the first term; and that, which is of the same name or quality with the answer required, the second term.

2. Multiply the second and third numbers together; divide the product by the first, and the quotient will be the answer to the question,

† *More* requiring *more*, is when the third term is greater than the first, and requires the fourth term to be greater than the second. And *less* requiring *less*, is when the third term is less than the first, and requires the fourth term to be less than the second.

Also, *more* requiring *less*, is when the third term is greater than the first, and requires the fourth term to be less than the second. And *less* requiring *more*, is when the third term is less than the first, and requires the fourth term to be greater than the second.

* This Rule, on account of its great and extensive usefulness, is sometimes called the *golden rule of Proportion*: for, on a proper application of it and the preceding rules, the whole business of Arithmetic, as well as every mathematical enquiry depends. The rule itself is founded on this obvious principle, that the magnitude or quantity of any effect varies constantly in proportion to the varying part of the cause: thus, the quantity of goods, bought, is in proportion to the money laid out;—the space, gone over by an uniform motion, is in proportion to the time, &c.

As the idea, annexed to the term, *proportion*, is easily conceived, the truth of the rule, as applied to ordinary enquiries, may be made evident by attending to principles, already explained.

It has been shewn, in Multiplication of Money, that the price of one, multiplied by the quantity, is the price of the whole; and in Division, that the price of the whole, divided by the quantity, is the price of one: Now, in all cases of valuing goods, &c. where one is the first term of the proportion, it is plain that the answer, found by this rule, will be the same as that, found by Multiplication of Money; and, where one is the last term of the proportion, it will be the same as that, found by Division of Money.

In like manner, if the first term be any number whatever, it is plain that the product of the second and third terms will be greater than the true answer, required, by as much as the price in the second term exceeds the price of one, or as the first term exceeds an unit; consequently this product, divided by the first term, will give the true answer required.

Direct and inverse proportion are properly only parts of the same general rule: but I have preserved the common distinction, and given some loose definitions, which, to young persons in general are more intelligible.

Note 1. When it can be done, multiply and divide as in Compound Multiplication, and Compound Division.

2. If the first term, and either the second or third can be divided by any number without a remainder, let them be divided and the quotient used instead of them.

The following methods of operation, when they can be used, perform the work in a much shorter manner than the general rule.

1. Divide the second term by the first: multiply the quotient into the third, and the product will be the answer.

2. Divide the third term by the first; multiply the quotient into the second, and the product will be the answer.

3. Divide the first term by the second, and the third by that quotient, and the last quotient will be the answer.

4. Divide the first term by the third, and the second by that quotient, and the last quotient will be the answer.

‡ *Note*, The term which asks or moves the question, has generally some words like these before it, *viz.* What will? What cost? How many? How far? How long? or, How much? &c.

queſtion, which (as alſo the remainder) will be in the ſame denomination you left the ſecond term in, and which may be brought into any other denomination required.

Two, or more ſtatings, are ſometimes neceſſary, which may always be known from the nature of the queſtion.

The method of proof is by inverting the queſtion.

But, that I may make the method of working this excellent Rule as intelligible as poſſible to the learner, I ſhall divide it into the ſeveral caſes following,

1. The fourth number is always found in the ſame name in which the ſecond is given, or reduced to; which, if it be not the higheſt denomination of its kind, reduce to the higheſt, when it can be done.

2. When the ſecond number is of divers denominations, bring it to the loweſt mentioned, and the fourth will be found in the ſame name to which the ſecond is reduced, which reduce back to the higheſt poſſible.

3. If the firſt and third be of different names, or one or both of divers denominations, reduce them both to the loweſt denomination mentioned in either.

4. When the product of the ſecond and third is divided by the firſt; if there be a remainder after the diviſion, and the quotient be not the leaſt denomination of its kind; then multiply the remainder by that number, which one of the ſame denomination with the quotient contains of the next leſs, and divide this product again by the firſt number; and thus proceed 'till the leaſt denomination be found, or 'till nothing remain.

5. If the firſt number be greater than the product of the ſecond and third; then bring the ſecond to a lower denomination.

6. When any number of barrels, bales, or other packages or pieces are given, each containing an equal quantity, let the content of one be reduced to the loweſt name, and then multiplied by the given number of packages or pieces.

7. If the given barrels, bales, pieces, &c. be of unequal contents, (as it moſt generally happens) put the ſeparate content of each properly under one another, then add them together, and you will have the whole quantity.

1. If 6 ℔. of Sugar coſt 9ſ. what will 30 ℔. coſt at the ſame rate?

Here, the firſt clauſe (if 6 ℔. of Sugar coſt 9ſ.) ſuppoſes the rate; then follows the queſtion: what will 30 ℔. coſt?

30 ℔. which moves the queſtion, is the 3d. term.—6 ℔. the ſame kind, is the 1ſt.—and 9 ſhillings the 2d.

℔. *s.* ℔.
As 6 : 9 :: 30 : *the Anſwer.*
9
6)270
45*s.* = £.2 5ſ. *Anſwer.*

Again—By inverting the order of the queſtion, it will be,

2.

2. If 9*s*. buy 6 ℔. of Sugar, how much will £.2 5*s*. buy at that rate?

s. ℔. s.
As 9 : 6 :: 45 : *the Answer.*
6
9)270
30 ℔. *Answer.*

Again 3. If 30 ℔. of Sugar be worth £.2 5*s*. how much may I buy for 9*s*.?

s. ℔. s.
As 45 : 30 :: 9 : *the Answer.*
9
45)270(6 ℔. *Answer.*
270

Again 4. Suppose £.2 5*s*. will buy 30 ℔. Sugar: what will 6 ℔. of the same Sugar cost?

℔. s. ℔.
As 30 : 45 :: 6 : *the Answer.*
6
3|0)27|0
9*s*. *Answer.*

N. B. The three last questions are only the first *varied*, being put purely to shew how any question, in this Rule, may be inverted.

5. If 5 *yds*. Cloth cost £.1 10*s*. what will 20 *yds*. ditto come to?

yds. s. yds.
As 5 : 30 :: 20
30 ÷ 5 = 6
20
5)30*s*.
6 *Quot*.
2,0)12,0*s*. = £6 *Ans*.

Here I divide the 2d term by the 1st, and multiply the quotient into the 3d for the answer.

yds. s. yds.
Again 6. As 5 : 30 :: 20
20 ÷ 5 = 4
120*s*. = £6.

Here I divide the 3d term by the 1st, and multiply the quotient into the 2d for an answer.

7. If 20*yds*. cost £120: how many yards may I have for £30?

£ yds. £
As 120 : 20 :: 30
120÷20=6*quo*. & 30÷6=5*yds*. *ans*.

Here I divide the 1st term by the 2d, and then, the 3d term by that quotient for the answer.

£. yds. £.
Again—8. As 120 : 20 :: 30
120÷30=4*quo*. 20÷4=5*yds ans*.

Here I divide the 1st term by the 3d, and then, the 2d term by that quotient for the Answer.

9. If 1*Cwt*. of Tobacco cost £5 12 9½: what will 8 *Cwt*. ditto cost?

Cwt. £ s. d. Cwt.
As 1 : 5 12 9½ :: 8
8
Ans. £45 2 4

Here is no need of reducing the middle term; because it can be performed by compound multiplication, the 1st term being an unit.

10. If 8 *Cwt*. of Tobacco cost £45 2*s*. 4*d*. what is that *per Cwt*?

£. s. d.
8)45 2 4
Ans. £5 12 9½

Here there is no need of reducing the middle term, because it may be performed by compound division only, the 3d term being an unit.

11. If

11. If 9*C.* 3*qrs.* Sugar cost £27 17*s.* 6*d.* what will 2*C.* 1*qr.* 11*lb.* cost?

9C. 3qrs.	£27 17s. 6d.	2C. 1qr. 11lb.
4	20	4
39	557	9
28	12	28
312	6690	73
78		19
1092		263

As 1092 lb : 6690 d. :: 263 lb : *the answer!*

263

2007
4014
1338

1092)1759470(1611 —— 12

1092

2|0)13|4 3*d.*

6674 — £6 14*s.* 3 *Ans.*

6552

1227
1092

1350
1092

258
4

1092)1032(0*qr.*

Note 1. If you look at the Stating, you will see that the first and third terms are of the same kind, but of different denominations, and therefore are reduced to the same name or denomination, and, that the demand of the question lies on the third term.

2. That the middle term, being given in pounds, shillings and pence, is reduced to pence.

12. If 57 *yds.* cost £69: what will 9 *yds.* cost at that rate?

yds. £ *yds.*
As 57 : 69 :: 9
9

57)621(10 £ *Ans.*
57

51
20

57)1020(17*s.*
57

450
399

51
12

57)612(10*d.*
57

42
4

57)168(2 $\frac{54}{57}$ *qrs.*
114

Here all the terms being whole numbers, there is no need of reducing the middle one 'till after stating.

13. If

13. If my income be 109 guineas *per annum*, I desire to know what I may spend *per* day, so that I may lay up £.45 at the year's end? *Answ.* £.0 5 $10\frac{3}{4}\frac{1}{365}$ *per day*.

Note 1. You must subtract £.45 from the value of 109 guineas.

2. There being 365 days in a year, your question must next be stated thus:

	D.		*Guin.*		*£.*		*D.*		
As	365	:	109	—	45	::	1	:	*the answer*.

14. If my Salary be £.43 12*s.* 5*d.* *per annum*, what does it amount to *per* week? *Answ.* £.0 16 $9\frac{18}{365}$.

The Stating.

	W.		*£.*	*s.*	*d.*		*W.*		
As	52	:	43	12	5	::	1	:	*the Answ.*

Note. As there are 52 weeks & 1 day in a year, you will get the true answer to the above quest. by the following *Ratio*.

	D.		*£.*	*s.*	*d.*		*D.*		
As	365	:	43	12	5	::	7	:	*the answer*.

15. Suppose my income to be 16*s.* $9\frac{18}{365}$*d.* *per* week, what is it *per annum*? *Answ.* £.43 12 5.

The Stating.

	D.		*s.*	*d.*		*D.*		
As	7	:	16	$9\frac{18}{365}$	::	365	:	*the answer above*.

Note 1. You must first reduce the middle term to pence.

2. You must multiply by 365 (the denominator of the fraction) and add to the product the 18 which remains; and remember always to do so in similar cases.

3. You must divide by 7, the first term, and the quotient will be the answer in 365ths of a penny, which (in all similar cases) must be first divided by the denominator and then brought into pounds.

16. If I am to pay 1*s.* 7*d.* *per* week for pasturing a cow; what must I give *per* week for 37 cows?

	C.		*s.*	*d.*		*C.*		*£.*	*s.*	*d.*	
As	1	:	1	7	::	37	:	2	18	7	*Answer*.

17. How many yards of Cloth may be bought for £.57 13*s.* whereof $9\frac{1}{2}$ yards cost £.3 15*s.* $5\frac{1}{2}$*d.*?

	£.	*s.*	*d.*		*Yds.*		*£.*	*s.*		*Yds.*	*qr.*	*n.*	
As	3	15	$5\frac{1}{2}$	:	$9\frac{1}{2}$	::	57	13	:	145	0	$2\frac{2004}{3622}$	*Answer*.

18. If I buy 57 yards of Cloth for 49 guineas; what did it cost *per* Ell-English?

	Yds.		*Guin.*		*Yds.*		
As	57	:	49	::	$1\frac{1}{4}$	:	£.1 10*s.* $1\frac{12}{228}$*d.* *Answer*.

19. A Merchant, failing in Trade, owes in all £.3475, and has in money and effects but £.2316 13 4: Now, supposing his effects are delivered up; Pray what will each Creditor receive on the pound?

	£.		*£.*	*s.*	*d.*		*£.*		
As	3475	:	2316	13	4	::	1	:	£.0 13*s.* 4*d.* *Answer*.

20. A

20. A owes B £.3475, but B compounds with him for 13*s.* 4*d.* on the pound; pray, what must he receive for his debt?

£. s. d. £. £. s. d.
As 1 : 13 4 :: 3475 : 2316 13 4.

21. If the distance from Newbury-port to York be 31 miles; I demand how many times a wheel, whose circumference is 15½ feet, will turn round in performing the journey?

Feet. Cir. M. Cir.
As 15½ : 1 :: 31 10560 *times, answer.*

22. Bought 9 Chests of Tea, each weighing 3 *C.* 2 *qrs.* 21 ℔. at £.4 9*s. per Cwt.* what came they to?

Cwt. £. s. C. qr. lb. £. s. d.
As 1 : 4 9 :: 3 2 21 × 9 : 147 13 8¼.

23. What will 37½ gross of buttons, at 9½*d. per* dozen, come to?

Doz. d. Grofs. £. s. d.
As 1 : 9½ :: 37½ : 17 16 3 *Answer.*

24. A Farm, containing 125*A.* 3*R.* 27*P.* is rented at £.3 9*s.* *per* acre; what is the yearly rent of that Farm?

A. £. s. A. R. P. £
As 1 : 3 9 :: 125 3 27 : 434 8/4½ $\frac{142}{160}$ *Answer.*

25. If a Ship cost £.537; what are ⅜ of her worth?

Eigh. £. Eigh. £. s. d.
As 8 : 537 :: 3 : 201 7 6 *Answer.*

26. If $\frac{7}{16}$ of a Ship cost £.349; what is the whole worth?

Sixt. £. Sixt. £. s. d.
As 7 : 349 :: 16 : 797 14 3¼ $\frac{5}{7}$ *Answer.*

27. Bought a Cask of Wine at 4/7 *per* gallon, for 125 dollars; how much did it contain?

s. d. Gal. Dol. Gal. qt. pt.
As 4 7 : 1 :: 125 : 162 2 1 $\frac{5}{55}$ *Answer.*

28. What comes the Insurance of £.537 15*s.* to, at £.4½ *per Centum?*

£. £. £. s. £. s. d.
As 100 : 4½ :: 537 15 : 24 3 11½ $\frac{8}{10}$ *Answer.*

29. What come the Commissions of £.785 to at 3½ guineas *per cent.?*

£. Guin. £. £. s. d.
As 100 ; 3½ :: 785 : 38 9 3½ $\frac{4}{10}$ *Answer.*

30. A Merchant bought 9 packages of Cloth, at 3 guineas for 7 yards; each package contained 8 parcels, each parcel, 12 pieces, and each piece, 20 yards; what came the whole to, and what *per* yard?

Yds. Guin. Pack. £.
As 7 : 3 : : 9 : 10368 *Answer, for the whole cost.*

Yds. Guin. Yd.
As 7 : 3 : : 1 : £.0 12*s.* *Answ. per yard.*

31. A Merchant bought 49 Tuns of Wine for £.273; Freight cost £.27—Duties £.12—Cellar £.9 10*s.* other charges £15, and he

he would gain £.55 10s. by the Bargain; what must I give him for 23 Tuns?

Tons.		£.		£.		£.		£. s.		£.		£. s.		Tons.		£.
As 49	:	273	+	27	+	12	+	9 10	+	15	+	55 10	::	23	:	184 *Ans.*

32. If £.100 gain £.6 in a year, what will £.475 gain in that time? As £.100 : £.6 :: £.475 : £.28 10s. *Answ.*

33. The Earth, being 360 degrees in circumference, turns round on its Axis in 24 hours; how far does it turn in one minute, in the 43d. parallel of Latitude:—the degree of Longitude, in this Latitude, being about 51 statute miles?

H.		D.		M.		M.		miles.
As 24	:	360×51	::	1	:	$12\frac{3}{4}$ *Answer.*

34. Shipt for the West-Indies 225 quintals of fish, at 15/6 *per* quintal; 37000 feet of boards, at $8\frac{1}{3}$ dollars *per* 1000; 12000 shingles, at a Half-guinea *per* 1000; 19000 hoops at $1\frac{1}{2}$ dollar *per* 1000, and 53 Half-joes; and, in return, I have had 3000 gallons of rum, at 1*s.* 3*d.* *per* gallon; 2700 gallons of molasses, at $5\frac{1}{2}$*d.* *per* gallon; 1500 lb. of coffee, at $8\frac{1}{2}$*d.* *per* lb. and 19 *Cwt.* of sugar, at 12*s.* 3*d.* *per Cwt.* and my charges on the voyage were £.37 12*s.* pray, did I gain or lose, and how much, by the voyage?

Answer, Lost £.134 9*s.* 9*d.*

35. If a Staff, 4 feet long, cast a shade (on level ground) 7 feet; what is the heighth of that Steeple, whose shade, at the same time, measures 198 feet?

F. Sh.		F. bei.		F. Sb.		F. bei.
As 7	:	4	::	198	:	$113\frac{1}{7}$ *Answer.*

† 36. Suppose a Tax of £.755 be laid on a town, and the inventory of all the estates in the town amounts to £.9345, what must A pay, whose estate is £.149?

£.		£.		£.		£.
As 9345	:	755	::	149	:	12 0 $9\frac{1095}{9345}$ *Ans.*

37. If

† It may not be amiss to shew the general method of assessing town or parish Taxes.

First, then, an Inventory of the value of all the Estates, both real and personal, and the number of polls, for which each person is rateable, must be taken in separate columns; The most concise way is then to make the total value of the Inventory the first term, the tax to be assessed, the second, and £.1 the third, and the quotient will shew the value on the pound:—2dly. Make a table, by multiplying the value on the pound by 1, 2, 3, 4, &c.—3dly. From the Inventory take the real and personal estates of each man, and find them separately in the table, which will shew you each man's proportional share of the tax for real and personal estates.

Note. If any part of the tax is averaged on the polls, or otherwise, before stating to find the value on the pound, you must deduct the sum of the average tax from the whole sum to be assessed: for which average, you must have a separate column, as well as for the real and personal estates.

Example.

Suppose the General Court should grant a tax of £.100000, of which the town of Newbury-port is to pay £.1321 17*s.* 6*d.* and, of which the polls, being 750, are to pay 5*s.* 3*d.* each:—The Town's inventory amounts to £.45000; what will it be on the

37. If 50 gallons of water, in one hour, fall into a ciſtern, containing 230 gallons, and by a pipe in the ciſtern 35 gallons run out in an hour; in what time will it be filled?

	Gal. Gal.		H.		Gal.		H.
As	50—35	:	1	::	230	:	$15\frac{1}{3}$ *Anſ.*

38. A Butcher went with £416, to buy cattle: oxen, at £22 each, cows at £4, ſteers at £3 10*s.* and calves at £2 10*s.* and of each a like number; how many of each could he purchaſe with that ſum?

	£.		£.		£. s.		£. s.		each.		£.		each.
As	22	+	4	+	3 10	+	2 10	:	1	::	416	:	13 *Anſwer.*

39. Said Harry to Dick, my purſe and money are worth $3\frac{1}{4}$ guineas; but the money is worth eleven times as much as the purſe; Pray, how much money is there in it?

				Guin.		s. d.	
As	12	:	1	:: $3\frac{1}{4}$	:	7 7.	*then* £.4 11*ſ*.—7*ſ*7 = £.4 3*ſ*5 *Anſw.*

40. How many dozen pair of gloves, at 13 groats *per* pair, may I have for 125 dollars?

	gr.		pr.		dol.		doz. pr.
As	13	:	1	::	125	:	14 $5\frac{4}{52}$ *Anſwer.*

41. There is a ciſtern, having 4 cocks; the firſt will empty it in 10 minutes; the ſecond in 20 minutes; the third in 40, and the fourth

the pound; and what is A's tax, whoſe eſtate (as by the inventory) is as follows, viz. real £.376, perſonal £.149, and he has 3 poles?

		Pol.	s. d.		Pol.		£. s. d.	
First.	As	1	: 5 3	::	750	:	196 17 6	the average part of the tax to be deducted from £.1321 17*s.* 6*d.* and there will remain £.1125.

		£.		£.		£.	
Secondly.	As	45000	:	1125	::	1	: 6*d.* on the pound.

T A B L E.

£.		£.	s.	d.	£.		£.	s.	d.	£.		£.	s.	d.
1	is	—	-	6	20	is	-	10	—	200	is	5	—	—
2	—	—	1	-	30	—	-	15	—	300	—	7	10	—
3	—	—	1	6	40	—	1	—	—	400	—	10	—	—
4	—	—	2	-	50	—	1	5	—	500	—	12	10	—
5	—	—	2	6	60	—	1	10	—	600	—	15	—	—
6	—	—	3	-	70	—	1	15	—	700	—	17	10	—
7	—	—	3	6	80	—	2	—	—	800	—	20	—	—
8	—	—	4	-	90	—	2	5	—	900	—	22	10	—
9	—	—	4	6	100	—	2	10	—	1000	—	25	—	—
10	—	—	5	-										

Now, to find what A's Rate will be.

His real eſtate being £.376, I find, by the Table, that £.300 is £.7 10*s.*
that £.70 is - - - 1 15
and that £.6 is - - - - 3

for his real eſtate - - - £.9 8

In like manner, I find his tax for perſonal eſtate to be 3 14 6
His 3 polls, at 5*s.* 3*d.* each, are - 15 9

} + £ 9 8*s.* = £ 13 18*ſ*3. *Anſwer.*

Real. £. s. d.	Perſonal. £. s. d.	Polls. £. s. d.	Total. £. s. d.
9 8 -	3 14 6	- 15 9	13 18 3

fourth in 80 minutes, in what time will all four, running together, empty it?

$$\text{As} \left\{\begin{matrix} \textit{Min.} \\ 10 \\ 20 \\ 40 \\ 80 \end{matrix}\right\} \overset{\textit{Cist.}}{:\ 1}\ ::\ \overset{\textit{Min.}}{60}\ :\ \left\{\begin{matrix} \textit{Cist.} \\ 6 \\ 3 \\ 1\frac{1}{2} \\ \frac{3}{4} \end{matrix}\right\} \text{As } \overset{\textit{Cist.}}{11\tfrac{1}{4}} : \overset{\textit{Min.}}{60} :: \overset{\textit{Cist.}}{1} : \overset{\textit{Min.}}{5\tfrac{1}{3}} \textit{ Answ.}$$

$11\frac{1}{4}$ *Cist.*

42. A and B depart from the same place, and travel the same road; but A goes 5 days before B, at the rate of 20 miles *per* day; B follows at the rate of 25 miles *per* day; In what time and distance will he overtake A?

As $\overset{M.\ \ M.}{25-20} : \overset{D.}{1} :: \overset{M.\ \ D.}{20\times 5} : \overset{D.}{20}$. *And*, As $\overset{D.}{1} : \overset{M.}{25} :: \overset{D.}{20} : \overset{M.}{500}$

43. If the earth revolves 366 times in 365 days, In what time does it perform one revolution?

As $\overset{\textit{Revol.}}{366} : \overset{\textit{Days}}{365} :: \overset{\textit{Revol.}}{1} : 23^h\ 56'\ 3''\ 56'''+ = 1$ Sidereal day. †

44. If the earth makes one complete revolution in $23^h\ 56'\ 3''+$, In what time does it pass through one degree?

As $366^\circ : 23^h\ 56'\ 3'' :: 1^\circ : 3'\ 59''\ 20'''$ *Answer.*

45. If the earth performs its diurnal revolution in a Solar ‖ day, or 24 hours; In what time does it move one degree?

As $360^\circ : 24^h : 1^\circ : 4'$ *Answer.*

46. Sold a cargo of flax-seed in Ireland, for £1795 10/ Irish money; what does that amount to, in Massachusetts currency, £81 5*s.* Irish being equal to £100 Massachusetts?

As $\overset{\textit{Irish}}{£81\tfrac{1}{4}} : \overset{\textit{Mass.}}{£100} :: \overset{\textit{Irish}}{£1795\tfrac{1}{2}} : \overset{\textit{Mass.}}{£2209\ 16/11}$ *Ans.*

Or, As $13 : 1795\frac{1}{2} :: 16 : £2209\ 16\ 11$ *as before, because* £13 *Irish are equal to* £16 *Massachusetts.*

47. My correspondent in Maryland purchased a cargo of flour for me, for £437 that currency, how much Massachusetts money must I remit him, £125 Maryland being equal to £100 Massachusetts? As £125 : £100 :: £437 : £349 12*s.* *Ans.*

Or, As $5 : \overset{£}{437} :: 4 : \overset{£\ \ \ s.}{349\ \ 12}$ $\left\{\begin{matrix}\text{Because £5 Maryland are equal} \\ \text{to £4 Massachusetts.}\end{matrix}\right.$

48. A Bill of Exchange was accepted at Newbury-port for the payment of £345 10, for the like value delivered in New-York, at £$133\frac{1}{3}$ New-York currency for £100 Massachusetts ditto; how much money was paid in New-York, £75 Massachusetts being equal to £100 of New-York?

As

† A Sidereal day is the space of time which happens between the departure of a Star from, and its return to the same meridian again.

‖ The Solar day is that space of time which intervenes between the Sun's departing from any one meridian, and its return to the same again.

£ Maſſ. £ N.Y. £ Maſſ. £ N.Y.
As 75 : 100 :: 345 10/ : 460 13/4 *Anſ.*

49. WHEN the exchange from Maſſachuſetts to Georgia is £83$\frac{1}{3}$ Georgia *per* £100 Maſſachuſetts, how much Maſſachuſetts money muſt be paid in Boſton to balance £457 Georgia currency ?

£ Geor. £ Maſſ. £ Geor. £ Maſſ.
As 83$\frac{1}{3}$: 100 :: 457 : 548 8*s.* *Anſwer.*

50. A MERCHANT delivered at Boſton £320 Maſſachuſetts currency, to receive £400 in Philadelphia ; what was the Maſſachuſetts pound valued at ?

B. P. B. P.
As £320 : £400 :: £1 : £1 5*s.* *Anſ.*

£ £ £ £ *s*
Or, As 80 : 100 :: 1 : 1 5 { Becauſe £80 Maſſachuſetts are equal to £100 Pennſylvania.

51. IF I draw a bill of exchange for £537 10/6 Maſſachuſetts, to be paid in Ireland, at £123$\frac{1}{13}$ Maſſachuſetts, *per* £100 Iriſh ; for how much Iriſh money muſt I draw the bill ?

Maſſ. Iriſh Maſſ. Iriſh
As £123$\frac{1}{13}$: £100 :: £537 10/6 : £436 14/9$\frac{1}{4}$

£ £ £ £
Or, As 16 : 537 10/6 :: 13 : 436 14/9$\frac{1}{4}$ { Becauſe £16 Maſſachuſetts are = £13 Iriſh.

52. SUPPOSE a bill is drawn in Ireland, and payable in Boſton, for £673 12/6 Iriſh ; how much Maſſachuſetts money comes it to, the exchange at £81$\frac{1}{4}$ Iriſh *per* £100 Maſſachuſetts ?

£ £ £ £
As 81$\frac{1}{4}$: 100 :: 673 12/6 : 829 1/6 $\frac{6}{13}$

THE value of any quantity of Silver in any of the currencies of the United States may be found by the following Proportion.

As the number of grains, contained in 1 £, is to 1 £ ; ſo are the grains, in any given quantity, to its value.

53. WHAT is the value of 1 ℔ of Silver in Maſſachuſetts currency ; the pound (or 20 ſhillings) containing 1393$\frac{1}{2}$ grains ?

As 1393$\frac{1}{2}$: 1 :: 5760 : 4 2/8.

ALL Queſtions in the Rule of Three, whether direct or inverſe, may be ſolved by the following Rule.

LET that number, which is of the ſame name or quality as the number ſought, be the third term ; then, conſider whether the number ſought ſhould be more or leſs than the third ; if *more*, let the *greater* of the two other terms be the *middle* term, and the *leſs*, the *firſt* ; but if the fourth number ought to be *leſs* than the third, then give the *leſs* the *ſecond* place, and the *greater*, the firſt.—The queſtion being thus ſtated, the proportion will be ; As the firſt term is to the ſecond : ſo is the third to the fourth, or number ſought,——

Euclid's Elements V. 14.

Note. The firſt and ſecond terms muſt always be brought into one name, and the third into the loweſt mentioned, then proceed as in

in the common method, by multiplying the second and third terms together, and dividing the product by the first, and the quotient will be the answer, in the same name as the third term was reduced into.

54. If 15 yards of cloth cost £6; how many yards may I have for £125?

	£	£		Yds.
As	6 :	125	::	15
		15		
		625		
		125		
	6)	1875		
		$312\frac{1}{2}$ *Yds. Ans.*		

55. If 12 men can do a Jobb in 20 days; in what time will 18 men do it?

	M.	M.		D.
As	18 :	12	::	20
		20		
	18)	240($13\frac{1}{3}$ *days. Ans.*		
		18		
		60		
		54		
		6		

56. If I give 7/9 for 3 yards; how many yards may I have for £39?

	s. d.		£		Yds.		Yds. qrs. N.
As	7 9	:	39	::	3	:	301 3 $2\frac{90}{93}$ *Answer.*

Or thus,

State the question in the usual way, and let the second term keep its proper, or natural place; then, multiply it by the greater or less extreme, that is, by the first or third number, accordingly as the answer ought to be greater or less; divide the product by the other term, and the quotient will be the answer.

RULE OF THREE DIRECT IN VULGAR FRACTIONS.

Rule.‖

Having made the necessary preparations, as directed in multiplication and division of vulgar fractions, state your question as in whole numbers, and invert the first term of the proportion; then, multiply the three terms continually together, and the product will be the answer.

1. If $\frac{5}{8}$ of a yard cost $\frac{5}{7}$ of a £. what will $\frac{9}{15}$ of an Ell Eng. cost?

$$\frac{5}{8}\,yd. = \frac{5}{8} \text{ of } \frac{4}{1} \text{ of } \frac{1}{5} = \frac{5\times4\times1}{8\times1\times5} = \frac{1}{2} \text{ Ell English.}$$

E. Eng. £. E. Eng.

As $\frac{1}{2} : \frac{5}{7} :: \frac{9}{15}$ And $\frac{2}{1}\times\frac{5}{7}\times\frac{9}{15} = \frac{2\times5\times9}{1\times7\times15} = \frac{90}{105}$£. $= 17/1\frac{1}{2}\ \frac{30}{35}$ *Ans.*

2. If $\frac{3}{5}$ *yd.* cost $\frac{7}{8}$£. what will $40\frac{1}{2}$ *yds.* come to?

Ans. £59 1/3.

3. If 70 bushels of corn cost £$12\frac{3}{5}$; what is it *per* bushel?

Ans. 3/$7\frac{1}{5}$.

4. If

‖ This Rule and the next, depend upon the same principle as the Rule of Three in whole numbers.

4. If $\frac{7}{16}$ of a ship cost £51; what are $\frac{3}{32}$ of her worth?

Ans. £10 18/6$\frac{3}{4}$ $\frac{3}{7}$.

5. At £3$\frac{5}{8}$ *per Cwt.* what will 9$\frac{2}{3}$ ℔ come to? *Ans.* 6/3$\frac{5}{56}$.

6. A Person having $\frac{4}{5}$ of a vessel, sells $\frac{2}{3}$ of his share for £319; what is the whole vessel worth? *Ans.* £598 2/6.

7. A Merchant sold 5$\frac{1}{2}$ pieces of cloth, each containing 12$\frac{2}{3}$ *yds.* at 9*s.* $\frac{1}{2}$*d.* *per yard*; what did the whole amount to?

Ans. £31 9/10$\frac{3}{4}$ $\frac{1}{3}$.

8. *A* buys of *B* £560$\frac{3}{4}$ Bank Stock at £85$\frac{2}{3}$ *per Cent.* what comes it to? *Ans.* £480 7/6$\frac{1}{5}$.

9. A Merchant makes Insurance upon a vessel and cargo, valued at £3750 16*s.* at 15$\frac{1}{2}$ guineas *per Cent.* what does the premium amount to? *Ans.* £813 18/5$\frac{1}{2}$.

10. A Merchant in Holland draws a Bill upon his correspondent in Boston for 3750 Ducats at 8/4$\frac{1}{5}$: How much Massachusetts currency must he receive? *Ans.* £1565 12/6.

11. A Gentleman from Boston being in England where the price of Silver is to that of Gold, as 1 to 15$\frac{1}{14}$, exchanged 158$\frac{1}{4}$ ℔. of Silver for Gold; on his return to Massachusetts, where the price of Silver is to that of Gold, as 1 to 15$\frac{15}{31}$, a friend, wanting his Gold, gave him the value thereof in Silver: what weight of Silver did he gain by the exchange?

	℔ *Sil.*		*G.*		℔ *Sil.*		℔ *Gold.*		*G.*		*Silv.*		*G.*		℔ *Silv.*
As	15$\frac{1}{14}$	:	$\frac{1}{1}$	::	158$\frac{1}{4}$	:	10$\frac{1}{2}$	As	$\frac{1}{1}$	:	15$\frac{15}{31}$	::	10$\frac{1}{2}$	:	162$\frac{36}{62}$

Ans. 4$\frac{41}{124}$*lb.*

12. A Merchant bought a number of bales of Velvet, each containing 129$\frac{17}{27}$ yards, at the rate of 7 dollars for 5 yards, and sold them out at the rate of 11 dollars for 7 yards; and gained 200 dollars by the bargain: how many Bales were there?

	Yds.		*Dol.*		*Yds.*		*Dol.*
As	7	:	11	::	5	:	7$\frac{6}{7}$

Sold 5 *yards for* 7$\frac{6}{7}$ *Dollars.*
Bought 5 *yds. for* 7 *Dollars.*
In 5 *yards gain'd* $\frac{6}{7}$ *Dollar.*

	Dol.		*Yds.*		*Dol.*		*Yds.*			*Yds.*		*B.*		*Yds.*		*Bales.*	
As	$\frac{6}{7}$	:	5	::	200	:	1166$\frac{2}{3}$	*and*	As	129$\frac{17}{27}$	:	$\frac{1}{1}$	::	1166$\frac{2}{3}$	:	9	*Ans.*

Although the method before laid down be universally applicable, yet there are other methods more ready and expeditious in some particular cases.

Rule 1.

If the first and third terms be fractions, and the second a whole number, reduce the first and third to one common denominator, then, rejecting the denominators, make the numerator of the first, the first term, and the numerator of the third, the third term, and work as in whole numbers.

If $\frac{5}{8}$ yard cost 9/: what cost $\frac{7}{12}$ yard at that rate?

$\frac{5}{8}=\frac{15}{24}$ and $\frac{7}{12}=\frac{14}{24}$. Now, As 15 : 9*s.* :: 14 : 8/4$\frac{3}{4}$ *Ans.*

RULE 2.

IF, of the first and third terms, one be 1, and the other a fraction; put the denominator of the fraction instead of 1, and the numerator in the place of the fraction, and work as in whole numbers, as before.

IF 1 Acre of land cost £12 : what will $\frac{5}{8}$ of an acre cost, at that rate?

	Den.		*£.*		*Num.*		*£.*	*s.*	
As	8	:	12	::	5	:	7	10	*Anʃ.*

RULE 3.

IF the ʃecond term be a fraction likewiʃe, (that is, if all the terms be fractions) : having reduced the firʃt and third to one common denominator, multiply the numerator of the firʃt term by the denominator of the ʃecond, for a diviʃor,—and the numerator of the third by the numerator of the ʃecond, for a dividend ; divide the laʃt product by the firʃt, and the quotient will be the anʃwer.

IF $\frac{1}{2}$ yard of cloth coʃt $\frac{3}{4}$£. what coʃt $\frac{7}{8}$ yard?

$\frac{1}{2} = \frac{4}{8}$, which reduces it to a common denominator, then

As 4 : $\frac{3}{4}$ :: 7
4 ... 3
—— ... ——
16 ... 16)21($1\frac{5}{16} = 26/3$. *Anʃ.*
16
——
5

To find the value of Gold in Maʃʃachuʃetts Currency.

PROBLEM 1. Given the weight of any quantity of Gold, to find its value.

THEOREM 1. As 1 *Oz.* : $5\frac{1}{3}$ *£.* :: 12 *Oz.* : 64 *£.* :: 1 *pwt.* : $5\frac{1}{3}$ *s.* :: 1 *gr.* : $2\frac{2}{3}$ *d.* (Caʃe 1.) $= \frac{2\frac{2}{3}}{1}$ (Caʃe 2.) $= \frac{5\frac{1}{3}}{2}$ (Caʃe 3.) $= \frac{8}{3}$. therefore,

Rule 1. IF the given quantity be in grains ; ʃay, As the denominator is to the number of grains : ʃo is the numerator to their value, in pence.

1. WHAT is the value of 18 grains of Gold?

By Caʃe 1.	*By Caʃe* 2.	*By Caʃe* 3.
Gr.	*Gr.*	*Gr.*
As 1 : 18 :: $2\frac{2}{3}$	As 2 : 18 :: $5\frac{1}{3}$	As 3 : 18 :: 8
$2\frac{2}{3}$	$5\frac{1}{3}$	8
36	90	3)144
12	6	48=4*s.*
12)48(4*s.* *Anʃ.*	2)96(48*d.*=4*s.*	
48		

Rule 2. IF the given quantity consist of ounces, penny-weights and grains, halve the grains, and then proceed as in multiplication of pounds, shillings and pence, making the numerator in Case 2d. the multiplier.

1. WHAT is the value of 7 *oz.* 8 *pwt.* 16 *gr.* of Gold?

$$\overset{gr.}{16} \div 2 = \overset{gr.}{8}, \text{ then, } \begin{array}{rrr} oz. & pwt. & gr. \\ 7 & 8 & 8 \\ & & 5\frac{1}{3} \\ \hline 37 & 3 & 4 \\ 2 & 9 & 6\frac{2}{3} \\ \hline \pounds 39 & 12 & 10\frac{2}{3} \end{array} \textit{ Ans.}$$

Rule 3. IF the given quantity consist of pounds only, multiply by 64, and the product will be the answer; but if it consist of pounds, ounces, &c. it will be most convenient to reduce the pounds to ounces, and proceed by Rule 2.

1. WHAT is the value of 36℔. of Gold, at £64 *per* ℔?

$$\begin{array}{r} 36 \\ 64 \\ \hline 144 \\ 216 \\ \hline \pounds 2304 \end{array} \textit{ Ans.}$$

2. WHAT is the value of 15℔. 9*oz.* 12*pwt.* 18*gr.* of Gold?

$$\begin{array}{rrrl} 15 & & & \\ 12 & & & \\ \hline & pwt. & gr. & \quad gr. \\ oz\ 189 & 12 & 9 & = 18 \div 2 \\ & & 5\frac{1}{3} & \\ \hline 948 & 3 & 9 & \\ 63 & 4 & 3 & \\ \hline \pounds 1011 & 8 & 0 & \textit{Ans.} \end{array}$$

PROB. 2. To ascertain the value of any given quantity of gold in Spanish milled dollars.

THEOREM 2. 1 *pwt.* of gold $= 5\frac{1}{3}s.$ 1 dollar $= 6s.$ And $\frac{5\frac{1}{3}}{6} = \frac{16}{18} = \frac{8}{9}$, therefore,

Rule. REDUCE the given quantity of gold to pennyweights; then, As the denominator is to the given quantity : so is the numerator to the answer in dollars. *Or,*

DIVIDE by the denominator and multiply the quotient by the numerator. *Or,*

DIVIDE

DIVIDE by the denominator and ſubtract the quotient from the dividend, in either caſe, you will have the anſwer.

1. WHAT is the value of 6*oz.* 6*pwt.* of gold, in Spaniſh dollars?

$$\begin{array}{r} 20 \\ \hline 126 \ pwt. \end{array}$$

$$\begin{array}{rl} & pwt. \\ \text{As } 9 : & 126 :: 8 \\ & \quad 8 \\ \hline 9) & 1008 \\ \hline \textit{Anſ.} & 112 \ \textit{Dollars.} \end{array}$$

Or,

$$9)126$$

$$14 \times 8 = 112 \ \textit{Anſ.}$$

Or,

$$\begin{array}{r} 9)126 \\ \text{Subt. } 14 \\ \hline 112 \ \textit{Anſ.} \end{array}$$

2. IN 7 *oz.* 13 *pwt.* 17 *gr.* how many dollars?

$$\begin{array}{r} 20 \\ \hline 153\frac{17}{24} \\ 24 \\ \hline 619 \\ 307 \\ \hline 3689 \end{array}$$

$$\text{As } \frac{9}{1} : \frac{3689}{24} :: \frac{8}{1} : \frac{29512}{216}$$

$$\begin{array}{r} 216)29512(136 \ \textit{Dollars.} \\ 216 \\ \hline 791 \\ 648 \\ \hline 1432 \\ 1296 \\ \hline 136 \\ 6 \\ \hline 216)816(3s. \\ 648 \\ \hline 168 \\ 12 \\ \hline 216)2016(9d. \\ 1944 \\ \hline 72 \\ 4 \\ \hline 216)288(1qr. \\ 216 \\ \hline 72 \end{array}$$

3. IN 9 *oz.* 8 *pwt.* 6 *gr.* how many dollars?

$$\begin{array}{r} 20 \\ \hline 188 \end{array}$$

$$\begin{array}{rl} & pwt. \\ \text{As } 9 : & 188\frac{1}{4} :: 8 \\ & \quad 8 \\ \hline 9) & 1506 \\ \hline \textit{Anſ.} & 167\frac{1}{3} \ \textit{or 3 eight pences.} \end{array}$$

OR, ſuppoſe it were required to reduce the quantity of gold to Dollars, 9oths and 8ths of a 9oth?—

FIND the value for ounces and pennyweights as in the firſt Example; the quotient will be dollars, and the remainder, (if any) 9ths of a dollar: then, as one grain is very nearly $\frac{3}{90}+\frac{2}{8}$ of $\frac{1}{90}$ of a dollar; divide the grains by 3, and the quotient will be 9ths. of a dollar:—then multiply this remainder into $\frac{3}{90}+\frac{2}{8}$, and add all to the other work.

4. IN

4. In 5*oz.* 19*pwt.* 17*gr.* how many dollars, 90ths and 8ths of a 90th?

$$9 : \overset{oz.}{5}\ \overset{pwt.}{19} :: 8$$

$$\begin{array}{r} 20 \\ \hline 119 \\ 8 \\ \hline 9)952 \\ \hline \end{array}$$

$$\begin{array}{l} \quad gr. \\ 3)17(5 \text{ ninths} = \tfrac{50}{90} \\ \quad 15 \\ \hline \quad 2 \times \tfrac{3}{90} + \tfrac{2}{8} = \tfrac{6}{90} + \tfrac{4}{8} \end{array}$$

$105\frac{7}{9} = \frac{70}{90}$. then $105\frac{70}{90} + \frac{50}{90} + \frac{6}{90} + \frac{4}{8} = 106\frac{36}{90}, \frac{4}{8}$ *Answer.*

Prob. 3. To ascertain the weight of gold equivalent to any given sum, currency.

Rule 1. If the given sum be in pence, reverse *Rule* 1. *Theor.* 1. that is; As the numerator 8 is to the given sum in pence: so is the denominator 3 to the weight required, in grains.

What weight of gold is equal to 4*s*?

$$\text{As } 8 : \overset{d.}{48} :: 3 \qquad \begin{array}{r} 12 \\ \hline 48d. \end{array}$$

$$\begin{array}{r} 3 \\ \hline 8)144 \\ \hline \end{array}$$

Answ. 18 *grains.*

Rule 2. If the given sum be in pounds, shillings and pence; As $\frac{5\frac{1}{3}}{1}$ is equal to $\frac{16}{3}$; therefore, divide the given sum by 8, and that quotient by 2; add the two quotients together, double the last denomination, and you will have the answer.

What quantity of gold is equivalent £45 13*s*4?

Mark the pounds, shillings and pence, as oz. pwt. and gr.

$$\begin{array}{rrrl} & oz. & pwt. & gr. \\ 8) & 45 & 13 & 4 \\ \hline 2) & 5 & 14 & 2 \\ & 2 & 17 & 1 \\ \hline & 8 & 11 & 3+3 \\ \hline Oz. & 8 & 11 & 6 \end{array} \quad \begin{array}{l} \\ \\ \\ \left.\right\} Add. \\ \\ \\ Answ. \end{array}$$

Prob. 4. To find the weight of gold equivalent to any given number of dollars.

Rule. As the numerator 8 is to the number of dollars; so is the denominator 9 to the answer in pennyweights:—*Or*, divide the dollars by the numerator 8 and add the quotient to the dividend.

Or,

Or, divide as before, and multiply the quotient by the denominator 9; in either caſe you will have the anſwer.

Required the weight of gold equal to 76 dollars.

As 8 : 76 :: 9 *Or thus* 8)76 *Or* $9\frac{1}{2} \times 9 = 85\frac{1}{2}$ *pwt.*

9 $9\frac{1}{2}$

8)684 *Anſw.* $85\frac{1}{2}$ *pwt.*

oz. pwt. gr.

Anſw. $85\frac{1}{2}$ *pwt.* = 4 5 12

RULE OF THREE DIRECT IN DECIMALS.

Rule.

Having reduced your fractions to decimals, and ſtated your queſtion as in whole numbers, multiply the ſecond and third together; divide by the firſt, and the quotient will be the anſwer.

1. If $\frac{5}{8}$ of a yard coſt $\frac{7}{12}$ of a pound; what will $9\frac{2}{3}$ yards come to?

$\frac{5}{8}$ = ,625 $\frac{7}{12}$ = ,583+ and $\frac{2}{3}$ = ,666+

Yd. £. *Yd.*

As ,625 : ,583 :: 9,666

,583

28998

77328

48330

,625)5,635278(9,0164+ = £9 0*s.* $3\frac{3}{4}$*d.* *Anſw.*

5 625

1027

625

4028

3750

2780

2500

280

2. If 1*oz.* of Silver coſt 6*s.* 8*d.* what is the price of a Bowl, which weighs 1 ℔. 7*oz.* 13*gr.*? *Anſw.* £6 6/10.

3. If $9\frac{3}{4}$ yards coſt £3 7 6; what will $\frac{1}{2}$ yard come to? *Anſw.* 3*s.* $5\frac{1}{2}$*d.*

4. If 1 *hhd.* Sugar, weighing 9*Cwt.* 3*qrs.* 14 ℔. coſt £27 13/7; what will 3*Cwt.* 1*qr.* 17 ℔. ditto come to? *Anſw.* £9 $10/8\frac{1}{4}$.

5. A Tobacconiſt bought 5 *hhds.* of Tobacco, each weighing 8*Cwt.* 2*qrs.* 19 ℔. for £161 16 8; what was it *per* ounce? *Anſw.* $\frac{1}{2}$*d.*

X 6. There is a Ciſtern, which has 3 cocks, the firſt will empty it in $\frac{1}{4}$ of an hour, the ſecond in $\frac{3}{4}$, and the third in $1\frac{1}{2}$ hour; in what time will it be emptied, if all three run together?

As

$$\text{As}\left\{\begin{array}{l} \overset{b.}{,25} : \overset{Cist.}{1} :: \overset{b.}{1} : \overset{Cist.}{4} \\ ,75 : 1 :: 1 : 1,333+ \\ 1,5 : 1 :: 1 : 0,666+ \end{array}\right.$$

6 *Cist.*

Cist. *h.* *Cist.*
As 6 : 1 :: 1 : ,1666+ = 10 *min. ans.*

7. A Conduit has a Cock, which, running into a Ciſtern, will fill it in 12 minutes:—This Ciſtern has 3 Cocks; the firſt will empty it in $1\frac{1}{4}$ hour, the ſecond in $37\frac{1}{2}$ minutes, and the third in $\frac{1}{2}$ an hour: In what time will the Ciſtern be emptied, if all four run together?

$$\text{As}\left\{\begin{array}{ll} \overset{h.}{,2} : \overset{Cist.}{1} :: \overset{h.}{1} : \overset{Cist.}{5} & \textit{the filling Cock.} \\ \left.\begin{array}{l} 1,25 : 1 :: 1 : 0,8 \\ ,625 : 1 :: 1 : 1,6 \\ ,5 : 1 :: 1 : 2 \end{array}\right\} & \textit{emptying Cocks.} \end{array}\right.$$

4,4

$\left\{\begin{array}{l} 5 \text{ Cisterns filled in an hour.} \\ 4,4\ \textit{do.}\text{ emptied in an hour.} \\ ,6 \text{ Cistern, difference.} \end{array}\right.$

Cist. *h.* *Cist.* *h.* *h. m.*
Then, as ,6 : 1 :: 1 : 1,66+ = 1 40 *Anſw.*

Dol. d. c.
8. If 19 yards coſt 25 ,7 5; what will $435\frac{1}{2}$ yards come to?

Yds. *dol. d.c.* *Yds.*
As 19 : 25,7 5 :: 435,5
25,75
———
217 75
3048 5
21775
8710
——— *D d c m*
19)11214,125(590,2 1 7$\frac{2}{19}$ *Anſ.*
95
——
171
171
——
41
38
——
32
19
——
135
133
——
2

D.d.c.m.
9. If 345 yards of Tape coſt 5, 1 7 5; what will one yard coſt?

Yds. *D. d. c. m.* *Yd.*
As 345 : 5, 1 7 5 :: 1
1
——— *d. c. m.*
345)5, 1 7 5(,0 1 5 *Anſw.*
3 4 5
———
1 7 2 5
1 7 2 5
———

Dol. d.c.m.
10. If I give 12, 8 2 5 for 675 Tops; how many Tops will 19 Mills buy?

D. d.c.m. *T.* *d.c.m.*
As 12, 8 2 5 : 675 :: ,0 1 9
,019
———
6075
675
———
12,825)12,825(1 *Top, Anſw.*
12,825
———

RULE

RULE OF THREE INVERSE,

OR

RECIPROCAL PROPORTION,

Teacheth, by having three numbers given, to find a fourth, which ſhall have the ſame proportion to the ſecond, as the firſt has to the third.

Therefore, the greater the third term is, in reſpect to the firſt, the leſs will the fourth term be, in reſpect to the ſecond; or, the leſs the third term is in proportion to the firſt, the greater the fourth muſt be in proportion to the ſecond; and this is called *reciprocal, inverted* or *indirect* Proportion.

The principal difficulty, that will embarraſs the learner, will be, to diſtinguiſh when the proportion is direct, and when indirect.—This is done by an attentive conſideration of the ſenſe and tenor of the queſtion propoſed: for if thereby it appears that when the third term of the ſtating is leſs than the firſt, the anſwer muſt be leſs than the ſecond; or when the third is greater than the firſt, the anſwer muſt be greater than the ſecond; then the proportion is direct: But, if the third be leſs than the firſt, and yet the ſenſe of the queſtion requires the fourth to be greater than the ſecond; or if the third being greater than the firſt, the anſwer muſt be leſs than the ſecond, the proportion is inverſe.

Rule. ‡

State and reduce the terms as in the Rule of Three direct; then, multiply the firſt and ſecond terms together, and divide the product by the third; the quotient will be the anſwer in the ſame denomination as the middle term was reduced into.

If there be fractions in your queſtion, they muſt be ſtated as before directed, and if they be vulgar, invert the third term: then multiply the three terms continually together, and the product will be the anſwer.

Examples.

1. How much Shalloon, that is $\frac{3}{4}$ yard wide, will line $6\frac{3}{4}$ yards of Cloth which is $1\frac{1}{4}$ yard wide?

	Yd.		*yds.*		*qr.*
As	$1\frac{1}{4}$	:	$6\frac{3}{4}$	::	3
	4		4		
	5		27		

	qrs.		*qrs.*		*qrs.*
As	5	:	27	::	3
			5		
			3)135		
			4)45		
			$11\frac{1}{4}$ *yds. Anſwer.*		

The

‡ The reaſon of this rule may be explained from the principles of Compound Multiplication and Compound Diviſion, in the ſame manner as the direct rule.—*For example.* If 4 men can do a piece of work in 12 days; in what time will 8 men do it?

As

The same by Vulgar Fractions.

First. $1\frac{1}{4}=\frac{5}{4}$, $6\frac{3}{4}=\frac{27}{4}$, and 3 *qrs.*$=\frac{3}{4}$, then

As $\overset{yd.}{\frac{5}{4}} : \overset{yds.}{\frac{27}{4}} :: \overset{yd.}{\frac{3}{4}}$ And $\frac{5}{4}\times\frac{27}{4}\times\frac{4}{3}=\frac{5\times27\times4}{4\times4\times3}=\frac{540}{48}=\frac{45}{4}=\overset{yds.}{11\frac{1}{4}}$ *Ans.*

The same by Decimal Fractions.

$1\frac{1}{4}$ = 1,25. $6\frac{3}{4}$ = 6,75. and 3*qrs.* = ,75, then

```
   yd.    yds.     yd.
As 1,25 : 6,75 : : ,75
          1,25
          ----
          3375
         1350
         675
         ------
,75)8,4375(11,25 yds. Ans.
    75
    --
     93
     75
     ---
     187
     150
     ---
      375
      375
```

2. What length of board $7\frac{1}{2}$ inches wide, will make a square foot?

In.br. *In.len.* *In.br.* *In.len.*
As 12 : 12 :: $7\frac{1}{2}$: $19\frac{1}{5}$ *Ans.*

3. How many yards of carpet, $2\frac{3}{4}$ feet wide, will cover a floor, which is 18 feet long and 16 feet wide?

As $\overset{ft.}{16} : \overset{ft.}{18} :: \overset{ft.\ \ ft.}{2\frac{3}{4}\times3} : \overset{yds.}{34\frac{10}{11}}$ *Ans.*   { Note, *I multiply* $2\frac{3}{4}$ *by* 3, *because* 3 *feet* = 1 *yard.*

4. Suppose I lend a friend £350 for 5 months, he promising the like kindness: but, when requested, can spare but £125, how long may I keep it to balance the favour?

As $\overset{£}{350} : \overset{Mo.}{5} :: \overset{£}{125} : \overset{Mo.}{14}$ *Answer.*

5. Suppose 450 men are in a garrison, and their provisions are calculated to last but 5 months; how many must leave the garrison, that the same provisions may be sufficient for those who remain, 9 months?

As $\overset{Mo.}{5} : \overset{Men.}{450} :: \overset{Mo.}{9} : \overset{Men.}{250}$, and $450 - \overset{Men.}{250} = 200$ Men. *Ans.*

6. If a man perform a journey in 15 days, when the day is 12 hours long; In how many will he do it, when the day is but 10 hours?

As $\overset{h.}{12} : \overset{d.}{15} :: \overset{h.}{10} : \overset{d.}{18}$ *Answer.*

As 4 *men* : 12 *days* :: 8 *men* : $\frac{4\times12}{8}$ = 6 *days, the Answer.*

And here the product of the first and second terms, that is, 4 times 12, or 48 is evidently the time in which one man would perform the work; therefore, 8 men will do it in one eighth part of the time, or 6 days.

7. If a piece of land, 40 rods in length, and 4 in breadth, make an acre; how wide must it be, when it is but 19 rods long, to make an acre?

Leng. *Br.* *Leng.* *Br. ft. in.*
As 40 : 4 :: 19 : 8 6 $11\frac{7}{19}$ *Answer.*

8. If, when wheat is *6s per* bushel, the two-penny loaf weigh 9,6*oz.* what ought it to weigh, when wheat is *7s6 per* bushel?

s. *oz.* *s. d.* *oz. pwt. gr.*
As 6 : 9,6 :: 7 6 : 7 13 14,4 *Ans.*

9. If a piece of board be 30 inches in length; what breadth will make $1\frac{1}{2}$ square foot?

sq. f. *in.* *in.* *in.*
As 1,5 : 1 :: 30 : 7,2 *Ans.*

10. If 9 men can build a house in 5 months, by working 14 hours *per* day; in what time will the same number of men do it, when they work only 10 hours *per* day?

h. *mo.* *h.* *mo.*
As 14 : 5 :: 10 : 7 *Ans.*

11. A Wall, which was to be built 24 feet high, was raised 8 feet by 6 men, in 12 days; how many men must be employed, to finish the wall in 4 days?

ft. *men.* *ft.* *men.*
As 8 : 6 :: $\overline{24-8}$: 12 to finish it in 12 days. And

days. *men.* *days.* *men.*
As 12 : 12 :: 4 : 36 to finish it in 4 days.

12. There is a cistern having a pipe, which will empty it in 6 hours; how many pipes, of the same capacity, will empty it in 20 minutes?

ho. *pi.* *min.* *pi.*
As 6 : 1 :: 20 : 18 *Ans.*

13. What number of men must be employed, to finish in 9 days, what 15 men would be 30 days about?

da. *men.* *da.* *men.*
As 30 : 15 :: 9 : 50 *Answer.*

14. If a Field will feed 6 cows 91 days; how long will it feed 21 cows?

cows. *d.* *cows.* *d.*
As 6 : 91 :: 21 : 26 *Ans.*

15. How much in length, that is $8\frac{5}{7}$ inches broad, will make a foot square?

in. *in.* *in.* *in.*
As $\frac{144}{1}$: $\frac{1}{1}$ :: $8\frac{5}{7}$: $16\frac{32}{61}$ *Answer.*

16. How much in length, that is $13\frac{7}{8}$ poles in breadth, will make a square acre?

po. *po.* *po.* *po.*
As $\frac{160}{1}$: $\frac{1}{1}$ :: $13\frac{7}{8}$: $11\frac{59}{111}$

17. A Regiment of soldiers, consisting of 745 men, is to be cloathed; each suit to contain $3\frac{1}{2}$ yards of cloth, which is $1\frac{3}{8}$ yard wide, and

and lined with shalloon $\frac{7}{8}$ yard wide; how many yards of shalloon will line them?

As $\overline{745 \times 3\frac{1}{2}}$: $1\frac{3}{8}$ :: $\frac{7}{8}$: $4097\frac{1}{2}$ *yards. Answer.*

18. If a Suit of Clothes can be made of $4\frac{5}{8}$ yards of cloth, $1\frac{3}{8}$ yard wide; how many yards of coating $\frac{7}{8}$ of a yard wide will it require for the same person?

	Yd.		Yd.		Yd.		Yds. qr. n.	
As	$1\frac{3}{8}$	:	$4\frac{5}{8}$	::	$\frac{7}{8}$	:	6 1 $3\frac{5}{7}$	*Answer.*

ABBREVIATIONS.

To know whether a fraction, when abbreviated, be equivalent in all respects to the original given fraction.

RULE.

As the numerator of the fraction, in its lowest terms, is to its denominator; so will the numerator of the original fraction be to its own denominator.

Or, as one numerator is to the other; so will one denominator be to the other, &c.

A owes B £75 13*s.* 6*d.* now £100 of A's money is equal to £140 of B's; what must A pay to satisfy the said debt?

$$\frac{100}{140} = \frac{5}{7} \textit{ therefore, } \begin{array}{rrr} \text{£.} & s. & d. \\ 75 & 13 & 6 \\ & & 5 \\ \hline 7)378 & 7 & 6 \\ \hline \text{£}54 & 1 & 0\frac{6}{7} \end{array} \textit{ Answer.}$$

Now, to prove whether $\frac{5}{7}$ be equal to $\frac{100}{140}$.

	Num.		Den.		Num.		Den.		Num.		Num.		Den.		Den.
As	5	:	7	::	100	:	140——*Or*, as		5	:	100	::	7	:	140.

COMPOUND PROPORTION,

OR,

DOUBLE RULE OF THREE

Teacheth to resolve such questions as require two, or more, statings by simple proportion; and that, whether direct or inverse:— It is composed (commonly) of 5 numbers to find a sixth, which, if the proportion be direct, must bear such proportion to the 4th and 5th as the 3d bears to the 1st and 2d: but if inverse, the 6th number must bear such proportion to the 4th an 5th as the first bears to the 2d and 3d.

FIRST

FIRST METHOD †

By two, or more, proportions in the Single Rule of three.

RULE.

1. LET either of the two numbers, of which the queſtion is raiſed, be put in the third place, and the correſpondent number, of the ſame name or kind, in the firſt; the ſecond will be that, which has no correſpondent number given.

2. THREE of the five given numbers being thus ſtated, find a fourth proportional.

3. PUT this fourth number for a ſecond number of a ſecond ſtating, the remaining number, of which the queſtion is raiſed, the third, and its correſpondent number of the ſame name, the firſt, then will the fourth number reſulting be the anſwer.

IF a principal of £100 gain £6 intereſt in a year; what will a principal of £400 gain in 9 months?

HERE, of the five given numbers, £100 principal, £6 intereſt and a year or 12 months, are conjoined in form of a ſuppoſition, and thereupon a queſtion is raiſed concerning £400 for 9 months; wherefore, let either of the two numbers £400 or 9 months be put for the third number of the firſt ſtating, and its correſponding term £100 or 12 months, for the firſt.

	£.		£.		£.			Mo.		£.		Mo.
As	100	:	6	::	400		As	12	:	24	::	9
					6					9		
				1,00)	24,00				12)	216		
					£24 *Anſ.*					£18 *Anſw.*		

Or thus,

	Mo.		£.		Mo.		£.			£.		£.		£.		£.
As	12	:	6	::	9	:	$4\frac{1}{2}$.	And	As	100	:	$4\frac{1}{2}$	::	400	:	18.

SUCH queſtions as, when ſtated, are found to have both ſtatings direct, may be ſolved more readily by one compound ſtating, thus: Place the two terms, of which the queſtion is raiſed, under one another in the third place, their correſpondent terms under each other in the firſt, and the remaining term in the middle: then multiply both theſe firſt terms together, and the third terms together, and ſo the double ſtating is reduced to a ſimple one of the *Rule of Three direct*; *viz.* the product of the two firſt terms is the firſt of a ſimple ſtating;

† THE reaſon of this rule may be ſhewn from the nature of direct and inverſe proportion:—For in this rule, every row is a particular ſtating in one of thoſe rules; and, therefore, if all the ſeparate dividends be collected into one dividend, and all the diviſors into one diviſor, their quotients muſt be the anſwer ſought: Thus, in example 1ſt,

$$\overset{\pounds.}{\text{As } 100} : \overset{\pounds.}{6} :: \overset{\pounds.}{400} : \frac{400 \times 6}{100}, \text{ and as } \overset{Mo.}{12} : \overset{\pounds.}{\frac{400 \times 6}{100}} :: \overset{Mo.}{9} : \frac{400 \times 6 \times 9}{100 \times 12}$$

by the Rule of Three direct.

ting; the second term is the second, and the product of the two third terms is the third, to find a fourth proportional—Thus

$$\text{As}\left\{\begin{matrix}100\\12\end{matrix}\right\} : 6 :: \left\{\begin{matrix}400\\9\end{matrix}\right\}$$

So the first example will stand thus.

$$\left.\begin{matrix}£.\ 100\\ Mo.\ 12\end{matrix}\right\} : £6 :: \left\{\begin{matrix}400\ £.\\ 9\ Mo.\end{matrix}\right\}$$

```
12|00        36|00
                6
            12)216
            £18 Answ.
```

SECOND METHOD.

ALWAYS place the three conditional terms in this order: That number, which is the principal cause of gain, loss or action, possesses the first place; that which denotes the space of time or distance of place, the second; and that, which is the gain, loss or action, the third: this being done, place the other two terms, which move the question, under those of the same name, and if the blank place, or term sought, fall under the third place, then the question is in direct proportion; therefore,

Rule 1. MULTIPLY the three last terms together, for a dividend, and the two first for a divisor:——But if the blank fall under the first or second place; then, the proportion is inverse; therefore,

Rule 2. MULTIPLY the first, second and last terms together for a dividend, and the other two for a divisor, and the quotient will be the answer.

1. IF £100 gain £6 in a year; what will £400 gain in 9 months?

P. £. Mo. Int. £.
100 : 12 :: 6 . *Terms in the supposition, or conditional Terms.*
400 : 9 *Terms which move the question.*

HERE, the blank falling under the third place, the question is in direct proportion, and the answer must be found by the first Rule; therefore, $400 \times 9 \times 6 = 21600$ *For the dividend, and*
$100 \times 12 = 1200$ *For the divisor.*

See the work at large.

```
      £.Pr.  Mo.  £.Int.
       100 : 12 :: 6
       400 : 9
               9          12,00)216,00(18£. Answer.
                                 12
  100      3600                  ---
   12         6                  96
 -----   ------                  96
12,00)216,00(                    ---
```

2. If £100 will gain £6 in a year; In what time will £400 gain £18?

```
£.    Mo.    £.
100 : 12 :: 6    Terms in the supposition.
400 :    :: 18   Terms which move the question.
```

Here, the blank falling under the 2d place, the question is in reciprocal or inverse Proportion, and the answer must be sought by the second Rule; therefore,

$100 \times 12 \times 18 = 21600$ *For the dividend.*

$400 \times 6 = 2400$ *For the divisor.*

```
Pr.£      mo.      Int.£
100   :   12   ::   6
400            ::  18
  6                12
----               ---
2400               216
                   100

            24|00)216|00(9 months, answer.
                  216
```

3. If £400 gain £18 in 9 months; what is the rate *per Cent. per Annum*?

```
Pr.      mo.     Int.
400  :   9   ::  18
100  :  12
        18
        --
        96
        12
        ---
400    216
  9    100
----  -----
36|00)216|00(£6 Answ.
      216
```

4. What principal, at 6 *per Cent. per Ann.* will gain £18 in 9 months?

```
Pr.   Mo.   Int.
100 : 12 :: 6
       9 :: 18
            12
      ---------
       9   216
       6   100
      --   ---£
      54)21600(400 Ans.
         216
         ---
          00
```

5. If 8 men spend £32 in 13 weeks; what will 24 men spend in 52 weeks?

```
M.   W.    £.
 8 : 13 :: 32
24 : 52        £384 Answ.
```

6. If the freight of 9 *hhds.* of Sugar, each weighing 12 *Cwt.* 20 leagues, cost £16; what must be paid for the freight of 50 tierces ditto, each weighing $2\frac{1}{2}$ *Cwt.* 100 leagues?

```
hhds. leag.   £.
  9 :  20 :: 16
 50 : 100
```

£23 $2/11\frac{1}{2}$ $\frac{2}{9}$ *Ans.*

7. There

7. There was a certain Edifice completed in a year by 20 workmen; but the same being demolished; it is necessary that just such an one should be built in 5 months; I demand the number of men to be employed about it?

men.		*mo.*		*Ed.*
20	:	12	::	1
		5	::	1 — 48 *men, answer.*

8. If 6 men build a wall 20 feet long, 6 feet high and 4 feet thick in 16 days; in what time will 24 men build one 200 feet long, 8 feet high and 6 feet thick?

men.		*da.*		*ft.*
6	:	16	::	$20\times6\times4$
24	:		::	$200\times8\times6$ 80 *days, answer.*

Third Method.

That number, which is of the same name as the number sought, must be the last term, on the right hand; then, take any two of the other numbers, which are of one kind, and if, when compared with the last number, *more* be required; set the greater in the second place, (next to the last term, with four dots between) and the less in the first, (at the left hand, with two dots between:) But, if *less* be required, let the less stand in the second place, and the greater in the first: When these three numbers are properly stated, take any two others, of one kind, which remain in the question, and compare them with the last number, as before, to find whether they require a greater or a less answer, and set them accordingly, immediately to the left hand of those, already stated, with dots, as before directed;—thus proceed with every two remaining numbers, 'till all stand in one continued line.

Place A over the first, third, fifth, &c. numbers, omitting the last, and call them *antecedents*; and C over the second, fourth, sixth, &c. and call them *consequents*; this being done, multiply all the antecedents continually together, for your first term; and all the consequents continually together, for the second: Then will the proportion be: As the product of the antecedents is to the product of the consequents; so will the last number be to the answer.

Euclid's Elements, V. 12.

Take the 8th question in the second method.

If 6 men build a wall 20 feet long, 6 high and 4 thick in 16 days; in what time will 24 men build one 200 feet long, 8 high, and 6 thick?

A	C		A	C		A	C		A	C		
ft.th.	*ft.th.*		*ft.hi.*	*ft.hi.*		*ft.lon.*	*ft.lon.*		*men.*	*men.*		
4	6	::	6	8	::	20	200	::	24	6	::	16

```
20          6
24        200
---       ----
80       1200
40          8
---       ----
480      9600
6           6
---      -----
2880     57600
4
---
11520
```

As 11520 : 57600 :: 16

```
     57600
        16
    ------
    345600
    57600
1152|0)92160|0(80 days, answer.
     9216
     ----
        0
```

Comparison *of* WEIGHTS *and* MEASURES.

Examples.

1. If 78 pence Massachusetts be worth 1 French Crown; how many Massachusetts pence are worth 320 French Crowns?

	Fr.Cr.		*d.*		*Fr.Cr.*
As	1	:	78	::	320

```
   320
    78
  ----
  2560
 2240
 -----
 24960 Answ.
```

2. If 24 yards at Boston make 16 Ells at Paris; how many Ells at Paris will make 128 yards at Boston?

	Bost.		*Par.*		*Bost.*		*Par.*
As	24 *yds.*	:	16 *ells*	::	128 *yds.*	:	$85\frac{1}{3}$ *ells, answ.*

3. If 60℔. at Boston make 56℔. at Amsterdam; how many ℔. at Boston will be equal to 350 at Amsterdam?

	Ams.		*Bost.*		*Ams.*		*Bost.*	
	℔.		℔.		℔.		℔.	
As	56	;	60	::	350	:	375	*Answ.*

4. If

4. If 95lb. Flemish make 100lb. American; how many American lbs. are equal to 550lb. Flemish?

	Flem.		*Amer.*		*Flem.*		*Amer.*	
As	95lb.	:	100lb.	::	550lb.	:	$578\frac{90}{95}$lb.	*Answ.*

CONJOINED PROPORTION

Is when the Coins, weights or measures of several countries are compared in the same question; or in other words, it is joining many proportions together, and by the relation, which several antecedents have to their consequents, the proportion between the first antecedent and the last consequent is discovered, as well as the proportion between the others in their several respects.

This Rule may generally be so abridged by cancelling equal quantities on both sides, and abbreviating commensurables, that the whole operation may be performed with very little trouble—and it may be proved by as many statings in the single Rule of Three as the nature of the question may require.

Case I.

When it is required to find how many of the first sort of coin, weight, or measure, mentioned in the question, are equal to a given quantity of the last.

Rule.

Place the numbers alternately, that is, the Antecedents at the left hand, and the Consequents at the right, and let the last number stand on the left hand; then multiply the left hand column continually for a dividend, and the right hand for a divisor, and the quotient will be the answer.

Examples.

1. Suppose 100 yards of America = 100 yards of England, and 100 yards of England = 50 Canes of Toulouse, and 100 Canes of Thoulouse = 160 Ells of Geneva, and 100 Ells of Geneva = 200 Ells of Hamburg: How many yards of America are equal to 379 Ells of Hamburg?

Antecedents.		*Consequents.*	*Abridged.* *Ant.*	*Con.*
100 of America	=	100 of England.		
100 of England	=	50 of Thoulouse.	5	8
100 of Thoulouse	=	160 of Geneva.	379	
100 of Geneva	=	200 of Hamburg.		
379 of Hamburg?				

Therefore $\frac{379 \times 5}{8} = 236\frac{7}{8}$ *yds.* of America = 379 *Ells* of Hamburg.

Illustration.

The two 100's on both sides cancel each other;—Let the last cyphers of the three next antecedents and consequents be cancelled, which is dividing by 10: then divide the second antecedent and

consequent by 5, and the quotients will be 2 on the side of the antecedents, and 1 on the side of the consequents; then 2 will measure the third antecedent and consequent, and the quotients will be 5 and 8.—10 will measure the 4th antecedent and consequent, and the quotients will be 1 and 2: now, there being 2 left on each side, they cancel each other, and as there is no further room for abridging by reason of the odd number 379, the operation is finished, and the answer found, as above.

2. If 20℔. at Boston make 23℔. at Antwerp, and 155 at Antwerp make 180 at Leghorn; how many at Boston are equal to 144 at Leghorn?

Antecedents.		*Consequents.*	
℔.		℔.	
20 of Boston	=	23 of Antwerp.	$20\times155\times144=446400$ *Divid.*
155 of Antwerp	=	180 of Leghorn.	$23\times180=4140$ *Divisor.*
144 of Leghorn.			$414\|0)44640\|0(107\frac{19}{23}$ *Answ.*

Or, abridged $\frac{155\times144}{23\times9} = 107\frac{19}{23}$

3. If 12℔ at Boston make 10 at Amsterdam, 100℔ at Amsterdam 120 at Paris; how many ℔ at Boston are equal to 80℔ at Paris? *Answ.* 80℔.

4. If 140 braces at Venice be equal to 150 braces at Leghorn, and 7 braces at Leghorn be equal to 4 American yards; how many Venetian braces are equal to 32 American yards? *Ans.* $52\frac{4}{15}$.

5. If 40℔ at Newbury-port make 36 at Amsterdam, and 90℔. at Amsterdam make 116 at Dantzick; how many ℔. at Newbury-port are equal to 260℔. at Dantzick? *Answ.* $224\frac{4}{29}$.

CASE 2.

When it is required to find how many of the last sort of Coin, Weight or Measure, mentioned in the question, are equal to a given quantity of the first.

RULE.

Place the numbers alternately, beginning at the left hand, and let the last number stand on the right hand; then multiply the first row for a divisor, and the second for a dividend.

EXAMPLES.

1. If 12℔. at Boston make 10℔. at Amsterdam, and 100℔. at Amsterdam 120 at Paris; how many at Paris are equal to 80 at Boston?

	Left.	*Right.*	
Boston	12	10	$10\times120\times80 = 96000$
Amsterdam	100	120	$\frac{96000}{1200} = 80$ *Ans.*
		80	$12\times100 = 1200$

2. If 40℔ at Newbury-port make 36 at Amsterdam, and 90℔. at Amsterdam make 116 at Dantzick; how many ℔ at Dantzick are equal to 244 at Newbury-port? *Answ.* $283\frac{12}{25}$.

ARBI-

ARBITRATION OF EXCHANGES.

By this term is underſtood how to chooſe, or determine the beſt way of remitting money from abroad with advantage; which is performed by conjoined proportion: thus,

Suppose a Merchant has effects at *Amſterdam* to the amount of 3530 dollars, which he can remit by way of *Liſbon* at 840 Rees *per* dollar, and thence to *Boſton*, at 8/1 *per* Milree (or 1000 Rees):—— Or, by way of *Nantz* at $5\frac{2}{5}$ livres *per* dollar, and thence to *Boſton* at 6/8 *per* crown: It is required to arbitrate theſe exchanges, that is, to chooſe that which is moſt advantageous?

1 Dollar at Amſterdam = 840 Rees at Liſbon.
1000 Rees at Liſbon = 97*d*. at Boſton.
3530 Dollars at Amſterdam.

$$\frac{840 \times 97 \times 3530}{1000 \times 1} = £1198\ 8/3\tfrac{4}{10} \text{ By way of Lisbon.}$$

1 Dollar at Amſterdam = $5\frac{2}{5}$ livres at Nantz.
6 Livres at Nantz = 80 pence at Boſton.
3530 dollars at Amſterdam.

$$\frac{5\frac{2}{5} \times 80 \times 3530}{1 \times 6} = £1059 \text{ By way of Nantz.}$$

Here it may be obſerved that the difference is £139 $8/3\frac{4}{10}$ in favor of remitting by way of *Liſbon* rather than by *Nantz*, which depends on the *courſe* of exchange, at that time; but the *courſe* may vary ſo, that, in a ſhort time, by way of *Nantz* may be better; hence appears the neceſſity and advantage of an extenſive correſpondence, to acquire a thorough knowledge in the *courſes* of exchange, to make this kind of remittance.

FELLOWSHIP.

The Rules of Fellowſhip are thoſe by which the accompts of ſeveral merchants, or other perſons, trading in partnerſhip, are ſo adjuſted, that each may have his ſhare of the gain, or ſuſtain his ſhare of the loſs, in proportion to his ſhare of the joint ſtock, together with the time of its continuance in trade.

SINGLE FELLOWSHIP

Is, when the ſtocks are employed for any certain equal time.

Rule.‡

As the whole ſtock is to the whole gain or loſs: So is each man's particular ſtock, to his particular ſhare of the gain, or loſs.

Proof.

‡ That their gain or loſs, in this rule, is in proportion to their ſtocks, is evident: for, as the times, in which the ſtocks are in trade, are equal, if I put in $\frac{1}{2}$ of the whole ſtock, I ought to have $\frac{1}{2}$ of the gain; if my part of the ſtock be $\frac{1}{4}$, my ſhare of the gain or loſs ought to be $\frac{1}{4}$ alſo—and generally the ſame ratio that the whole ſtock has to the whole gain or loſs, muſt each perſon's particular ſtock have to his reſpective gain or loſs.

PROOF. Add all the particular ſhares of the gain or loſs together, and, if it be right, the ſum will be equal to the whole gain or loſs.

EXAMPLES.

1. DIVIDE the number 360 into 4 ſuch parts, which ſhall be to each other as 3, 4, 5 & 6.

$$\text{As } 3+4+5+6 : 360 :: \left\{\begin{array}{l} 3 : 60 \\ 4 : 80 \\ 5 : 100 \\ 6 : 120 \end{array}\right\} \textit{Anſwer.}$$

360 *Proof.*

2. A, B, C and D companied ;—A put in £145 ; B, £219 ; C, £378, and D, £417, with which they gained £569 : what was the ſhare of each ?

$$\begin{array}{c} \textit{Whole ſtock.} \qquad \textit{Gain.} \\ \text{As } 145+219+378+417 : 569 :: \end{array} \left\{\begin{array}{rrrll} & £. & s. & d. & \\ 145 : & 71 & 3 & 8\frac{1}{2} & \frac{1058}{1159} \textit{A's ſha.} \\ 219 : & 107 & 10 & 3\frac{3}{4} & \frac{375}{1159} \textit{B's dit.} \\ 378 : & 185 & 11 & 6- & \frac{552}{1159} \textit{C's dit.} \\ 417 : & 204 & 14 & 5\frac{1}{4} & \frac{333}{1159} \textit{D's dit.} \end{array}\right.$$

£569 — — — *Proof.*

3. A, B, C and D are concerned in a joint ſtock of £168 2/6 : of which A's part is £25 10/ : B's £37 15/ : C's £49, and D's £55 17/6 :—Upon the adjuſtment of their accompts, they have loſt £73 13/4 : what is the loſs of each ?
Anſ. A's loſs £11 3/5½. B's £16 10/9¾. C's £21 9/4¾. & D's £24 9/7¾.

4. A and B companied :—A put in £45, and took ⅗ of the gain ; what did B put in ? 5—3=2, Then, As 3 : 45 :: 2 : 30 *Anſw.*

5. A, B and C freighted a ſhip with 68900 feet of boards : A put in 16520 feet ; B 28750 ; and C the reſt : but, in a ſtorm, the Captain threw overboard 26450 feet ; how much muſt each ſuſtain of the loſs ? *Anſ.* A, 6341¾ feet. B, 11036¾ and C, 9071½ ditto.

6. A gentleman died, leaving three ſons and a daughter, to whom he bequeathed his eſtate in the following manner :—To the eldeſt ſon he gave 312 moidores, to the ſecond 312 guineas, to the third 312 piſtoles, and to the daughter 312 dollars ; but when his debts were paid, there were but 312 Half-joes left ; what muſt each have in proportion to the legacies which had been bequeathed them ?
Anſw. 1*ſt* Son £293 0/2.—2*d* Son £227 17/10¾.—3*d* Son £179 1/2½. and the Daughter £48 16/8¼.

7. A Ship, worth £780, being loſt at ſea, of which ⅙ belonged to A, ½ to B, and the reſt to C ; what loſs will each ſuſtain, ſuppoſing £450 to have been inſured upon her ?

780—450=330, then ⅙)330 ½)330 1)330

£55=A's £165=B's £110=C's ſhare.

8. A

8. A and B venturing equal sums of money, cleared by joint trade £140 :—By agreement, as A executed the business, he was to have 8 *per Cent*. and B was to have 5 *per Cent*. What was A allowed for his trouble ?

$$\begin{array}{l} \text{£. £.}\quad\text{£.}\quad\quad\text{£.}\quad\text{£.}\qquad\qquad\qquad\text{£. £.}\quad\text{£.}\quad\quad\text{£.}\quad\text{£.} \\ \text{As } 8+5 : 140 :: 8 : 86\tfrac{2}{13} \quad \text{And As } 8+5 : 140 :: 5 : 53\tfrac{11}{13} \end{array}$$

Ans. £32 6/1$\frac{3}{4}$ $\frac{5}{13}$

9. A Bankrupt is indebted to A £120, to B £230, to C £340, and to D £450, and his whole estate amounts only to £560 : How must it be divided among the creditors ?

Ans. A, £58 18/11$\frac{1}{4}$. B, £112 19/7$\frac{3}{4}$. C, £167 0/4. & D, £221 1/0$\frac{1}{2}$.

10. A, B and C put their money into a joint stock : A put in £40; B and C together, £170 : they gained £126, of which B took £42; what did A and C gain, and B and C put in respectively ?

As £210 the whole stock : £126 the whole gain :: £40 A's stock : £24 A's gain.

As £24 A's gain : £40 A's stock :: £42 B's gain : £70 B's stock. Then £170—£70=£100 C's stock; and the whole gain £126 — £66 A's and B's gain=£60 C's gain.

11. A, B and C companied :—A put in £40; B £60, and C, a sum unknown ;—They gained £72; of which C took £32 for his share : what did A and B gain, and C put in ?

The whole gain £72—C's gain £32=£40, A's and B's gain : Then, As £100, A's and B's stock : £40 their gain : : £40 A's stock : £16, his gain.—Again, As £16, A's gain : £40, his stock : : £32, C's gain : £80, his stock.

12. A, B and C put in £720, and gained £540, of which, so often as A took up £3, B took 5, and C 7 : what did each put in, and gain ?

$$\text{As } \overset{\text{£.}}{3}+\overset{\text{£.}}{5}+\overset{\text{£.}}{7} : \overset{\text{£.}}{540} :: \left\{\begin{array}{lcl} \overset{\text{£.}}{3} & : & \overset{\text{£.}}{108} \text{ A's gain.} \\ 5 & : & 180 \text{ B's ditto.} \\ 7 & : & 252 \text{ C's ditto.} \end{array}\right\}$$

$$\text{And As } 3+5+7 : \overset{\text{£.}}{720} :: \left\{\begin{array}{lcl} 3 & : & 144 \text{ A's Stock.} \\ 5 & : & 240 \text{ B's ditto.} \\ 7 & : & 336 \text{ C's ditto.} \end{array}\right\}$$

Or, you may find a common multiplier to multiply the proportions by, or multiplicand to be multiplied by the given proportions, thus, 15)720(48 multiplicand to find the stocks,—And 15)540(36 multiplicand to find the gains.

$$\left.\begin{array}{l} 48\times3=\overset{\text{£.}}{144} \text{ A's stock} \\ 48\times5=240 \text{ B's ditto.} \\ 48\times7=336 \text{ C's ditto.} \end{array}\right\} \text{ And } \left\{\begin{array}{l} 36\times3=\overset{\text{£.}}{108} \text{ A's Gain,} \\ 36\times5=180 \text{ B's ditto.} \\ 36\times7=252 \text{ C's ditto. as before.} \end{array}\right.$$

13. A,

13. A, B, C and D companied; and gained a ſum of money, of which A, B and C took £120, B C and D, £180, C, D and A, £160, and D, A and B, £140; what diſtinct gain had each?

The ſum of theſe 4 numbers is £600, and as each man's money is named 3 times, therefore $\frac{1}{3}$, *viz.* £200 is the whole gain—Therefore £200 — £120 A's, B's and C's gain = £80 D's gain;----And £200 —£180 B's C's and D's gain=£20 A's gain.---£200 —£160 C's, D's and A's gain=£40 B's gain.---And £200 —£140 D's, A's and B's gain =£60 C's gain.

14. Two merchants companied: A put in £40, and B 288 ducats; They gained £135, of which A took £60; what was the value of a ducat?

As £60, A's gain : £40, his ſtock :: £135, the whole gain—£60, A's gain : £50, B's gain,

$$\text{And, As } \overset{Duc.}{288} : \overset{£.}{50} :: \overset{Duc.}{1} : \overset{s.}{3}\ \overset{d.}{5\tfrac{2}{3}} \ \textit{Anſwer.}$$

15. Four men ſpent, at a reckoning, 20 ſhillings, of which they agreed that A ſhould pay $\frac{3}{4}$, B, $\frac{1}{2}$, C, $\frac{1}{4}$, and D, $\frac{1}{8}$. what muſt each pay in that proportion?

$$\text{As } \overset{£.}{\tfrac{3}{4}}+\overset{£.}{\tfrac{1}{2}}+\overset{£.}{\tfrac{1}{4}}+\overset{£.}{\tfrac{1}{8}} : \overset{s.}{20} :: \left\{\begin{array}{lcl} \overset{s.\quad d.}{15} & : & \overset{s.\ \ d.}{9\ \ 2\tfrac{10}{13}} \\ 10 & : & 6\ \ 1\tfrac{11}{13} \\ 5 & : & 3\ \ 0\tfrac{12}{13} \\ 2\ \ 6 & : & 1\ \ 6\tfrac{6}{13} \end{array}\right\} \textit{Anſwer.}$$

DOUBLE FELLOWSHIP¶

Or *Fellowſhip with Time,* is occaſioned by the ſhares of partners being continued unequal times.

Rule.

Multiply each man's ſtock, or ſhare, by the time it was continued in trade, Then,

As the whole ſum of the products, is to the whole gain or loſs:
So is each man's particular product, to his particular ſhare of the gain or loſs.

1. A, B and C hold a paſture in common, for which they pay £40 *per annum.* A put in 9 oxen for 5 weeks; B, 12 oxen for 7 weeks, and C 8 oxen 16 weeks; what muſt each pay of the rent?
$9\times5=45$. $12\times7=84$, and $8\times16=128$, then $128+84+45=257$
As

¶ When the times are equal, the ſhares of the gain or loſs are evidently as the ſtocks, as in Single Fellowſhip; and when the ſtocks are equal, the ſhares are as the times, wherefore, when neither are equal, the ſhares muſt be as their products.

As 257 : 40 :: 45	As 257 : 40 :: 84	As 257 : 40 :: 128
45	84	40
200	160	257)5120(19
160	320	257
257)1800(7	257)3360(13	2550
1799	257	2313
1	790	237
20	771	20
257)20(0	19	257)4740(18
12	20	257
257)240(0	257)380(1	2170
4	257	2056
257)960(3	123	114
771	12	12
189	257)1476(5	257)1368(5
	1285	1285
	191	83
	4	4
	257)764(2	257)332(1
	514	257
	250	75

2. FOUR Merchants traded in company, A put in £100 for five months, B, £150 for 7 months, C, 220 for 8 months, and D, £310 for 9 months; but by misfortunes at sea, they lost £145 : what must each man sustain of the loss?

Answ. { A, £11 17/8$\frac{1}{4}$ $\frac{51}{61}$. C, £41 16/8$\frac{1}{2}$ $\frac{38}{61}$. B, £24 19/2 0$\frac{40}{61}$. D, £66 6/4$\frac{1}{2}$ $\frac{54}{61}$. }

3. A, with a capital of £100 began trade January 1st 1787, and meeting with success in his business, he took in B as a partner, on the 1st day of March following, with a capital of £150. Three months after that, they admit C as a third partner, who brought into stock £180, and after trading together 'till the 1st of January 1788, they found there had been gained since A's commencing business, 177 13/: how must this be divided among the partners?

Answ. A, £45 14/10$\frac{1}{4}$. B, £57 3/8. C, £74 14/4$\frac{3}{4}$.

4. Two Merchants entered into partnership for 18 months, A, at first, put into stock £100, and at the end of 8 months he put in £50 more; B, at first, put in £275, and at 4 month's end took out £70. Now, at the expiration of the time, they found they had gained £263; what is each man's just share?

Answer. A, £96 9/6. B, £166 10/6.

5. A and B companied; A put in the 1st of January £150; but B could not put in any 'till the first of May; what did he then put in, to have an equal share with A at the year's end?

As

$$\text{As } \overset{M.}{12} : \overset{£.}{150} :: \overset{M.}{8} : \frac{150 \times 12}{8} = £225 \textit{ Answ.}$$

6. A, B and C companied; A put in, the 1st of March, £30; B, the 1st of May, put in 80 yards of broad cloth; and on the first of June C put in 120 dollars. On the first of January following, they reckoned their gains, of which A and B took £228; B and C, £215 10*s*. and C and A £187 10*s*: what was the whole gain, and the gain of each; what did they value a yard of cloth at, and what was C's dollar worth?

228+215 10*s*+187 10*s*=£631, and 631÷2=315 10*s* the whole gain, then, £315 10*s*—228=£87 10*s* C's gain. £315 10*s*—£215 10*s* =£100 A's gain, and £315 10*s*—£187 10*s*=£128 B's gain.---To find the value of one yard of cloth, say, As £100 A's gain : £30 his stock :: £128 B's gain : £38 8*s*. then, inversely, As 10 months : £38 8*s* :: 8 months : £48 the value of the whole cloth.

As 80 *yds*. : £48 :: 1 *yd*. : 12*s* answer Now, to find the value of a Dollar,—As £100 A's gain : £30 his stock :: £87 10*s* C's gain : £26 5*s* then, inversely, As 10 months : £26 5*s* :: 7 months : £37 10*s*= 120 dollars. Lastly, As 120 dollars : £37 10*s* :: 1 dollar : 6/3 answer.

7. A, B and C companied and put in together £1911 : A's money was in 3 months, B's 5 months, and C's 7 months; they gained £117, which was so divided, as that the $\frac{1}{2}$ of A's gain was equal to $\frac{1}{3}$ of B's, and $\frac{1}{4}$ of C,s gain; what did each gain and put in?

Suppose A's gain was £2, then must B have £3, and C £4, by the question :——

$$\text{Then, as } 2+3+4 : £117 \text{ the whole gain } :: \begin{cases} 2 : 26£ \text{ A's gain.} \\ 3 : 39£ \text{ B's ditto.} \\ 4 : 52£ \text{ C's ditto.} \end{cases}$$

Then, multiply each man's gain by his time, and dividing the whole stock by the sum of those products, your quotient will be a *common multiplier*, by which multiplying the product of each man's gain and time, you will have each man's stock.

A's stock £234, B's ditto £585, and C's ditto £1092.

FELLOWSHIP BY DECIMALS.

Rule.*

Divide the whole gain, or loss, by the whole stock, and the quotient multiplied severally by each man's stock, will give the gain, or loss, of each.

Examples.

1. A, B and C companied, A put in £40 5*s*; B, £80 10*s*, and C, £161. with which they gained £120; what is each man's share of the Gain?

A's

* This is no more than Division of Decimals.

A's Stock = 40,25
B's ditto = 80,5
C's ditto = 161

Sum total = 281,75)120,000000(,4259+

,4259	,4259	,4259
40,25	80,5	161
21295	21295	4259
8518	340720	25554
170366		4259
	£34,28495	
£17,142475	20	£68,5699
20		20
	5,69900	
2,849500	12	11,3980
12		12
	8,388	
10,194000	4	4,776
4		4
	1,552	
0,776000		3,104

Proof. A's gain £17 2*s.* 10*d.*+ B's gain £34 5*s.* 8¼*d.* + C's gain £68 11*s.* 4¾*d.* = £119 19*s.* 11*d.*

2. A, B and C companied; A put in £200, B, £150, and C, £50, with which they gained £800; what is the share of each? 800 ÷ 400 = 2; then 200 × 2 = £400 = A's gain, 150 × 2 = £300 = B's gain, and 50 × 2 = 100 = C's gain.

3. A, B, C and D trade, and gain £200, which is to be divided in the following manner, *viz.* So often as A has £6, B must have £10, C, £14, and D, £20; what is the share of each? 6+10+14+20=50, and $\frac{200}{50}$ = 4, quotient: then 6×4=£24 A's gain: 10×4=£40 B's gain; 14×4=56 C's; and 20×4=£80 D's gain.

P R A C T I C E*

Is a contraction of the Rule of Three direct, when the first term happens to be an unit, or one; and has its name from its daily use among Merchants and Tradesmen, being an easy and concise method of working most questions which occur in trade and business.

The method of proof is by the Rule of Three, Compound Multiplication, or by varying the order of them.

Before the questions, hereafter given, can be wrought, the following Tables must be perfectly gotten by heart.

X TABLES.

* General Rule.

1. Suppose the price of the given quantity to be 1£. or 1*s.* &c. then will the quantity itself be the answer at the supposed price.

2. Divide

TABLES.

Aliquot, or even, Parts of Money.

Pts. of a shil. of a £.

d.		s.		£.
6	=	$\frac{1}{2}$	=	$\frac{1}{40}$
4	=	$\frac{1}{3}$	=	$\frac{1}{60}$
3	=	$\frac{1}{4}$	=	$\frac{1}{80}$
2	=	$\frac{1}{6}$	=	$\frac{1}{120}$
$1\frac{1}{2}$	=	$\frac{1}{8}$	=	$\frac{1}{160}$
1	=	$\frac{1}{12}$	=	$\frac{1}{240}$
$\frac{3}{4}$	=	$\frac{1}{16}$	=	$\frac{1}{320}$
$\frac{1}{2}$	=	$\frac{1}{24}$	=	$\frac{1}{480}$
$\frac{1}{4}$	=	$\frac{1}{48}$	=	$\frac{1}{960}$

Parts of 2 Shill.

d.		2 s.
1	is	$\frac{1}{24}$
$1\frac{1}{2}$	—	$\frac{1}{16}$
2	—	$\frac{1}{12}$
3	—	$\frac{1}{8}$
4	—	$\frac{1}{6}$
6	—	$\frac{1}{4}$
8	—	$\frac{1}{3}$

Parts of a Pound.

s.	d.		£.
10	0	is	$\frac{1}{2}$
6	8	—	$\frac{1}{3}$
5	0	—	$\frac{1}{4}$
4	0	—	$\frac{1}{5}$
3	4	—	$\frac{1}{6}$
2	6	—	$\frac{1}{8}$
1	8	—	$\frac{1}{12}$
1	4	—	$\frac{1}{15}$
1	3	—	$\frac{1}{16}$
1	0	—	$\frac{1}{20}$
0	10	—	$\frac{1}{24}$
0	8	—	$\frac{1}{30}$
0	5	—	$\frac{1}{48}$
0	$2\frac{1}{2}$	—	$\frac{1}{96}$

Aliquot, or even, Parts of Weight.

Parts of a Cwt.

Qrs.	℔		Cwt.
2	0	is	$\frac{1}{2}$
1	0	—	$\frac{1}{4}$
0	16	—	$\frac{1}{7}$
0	14	—	$\frac{1}{8}$
0	8	—	$\frac{1}{14}$
0	7	—	$\frac{1}{16}$
0	4	—	$\frac{1}{28}$

Pts. of $\frac{1}{2}$ Cwt.

℔.		$\frac{1}{2}$ Cwt.
28	is	$\frac{1}{2}$
14	—	$\frac{1}{4}$
8	—	$\frac{1}{7}$
7	—	$\frac{1}{8}$
4	—	$\frac{1}{14}$

Pts. of $\frac{1}{4}$ Cwt.

℔.		$\frac{1}{4}$ Cwt.
14	is	$\frac{1}{2}$
7	—	$\frac{1}{4}$
4	—	$\frac{1}{7}$
2	—	$\frac{1}{14}$

Parts of a Tun.

Cwt.	qr.		T.
10	0	is	$\frac{1}{2}$
5	0	—	$\frac{1}{4}$
4	0	—	$\frac{1}{5}$
2	2	—	$\frac{1}{8}$
2	0	—	$\frac{1}{10}$
1	1	—	$\frac{1}{16}$
1	0	—	$\frac{1}{20}$

Parts of 240.

180	is	$\frac{3}{4}$
120	—	$\frac{1}{2}$
80	—	$\frac{1}{3}$
60	—	$\frac{1}{4}$
40	—	$\frac{1}{6}$

Parts of 480.

360	is	$\frac{3}{4}$
160	—	$\frac{1}{3}$

Another TABLE *of* Aliquot *parts of* Money.

Parts of a Shilling.

d.		s.
10	is	$\frac{5}{6}$
9	—	$\frac{3}{4}$
8	—	$\frac{2}{3}$
$7\frac{1}{2}$	—	$\frac{5}{8}$
$4\frac{1}{2}$	—	$\frac{3}{8}$

Parts of a Pound.

s.	d.		£.
18	0	is	$\frac{9}{10}$
17	6	—	$\frac{7}{8}$
16	8	—	$\frac{5}{6}$
16	0	—	$\frac{8}{10}$
15	0	—	$\frac{3}{4}$
14	0	—	$\frac{7}{10}$

Parts of a Pound.

s.	d.		£.
13	4	is	$\frac{2}{3}$
12	6	—	$\frac{5}{8}$
12	0	—	$\frac{6}{10}$
8	0	—	$\frac{4}{10}$
7	6	—	$\frac{3}{8}$
6	0	—	$\frac{3}{10}$

A TABLE *of* DISCOUNT *per Cent.*

£.		s.	d.	
$1\frac{1}{4}$	*per Cent.* is	0	3	*on the Pound.*
$2\frac{1}{2}$	——	0	6	
$3\frac{3}{4}$	——	0	9	
5	——	1	0	
$6\frac{1}{4}$	——	1	3	
$7\frac{1}{2}$	——	1	6	

£.		s.	d.	
$8\frac{3}{4}$	*per Cent.* is	1	9	*on the Pound.*
10	——	2	0	
$12\frac{1}{2}$	——	2	6	
15	——	3	0	
$17\frac{1}{2}$	——	3	6	
20	——	4	0	

£.		s.	d.	
$22\frac{1}{2}$	*per Cent.* is	4	6	*on the Pound.*
25	——	5	0	
30	——	6	0	
35	——	7	0	
40	——	8	0	
45	——	9	0	
50	——	10	0	

2. DIVIDE the given price into aliquot parts, either of the supposed price, or of one another, and the sum of the quotients belonging to each will be the true answer required.

EXAMPLE

CASE I.

When the price of 1 *yd.* ℔. *&c. is an even part of one shilling*;—Find the value of the given quantity at 1*s. per* yard, ℔. &c. then draw a line underneath, and divide it by *that even part*, and the quotient will be the anſwer in ſhillings, which muſt always be brought into pounds.

EXAMPLES.

1. WHAT will $354\frac{1}{2}$ yards coſt, at $\frac{1}{4}$*d. per* yard?

| $\frac{1}{4}$*d.* | $\frac{1}{48}$ | 354 *s.* 6 *d.* *Value of* $354\frac{1}{2}$ *yards, at* 1*s. per yard.*

Anſw. £0 7 $4\frac{1}{2}$ *Value of* $354\frac{1}{2}$ *yds. at* $\frac{1}{4}$*d. per yard.*

Or thus.

£ *s.* *d.* = *s.* *d.*

8)17 14 6 = 354 6

6)2 4 $3\frac{3}{4}$

7 $4\frac{1}{2}$ *Anſ. as before.*

Or divide by 8 *&* 6, *thus,* 8)354 6

6)44 $3\frac{3}{4}$

7 $4\frac{1}{2}$ *Anſ. as before.*

2. WHAT will $759\frac{3}{4}$ yards come to, at 3*d. per* yard?

| 3*d.* | $\frac{1}{4}$ | 759 *s.* 9 *d.* *value at* 1*s. per yard.*

2|0)18|9 $11\frac{1}{4}$

Anſ. £9 9 $11\frac{1}{4}$ *value at* 3*d. per yd.*

Or thus, | 3*d.* | $\frac{1}{4}$ | 37 £ 19 *s.* 9 *d.* *value at* 1*s. per yd.*

Anſ. £9 9 $11\frac{1}{4}$ *value of* $759\frac{3}{4}$ *yds. at* 3*d. per yd.*

Queſtions. Yds.		*Anſwers.* £. s. d.	*Queſtions.* Yds.		*Anſwers.* £. s. d.
3d 642	at $\frac{1}{4}$*d. per yd.*	0 13 $4\frac{1}{2}$	7th $685\frac{3}{4}$	at 2*d.*	5 14 $3\frac{1}{2}$
4th $918\frac{1}{4}$	— $\frac{1}{2}$*d.*	1 18 $3\frac{1}{8}$	8th $475\frac{1}{4}$	— 4*d.*	7 18 5
5th $739\frac{1}{2}$	— 1*d.*	3 1 $7\frac{1}{2}$	9th $913\frac{1}{2}$	— 6*d.*	22 16 9
6th $567\frac{1}{2}$	— $1\frac{1}{2}$*d.*	3 10 $11\frac{1}{4}$			

CASE 2.

EXAMPLE.

WHAT is the value of 468 yards, at $2/9\frac{1}{4}$ per yard?

	£468	*s.*	*d.*	*Anſwer at*	£1	*s.*	*d.*
2*s.* 6*d.* *is* $\frac{1}{8}$ =	58	10	—	*ditto at*	—	2	6
3*d.* *is* $\frac{1}{10}$ =	5	17	—	*ditto at*	—	—	3
$\frac{1}{4}$*d.* *is* $\frac{1}{12}$ =	—	9	9	*ditto at*	—	—	$\frac{1}{4}$
The full price. =	£64	16	9		—	2	$9\frac{1}{4}$

IN this example it is plain that the quantity 468 is the anſwer at £1; conſequently, as 2/6 is $\frac{1}{8}$ of a pound, $\frac{1}{8}$ part of that quantity, or £58 10*s.* is the price at 2/6; in like manner, as 3*d.* is the $\frac{1}{10}$ part of 2/6, ſo $\frac{1}{10}$ part of £58 10*s.* or £5 17*s.* is the anſwer at 3*d.* and as $\frac{1}{4}$*d.* is $\frac{1}{12}$ of 3*d.* ſo $\frac{1}{12}$ of £5 17*s.* or 9*s.* 9*d.* is the anſwer at $\frac{1}{4}$*d.*—Now, the ſum of all theſe parts is equal to the whole price (2*s.* $9\frac{1}{4}$*d.*) ſo the ſum of the anſwers belonging to each price will be the anſwer at the full price required, and the ſame will be true in any example whatever.

CASE 2.

When the price is pence, and no even part of a shilling;—Find the value of the given quantity at 1*s*. *per* yard; divide the pence into aliquot parts, for divisors, and the sum of the quotients, arising from them, will be the answer.

EXAMPLES.

1. WHAT will $487\frac{1}{2}$ yards come to at 5*d*. *per* yard?

d.		£.	s.	d.	
3*d*.	$\frac{1}{4}$	24	7	6	*Value of* $487\frac{1}{2}$ *yards, at* 1*s*. *per yard*.
2*d*.	$\frac{1}{6}$				
		6	1	$10\frac{1}{2}$	*Value of ditto, at* 3*d*. *per yard*.
		4	1	3	*Value of ditto, at* 2*d*. *per yard*.
Answ.		£10	3	$1\frac{1}{2}$	*Value of ditto, at* 5*d*. *per yard*.

Questions. Yds.		*Answers.* £.	s.	d.	*Questions.* Yds.		*Answers.* £.	s.	d.
2d $568\frac{1}{4}$	at 7*d*.	16	11	$5\frac{3}{4}$	5th $649\frac{1}{4}$	at 10*d*.	27	1	$0\frac{1}{2}$
3d $683\frac{3}{4}$	— 8*d*.	22	15	10	6th $745\frac{3}{4}$	— 11*d*.	34	3	$7\frac{1}{4}$
4th $912\frac{1}{2}$	— 9*d*.	34	4	$4\frac{1}{2}$					

CASE 3.

WHEN the price is pence or farthings, and an even part of a pound, cut off the right hand figure of the given quantity, and the cypher, in the aliquot part, (if it has one) and divide by the remaining figure or figures: When you come to the remainder, double it, and divide as before: The answer will be pounds, shillings, &c. If there be no cypher in the divisor, then none should be cut off from the dividend.

d.
| $\frac{1}{4}$ | $\frac{1}{96.0}$ | 3795 lb. *at* $\frac{1}{4}$*d*.

96=8×12)379|5

8)31 12 6

Answ. £3 19 $0\frac{3}{4}$

d.
| $\frac{1}{2}$ | $\frac{1}{48.0}$ | 3795 *yds. at* $\frac{1}{2}$*d*.

48=6×8)379|5

6)47 8 9

Answ. £7 18 $1\frac{1}{2}$

d.
| $\frac{3}{4}$ | $\frac{1}{32.0}$ | 3795 *yds. at* $\frac{3}{4}$*d*.

32=4×8)379|5

4)47 8 9

Answ. £11 17 $2\frac{1}{4}$

d.
| 1 | $\frac{1}{24.0}$ | 3795 *yds. at* 1*d*.

24=4×6)379|5

4)63 5

Answ. £15 16 3

d.
| $1\frac{1}{2}$ |

d.
| 1½ | $\frac{1}{160}$ | 3795 *yds. at* 1½*d.*

16=4×4)379|5

4)94 17 6

£23 14 4½ *Answ.*

d.
| 4 | $\frac{1}{60}$ | 379|5 *yds. at* 4*d.*

£63 5 *Answ.*

d.
| 8 | $\frac{1}{30}$ | 379|5 *yds. at* 8*d.*

£126 10 *Answ.*

d.
| 2 | $\frac{1}{120}$)379|5 *yds. at* 2*d.*

£31 12 6 *Ans.*

d.
| 3 | $\frac{1}{80}$ | 379|5 *yds. at* 3*d.*

£47 8 9 *Answ.*

d.
| 6 | $\frac{1}{40}$ | 379|5 *yds. at* 6*d.*

£94 17 6 *Answ.*

d.
| 10 | $\frac{1}{24}$ | 3795 *yds.*

24=4×6)3795

4)632 10

£158 2 6 *Ans.*

CASE 4.

When the price is between one and two shillings;—Find the value of the quantity at 1*s.* *per* yard, &c. which value being divided by *those even parts* which the pence are of 1*s.* and the quotient or quotients, arising therefrom, added thereto;—the sum will be the answer.—

EXAMPLES.

1. WHAT will 758½ yards, at 1*s.* 9*d.* *per* yard, come to?

		£.	s.	d.	
6*d.*	½	37	18	6	*Value at* 1*s.* *per yard.*
3*d.*	¼	18	19	3	*Value at* 6*d.* *per yard.*
		9	9	7½	*Value at* 3*d.* *per yard.*
Answ.		£66	7	4½	*Value of* 758½ *yds. at* 1*s.* 9*d.* *per yd.*

Questions. Yds.		*Answers.* £.	s.	d.	*Questions.* Yds.		*Answers.* £.	s.	d.
2d 987½	at 12½*d.* —	51	8	7¾	9th 647¾	at 1*s* 5*d.* —	45	17	7¾
3d 793	— 12¾*d.* —	42	2	6¾	10th 896¼	— 1*s* 6*d.* —	67	4	4½
4th 847½	— 1*s* 1*d.* —	45	18	1½	11th 458	— 1*s* 7*d.* —	36	5	2
5th 686¾	— 1*s* 1½*d* —	38	12	7	12th 964	— 1*s* 8*d.* —	80	6	8
6th 591¼	— 1*s* 2*d.* —	34	9	9½	13th 752½	— 1*s* 10*d.* —	68	19	7
7th 573½	— 1*s* 3*d.* —	35	16	10½	14th 649¾	— 1*s* 11*d.* —	62	5	4¼
8th 846½	— 1*s* 4*d.* —	56	8	8					

CASE 5.

When the price is any even number of shillings under 40;—Multiply the given quantity by half the price, and double the first figure of the product for shillings; the rest of the product will be pounds.

N. B. If the price be 2*s.* you need only double the unit figure for shillings; the other figures will be pounds.

EXAMPLES.

1. WHAT will 746 yards cost at 2*s.* *per* yard?

746

Answer £74 12 *Value at* 2*s.* *per yard.*

Note. The above is done, by saying, twice 6 (the unit figure) is 12; the other figures, viz. 74, are pounds.

2. WHAT will $567\frac{3}{4}$ *yds.* at 2*s.* *per yd.* come to? *Ans.* £56 15/6.

N. B. Before I double the unit figure, viz. 7, I consider that $\frac{3}{4}$ of a yard, at 2*s.* *per* yard, will amount to 1/6, then I double 7, which makes 14/ and 1/6 added, makes 15/6; the other figures are pounds.

Questions.	*Yds.*					*Answers.* £.	s.	d.
3d	$129\frac{1}{2}$	at	4*s.*	*per yard.*		25	18	0
4th	697	—	6*s.*	—	—	209	2	0
5th	845	—	8*s.*	—	—	338	0	0
6th	$917\frac{1}{4}$	—	10*s.*	—	—	458	12	6
7th	$528\frac{1}{2}$	—	12*s.*	—	—	317	2	0
8th	$646\frac{3}{4}$	—	14*s.*	—	—	452	14	6
9th	591	—	16*s.*	—	—	472	16	0
10th	$845\frac{1}{2}$	—	18*s.*	—	—	760	19	0
11th	$645\frac{1}{4}$	—	24*s.*	—	—	774	6	0

CASE 6.

When the price wants an even part of 2*s.*—First find the value of the whole quantity at 2*s.* *per* ℔. yard, &c. then divide it by *that even part* which is wanting, and subtract this quotient from the value at 2*s.* the remainder will be the answer.

EXAMPLES.

1. WHAT will $95\frac{1}{2}$ yards cost at 22*d.* *per* yard?

		£.	s.	d.	
2*d.*	$\frac{1}{12}$	9	11	0	*Value at* 2*s.* *per yard.*
		0	15	11	*ditto at* 2*d.* *per yard.*
Answ.		£8	15	1	*Value at* 1*s.* 10*d.* *per yard.*

Questions.	*Yds.*			*Answers.* £.	s.	d.	*Questions.*	*Yds.*			*Answers.* £.	s.	d.
2d	64	at 23*d.*	*per yd.*	6	4	$1\frac{1}{4}$	5th	$375\frac{1}{4}$	at 20*d.*	*per yd.*	31	5	5
3d	128	— $22\frac{1}{2}$*d.*	—	12	0	0	6th	486	-- 18*d.*	—	36	9	0
4th	$246\frac{1}{2}$	-- 21*d.*	—	21	11	$4\frac{1}{2}$	7th	754	-- 16*d.*	—	50	5	4

Case 7.

When the price is between 2s. and 3s.—First find the value of the quantity at 2*s.* *per* yard, &c. which value being divided by *those even parts*, which the pence are of 2*s.* and, those quotients added thereto, the sum will be the answer.

Examples.

1. What will $148\frac{1}{2}$ yards come to at 2*s.* 7*d.* *per* yard?

		£.	s.	d.	
4*d.*	$\frac{1}{6}$	14	17	0	*Value at 2s. per yard.*
3*d.*	$\frac{1}{8}$	2	9	6	*ditto at 4d. per yard.*
		1	17	$1\frac{1}{2}$	*ditto at 3d. per yard.*
Answ.		£19	3	$7\frac{1}{2}$	*Value at 2s. 7d. per yard.*

Questions.	Yds.				*Answers.* £.	s.	d.
2d	$266\frac{1}{4}$	at 2*s.* 1*d.*	*per yard.*		27	14	$8\frac{1}{4}$
3d	344	— 2*s.* $1\frac{1}{2}$*d.*	—	—	36	11	0
4th	$543\frac{1}{2}$	— 2*s.* 2*d.*	—	—	58	17	7
5th	$655\frac{3}{4}$	— 2*s.* 3*d.*	—	—	73	15	$5\frac{1}{4}$
6th	716	— 2*s.* 4*d.*	—	—	83	10	8
7th	813	— 2*s.* 5*d.*	—	—	98	4	9

Case 8.

When there are pence in the price which are an even part of a shilling, besides an even number of shillings under 20;—First find the value of the quantity at the shillings *per* yard, &c. according to Case 5th; then suppose the quantity to stand as shillings *per* yard; divide it by *that even part*, which the pence are of 1*s.* and this quotient being added to the value before found, the sum will be the answer.

Examples.

1. What will $156\frac{1}{2}$ yards come to, at 6*s.* 4*d.* *per* yard?

Yds.
$156\frac{1}{2}$
3

d.		s. d.				
4	$\frac{1}{3}$	156 6				
			£46	19	0	*Value of $156\frac{1}{2}$ yards, at 6s. per yd.*
		52*s.* 2*d.* =	2	12	2	*Value of ditto, at 4d. per yd.*
		Answ.	£49	11	2	*Value of ditto, at 6s. 4d. per yd.*

Questions,

Questions.	Yds.		s. d.		£.	s.	d.
2d	$17\frac{1}{2}$	at	$4\frac{1}{2}$	per yard.	3	10	$8\frac{3}{4}$
3d	$59\frac{3}{4}$	—	$6\frac{3}{4}$	— —	18	2	$2\frac{3}{4}$
4th	$68\frac{1}{4}$	—	8 1	— —	27	11	$8\frac{1}{4}$
5th	96	—	10 $1\frac{1}{2}$	— —	48	12	0
6th	$67\frac{1}{2}$	—	12 2	— —	41	1	3

CASE 9.

When the price is any odd number of shillings under 40;—Find the value of the greatest even number contained in the price, according to Case 5th, and add thereto the value of the quantity at 1*s* *per* yard, &c. which sum will be the answer: *Or*, Multiply the quantity by the price, according to the 1st or 2d Case in Simple Multiplication, and divide the product by 20, the quotient will be the answer: *Or*, lastly, if the price be under 13*s* find the value of the quantity at 1*s* *per* yard, &c. and multiply it by the number of shillings in the price of 1 yard: the product will be the answer.

EXAMPLES.

1. WHAT will 186 yards cost, at 3*s.* *per* yard?

£.	s.	
18	12	*Value at 2s. per yard.*
9	6	*Value at 1s. per yard.*
£27	18	*Answ.*

Or thus:

	£.	s.	
	9	6	*Value at 1s. per yd.*
		3	
Product	£27	18	*Answ.*

2. WHAT will 647 yards cost, at 17*s.* *per* yard?

	647		
	8		
	£517	12	*Value at 16s. per yard.*
	32	7	*Value at 1s. per yard.*
Answ.	£549	19	*Value at 17s. per yard.*

Questions.	Yds.		s.		Answers. £.	s.	d.
3d	$169\frac{1}{4}$	at	5	per yd.	42	6	3
4th	$248\frac{3}{4}$	—	7	——	87	1	3
5th	139	at	9	——	62	11	0
6th	782	—	25	——	977	10	0

CASE 10.

When the price is an even part of a pound;—Find the value of the given quantity, at one pound *per* yard, &c. then draw a line underneath, and divide it by *that part*:—the quotient will be the answer.

EXAMPLES.

EXAMPLES.

1. WHAT will 156¾ yards of Cloth come to, at 3*s.* 4*d.* *per* yard?

s. d.		£.	s.	d.	
3 4	$\frac{1}{6}$	156	15	0	*Price at £1 per yard.*
	Answ.	£26	2	6	*Price at 3s. 4d. per yard.*

Questions. Yds.	s. d.		*Answers.* £.	s.	d.	*Questions.* Yds.	s. d.	*Answers.* £.	s.	d.
2d 516¾	at 1	*per yd.*	25	16	9	7th 429¼	— 4	85	17	0
3d 624	— 1 3		39	0	0	8th 687½	— 5	171	17	6
4th 719½	— 1 4		47	19	4	9th 843	— 6 8	281	0	0
5th 648	— 1 8		54	0	0	10th 486¾	— 10	243	7	6
6th 419¾	— 2 6		52	9	4½					

CASE 11.

When the price wants an even part of a pound;—First find the value of the given quantity at £1 *per* yard, &c. then divide it by *that even part* which is wanting, and subtract this quotient therefrom:—the remainder will be the answer.

EXAMPLES.

1. WHAT will 167½ yards cost, at 17*s.* 6*d.* *per* yard?

s. d.		£.	s		
2 6	$\frac{1}{8}$	167	10	0	*Value at £1 per yard.*
		20	18	9	*Value at 2s. 6d. per yard.*
	Answ.	£146	11	3	*Value at 17s. 6d. per yard.*

Questions. Yds.	s. d.		*Answers.* £.	s.	d.	*Questions.* Yds.	s. d.	*Answers.* £.	s.	d.
2d 347½	at 13 4	*per yd.*	231	13	4	4th 614	— 16	491	4	0
3d 485¾	-- 15		363	6	3	5th 912¼	— 17 6	798	4	4½

CASE 12.

When the price is shillings, pence and farthings, and not an even part of a pound;---Multiply the given quantity by the shillings in the price of 1 yard, &c. and take parts of parts from the quantity for the pence, &c. then add them together, and their sum will be the answer, in shillings, &c. *Or*, you may let the given quantity stand as pounds *per* yard, &c. then draw a line underneath, and take parts of parts therefrom; which add together, and their sum will be the answer.

N. B. I advise the learner to work the following examples both ways, by which means he will be able to discover the most concise method of performing such questions, in business, as may fall under this case.

1. WHAT

1. What will 248½ yards, at 7*s.* 6*d. per* yard, come to?

d.		s.	d.	
6	½	248	6	*Value of 248½ yards, at 1s. per yard.*
			7	
		1739	6	*Value of ditto, at 7s. per yard.*
		124	3	*Value of ditto, at 6d. per yard.*
		2|0)186|3	9	
Answ.		£93 3	9	*Value of ditto, at 7s. 6d. per yard.*

Or thus:

d.		£.	s.	d.	
6	½	12	8	6	*Value of 248½ yards, at 1s. per yard.*
Multiply by				7	
		86	19	6	*Value of ditto, at 7s. per yard.*
		6	4	3	*Value of ditto, at 6d. per yard.*
Answ.		£93	3	9	

By the latter part of this case:

s. d.		£.	s.	d.	
5 0	¼	248	10	0	*Value of 248½ yards, at £1 per yard.*
2 6	½	62	2	6	*Value of ditto, at 5s. per yard.*
		31	1	3	*Value of ditto, at 2s. 6d. per yard.*
Answ.		£93	3	9	*Value of ditto, at 7s. 6d. per yard.*

Questions.	Yds.		s. d.		*Answers.* £. s. d.
2d	68½	at	4 6	per yard.	15 8 3
3d	124	—	5 8	———	35 2 8
4th	146	—	14 9	———	107 13 6
5th	218½	—	12 6	———	136 11 3
6th	645	—	4 1½	———	133 00 7½

Case 13.

When the price of the yard, ℔, *&c. is pounds, shillings and pence*;—First multiply the quantity by the pounds; and if the shillings and pence be an even part of a pound, divide the given quantity by *that part*, and add the quotient to the product for the answer: But if they be not an even part of a pound, you must take parts of parts, and add them together as before.——*Or*, Reduce the pounds and shillings into shillings, and multiply the quantity thereby, after which, take parts for the pence, and add the whole together, and their sum will be the answer in shillings, &c.

N. B. The learner should work the following questions both ways.

Exam-

EXAMPLES.

1. WHAT will 156 yards of broad-cloth come to, at £3 6/8 *per* yard?

		£.	
6/8	$\frac{1}{3}$	156 0 0	*value at* 1£
		3	*per yd.*
		468 0 0	
		52 0 0	
	Answ.	£520 0 0	

Or thus.

d.		s.	
4	$\frac{1}{3}$	156	*value at* 1*s per yd.*
4	$\frac{1}{3}$	66	*Shill. in the price of* 1 *yd.*
		936	
		936	
		10296	*value at* 3£ 6*s per yd.*
		52	
		52	
	2|0)	1040|0	
		£520 0 0	*Answer.*

Questions. Yds.		£. s. d.		*Answers.* £. s. d.	*Questions.* Yds.		£. s. d.		*Answers.* £. s. d.
2d 345½	at	6 5 0	*per Yd.*	2159 7 6	6th 59	—	6 7 6	—	376 2 6
3d 59¾	—	3 6 8	—	199 3 4	7th 112½	—	3 8 8	—	386 5 0
4th 75	—	5 3 4	—	387 10 0	8th 125	—	4 9 7	—	559 17 11
5th 68	—	4 6 0	—	292 8 0					

CASE 14.

When the quantity is any number less than 1000, *and the price not more than* 12*d. per yard, &c.*—Find the value of the whole quantity at 1*d. per* yard, which may be done by dividing it by 12, *mentally,* setting down the quotient only in pounds, or shillings, or both: then multiply this sum by the pence in the price of 1 yard, and the product will be the answer.

1. WHAT will 759½ yards cost, at 7*d. per* yard?

	£. s. d.	
	0 63 3½	*value at* 1*d. per yard.*
Or,	3 3 3½	*value at* 1*d. per yard.*
Multiply by	7	
Answer.	£22 3 0½	*value of* 759½ *yards, at* 7*d. per yard.*

Questions. Yds.		d.		*Answers.* £. s. d.	*Questions.* Yds.		d.		*Answers.* £. s. d.
2d 975½	at	2	*per Yd.*	8 2 7	5th 684	—	5½	—	15 13 6
3d 846	—	3½	—	12 6 9	6th 984½	—	6¾	—	27 13 9¼
4th 793¾	—	4¾	—	15 14 2¼	7th 440½	—	9	—	16 10 4½

CASE 15.

When the price is such a number of shillings and pence, as, when reduced into pence, may be produced by any two numbers in the Multiplication Table,

Table, and when the quantity does not exceed 1000;—First find the value of the whole at 1*d. per* yard, &c. according to the last case; then multiply this sum by the component parts of the pence in the price, and the last product will be the answer.

EXAMPLES.

1. WHAT will $439\frac{1}{2}$ yards cost, at 6*s.* 9*d. per* yard?

	£.	s.	d.	
	1	16	$7\frac{1}{2}$	*Value at* 1*d. per yard.*
			9	
	16	9	$7\frac{1}{2}$	
			9	
Answer	£148	6	$7\frac{1}{2}$	*Value at* 81*d. or* 6*s.* 9*d. per yard.*

N. B. In 6*s.* 9*d.* there being 81 pence, I multiply by 9 twice, because 9 times 9 is 81.

Questions. Yds.	s. d.		*Answers.* £. s. d.	*Questions.* Yds.	s. d.		*Answers.* £. s. d.
2d $984\frac{1}{2}$	at 1 2	*per Yard.*	57 8 7	4th 657	— 3 6	——	114 19 6
3d $849\frac{3}{4}$	— 2 8	——	113 6 0	5th 593	— 4 8	——	138 7 4

CASE 16.

When the quantity is 240;---As many pence as there are in the price of 1 yard, &c. so many pounds will the quantity amount to.

N. B. One farthing *per* yard will come to 5*s.* at half-penny *per* yard to 10*s.*----and at three farthings, to 15*s.*

EXAMPLES.

1. WHAT will 240 yards come to, at $2/7\frac{1}{2}$ *per* yard?

Answer £31 10/0.

N. B. The price is $2/7\frac{1}{2}$ *per* yard :--- Now, as in 2/7 there are 31 pence, so the quantity being 240 comes to 31 pounds; then according to the same rule, the half-penny *per* yard comes to 10*s.*---therefore the answer to the question is £31 10*s.*

Questions. Yds.	s. d.		*Answers.* £. s. d.	*Questions.* Yds.	s. d.		*Answers.* £. s. d.
2d 240	at 1 $7\frac{3}{4}$	*per yard*	19 15 0	5th 240	at 7 $8\frac{1}{2}$	——	92 10 0
3d 240	— 2 9	——	33 00 0	6th 240	— 8 $3\frac{3}{4}$	——	99 15 0
4th 240	— 3 4	——	40 00 0				

CASE 17.

When the quantity is not less than 228, *nor more than* 252;—First find the value of 240 yards, &c. by the last Case; then multiply the price of 1 yard by the number *above* or *under* 240, and add or subtract

tract this product to or from the value of 240 yards, as the question may require; and the sum or remainder will be the answer.

EXAMPLES.

1. WHAT will 248 yards come to, at 16/5½ *per* yard?

	£.	s.	d.	
	197	10	0	*Value of* 240 *yards.*
16/5½ *multiplied by* 8 =	6	11	8	*Value of* 8 *yards.*
Answ.	£204	1	8	*Value of* 248 *yards.*

2. WHAT will 229 yards cost, at 5/9¾ *per* yard?

	£.	s.	d.	
	69	15	0	*Value of* 240 *yards.*
5/9¾ *multiplied by* 11 =	3	3	11¼	*Value of* 11 *yards.*
Answ.	£66	11	0¾	*Value of* 229 *yards.*

		Yds.		s.	d.		£.	s.	d.	
Questions.	3d	228	at	6	10½	*per yard.*	78	7	6	*Answers.*
	4th	252	—	4	6	———	56	14	0	
	5th	230	—	9	8¾	———	111	17	8½	
	6th	251½	—	3	10½	———	48	14	6¾	
	7th	231¼	—	8	2	———	94	8	6½	
	8th	250	—	5	9¼	———	72	2	8½	

CASE 18.

When the quantity is 480;—Find the value of 240 yards, &c. by Case the 16th, and multiply this sum by 2:—the product will be the answer.

N. B. If the quantity be 12 over, or under 480, proceed according to the directions given in the last case.

EXAMPLES.

1. WHAT will 480 yards cost at 2/9½ *per* yard?

	£.	s.	d.	
	33	10	0	*Value of* 240 *yards.*
Multiply by			2	
Answer	£67	0	0	*Value of* 480 *yards.*

2. WHAT will 468 yards come to at 5/8½ *per* yard?

	£.	s.	d.	
	68	10	0	*Value of* 240 *yards.*
			2	
	137	00	0	*Value of* 480 *yards.*
Subtract	3	8	6	*Value of* 12 *yards.*
Answer	£133	11	6	*Value of* 468 *yards*

3. WHAT

3. WHAT will 492 yards come to, at 3/8¾ *per* yard ?

	£.	s.	d.		
	44	15	0	*Value of*	240 *yards.*
			2		
	89	10	0	*Value of*	480 *yards.*
Add	2	4	9	*Value of*	12 *yards.*
Answer	£91	14	9	*Value of*	492 *yards.*

N. B. Any person, who is expert in figures, may find the value, *mentally*, of 480 yards, almost as easily as 240, it being nothing more than doubling the amount of 240.

		Yds.		*s. d.*		*£. s. d.*	
Questions.	4th	469	at	6 9½	*per yard.*	159 5 3½	*Answers.*
	5th	470	—	3 4¾	———	79 16 0½	
	6th	471	—	5 3½	———	124 12 4½	
	7th	472	—	4 9	———	112 2 0	
	8th	483	—	8 10¾	———	214 16 8¼	

CASE 19.

When the quantity is 160 ;—Find the value of 480 yards, and divide it by 3 :—the quotient will be the answer.

Note. If there be 12 yards over, or under, 160, proceed as before directed.

EXAMPLES.

1. WHAT will 160 yards come to, at 3/4½ *per* yard ?

	£.	s.	d.		
Divide by 3)	81	0	0	*Value of*	480 *yards.*
Answer	£27	0	0	*Value of*	160 *yards.*

2. WHAT cost 148 yards, at 4/2 *per* yard ?

	£.	s.	d.		
Divide by 3)	100	0	0	*Value of*	480 *yards.*
	33	6	8	*Value of*	160 *yards.*
Subtract	2	10	0	*Value of*	12 *yards.*
Answer	£30	16	8	*Value of*	148 *yards.*

3. WHAT will 172 yards amount to, at 5/7¾ *per* yard ?

	£.	s.	d.		
Divide by 3)	135	10	0	*Value of*	480 *yards.*
	45	3	4	*Value of*	160 *yards.*
Add	3	7	9	*Value of*	12 *yards.*
Answer	£48	11	1	*Value of*	172 *yards.*

Questions.

		Yds.		s.	d.		£.	s.	d.	
Questions.	4th	149	at	12	$6\frac{1}{2}$	*per yard.*	93	8	$8\frac{1}{2}$	*Answers.*
	5th	150	—	3	8	———	27	10	0	
	6th	166	—	12	8	———	105	2	8	
	7th	152	—	5	$2\frac{3}{4}$	———	39	14	10	
	8th	153	—	6	3	———	47	16	3	
	9th	171	—	7	9	———	66	15	3	

CASE 20.

When the quantity is 120;—First find the value of 240 yards, &c. then divide it by 2, and the quotient will be the answer.

Note. If there be 12 over, or under, 120, proceed as before directed.

EXAMPLES.

1. WHAT will 120 yards cost, at $3/7\frac{1}{2}$ *per* yard?

	£.	s.	d.	
Divide by 2)	43	10	0	*Value of* 240 *yards.*
Answer	£21	15	0	*Value of* 120 *yards.*

2. WHAT will 108 yards come to, at 4/7 *per* yard?

	£.	s.	d.	
Divide by 2)	55	0	0	*Value of* 240 *yards.*
	27	10	0	*Value of* 120 *yards.*
Subtract	2	15	0	*Value of* 12 *yards.*
Answer	£24	15	0	*Value of* 108 *yards.*

3. WHAT is the value of 132 yards, at $4/3\frac{1}{2}$ *per* yard?

	£.	s.	d.	
Divide by 2)	51	10	0	*Value of* 240 *yards.*
	25	15	0	*Value of* 120 *yards.*
Add	2	11	6	*Value of* 12 *yards.*
Answer	£28	6	6	*Value of* 132 *yards.*

		Yds.		s.	d.		£.	s.	d.	
Questions.	4th	109	at	3	$9\frac{1}{2}$	*per yard.*	20	13	$3\frac{1}{2}$	*Answers.*
	5th	110	—	4	3	———	23	7	6	
	6th	125	—	15	7	———	97	7	11	
	7th	112	—	4	9	———	26	12	0	
	8th	113	—	5	6	———	31	1	6	
	9th	130	—	6	4	———	41	3	4	

CASE 21.

When the quantity is 80 *yards, &c.*---One third part of the value of 240 will be the answer.

N. B.

N. B. If there be 12 over, or under, proceed as before directed; except when the quantity is found in the Multiplication-Table: for, then, Case the 2d. of Compound Multiplication will be more concise; or, when the price is an even part of a pound, Case 10th of Practice, is to be preferred.

EXAMPLES.

1. WHAT cost 80 yards of Cloth, at 7/9¾ *per* yard?

	£.	s.	d.	
Divide by 3)	93	15	0	*Value of* 240 *yards.*
Answer	£31	5	0	*Value of* 80 *yards.*

2. WHAT will 68 yards cost, at 4/9¼ *per* yard?

	£.	s.	d.	
Divide by 3)	57	15	0	*Value of* 240 *yards.*
	19	5	0	*Value of* 80 *yards.*
Subtract	2	17	9	*Value of* 12 *yards.*
Answer	£16	7	3	*Value of* 68 *yards.*

3. WHAT will 92 yards cost, at 6/4 *per* yard?

	£.	s.	d.	
Divide by 3)	76	0	0	*Value of* 240 *yards.*
	25	6	8	*Value of* 80 *yards.*
Add	3	16	0	*Value of* 12 *yards.*
Answer	£29	2	8	*Value of* 92 *yards.*

		Yds.		s.	d.		£.	s.	d.	
Questions.	4th	69	at	10	7½	*per yard.*	36	13	1½	*Answers.*
	5th	70	—	3	7¾	——	12	15	2½	
	6th	71	—	4	5	——	15	13	7	
	7th	84	—	16	10	——	70	14	0	
	8th	86	—	14	9	——	63	8	6	
	9th	90	—	2	9	——	12	7	6	

CASE 22.

When the quantity is 60 *yards, &c.*---One fourth of the value of 240 will be the answer.

Note. If there be 12 over, or under, proceed as before directed, and observe the exception made in the last case.

EXAMPLES.

1. WHAT will 60 yards of Cloth cost, at 3/9¾ *per* yard?

	£.	s.	d.	
Divide by 4)	45	15	0	*Value of* 240 *yards.*
Answer	£11	8	9	*Value of* 60 *yards.*

2. WHAT

2. What will 48 yards, at 4/5$\frac{1}{2}$ *per* yard, come to?

	£.	s.	d.		
Divide by 4)	53	10	0	*Value of* 240	*yards.*
	13	7	6	*Value of* 60	*yards.*
Subtract	2	13	6	*Value of* 12	*yards.*
Answer	£10	14	0	*Value of* 48	*yards.*

3. What will 72 yards come to, at 5/4$\frac{1}{2}$ *per* yard?

	£.	s.	d.		
Divide by 4)	64	10	0	*Value of* 240	*yards.*
	16	2	6	*Value of* 60	*yards.*
Add	3	4	6	*Value of* 12	*yards.*
Answer	£19	7	0	*Value of* 72	*yards.*

		Yds.		s.	d.		£.	s.	d.	
Questions.	4th	49	at	6	10$\frac{1}{2}$	*per yard.*	17	16	10$\frac{1}{2}$	*Answers.*
	5th	50	—	9	4	———	23	6	8	
	6th	51	—	10	3	———	26	2	9	
	7th	66	—	6	7	———	21	14	6	
	8th	69	—	17	5	———	60	1	9	
	9th	70	—	9	3	———	32	7	6	

Case 23.

When the quantity is 180;---Three-fourths of the value of 240 will be the anſwer.

Examples.

1. What will 180 yards coſt, at 5/5$\frac{1}{2}$ *per* yard?

	£.	s.	d.		
Divide by 2)	65	10	0	*Value of* 240	*yards.*
Divide by 2)	32	15	0	*Value of* 120	*yards.*
	16	7	6	*Value of* 60	*yards.*
Answer	£49	2	6	*Value of* 180	*yards.*

		Yds.		s.	d.		£.	s.	d.	
Questions.	2d	180	at	3	9$\frac{1}{4}$	*per yard.*	33	18	9	*Answers.*
	3d	180	--	4	9$\frac{1}{2}$	———	43	2	6	
	4th	180	--	6	4$\frac{3}{4}$	———	57	11	3	
	5th	180	--	7	3	———	65	5	0	
	6th	180	--	8	9$\frac{1}{2}$	———	79	2	6	
	7th	180	--	13	9$\frac{1}{2}$	———	124	2	6	

Case 24.

When the price of one hundred weight is of ſeveral denominations, and the quantity likewiſe;---Multiply the price by the integers, (that is

 the

the whole numbers) and take parts for the reſt from the price of an integer: which, added together, will be the anſwer.

EXAMPLES.

1. WHAT will 9 *Cwt.* 3 *qrs.* 14 lb. of ſugar come to, at £4 17*s.* 4*d.* *per* *Cwt.*?

qrs.	*lb.*		£.	*s.*	*d.*				
2	0	½	4	17	4	*Price of*	1 *Cwt.*		
1	0	½			9				
0	14	½					*Cwt.*	*qr.*	lb.
			43	16	0	*Price of*	9	0	0
			2	8	8	*Price of*	0	2	0
			1	4	4	*Price of*	0	1	0
			0	12	2	*Price of*	0	0	14
		Anſwer	£48	1	2	*Price of*	9	3	14

Questions.	*Cwt.*	*qr.*	*lb.*		£.	*s.*	*d.*		£.	*s.*	*d.*
2d	8	1	16	Tobacco, at	5	17	9	*per Cwt.*	49	8	2¾
3d	7	3	19	———	7	12	8	———	60	9	0½
4th	12	1	24	———	3	18	10	———	49	2	7¼
5th	16	2	17	———	2	15	11	———	46	11	1
6th	72	3	27	———	8	11	5	———	625	11	10¾
7th	59	1	14	Sugar at	1	8	7	———	84	17	1½
		lb.	*oz.*								
8th		27	10	Coffee at	0	1	4	*per* lb.	1	17	1⅓
	lb.	*oz.*	*pwt.* *gr.*								
9th	13	10	12 8	Silv. at	4	7	6	*per* lb.	60	14	11
	oz.	*pwt.*	*gr.*								
10th	17	6	16	Gold at	3	16	8	*per oz.*	66	8	10⅔

Anſwers.

CASE 25.

When the price is at any of the rates in the ſecond Practice-Table of aliquot parts;---Multiply the given quantity by the numerator, and divide that product by the denominator;---if the price be pence, the quotient will be the anſwer in ſhillings;---if ſhillings, the anſwer will be pounds.

EXAMPLES.

1. WHAT will 379 yards, at 4½*d.* *per* yard, come to?

```
       379
         3
      ----
    8)1137
    ------
  2|0)14|2  1½
    ------
Anſwer  £7  2  1½
```

2. WHAT will 149 yards, at 6*s.* *per* yard, come to?

```
        149
          3
       ----
    1|0)44|7
Anſwer £44 14
```

Queſtions.

		Yds.		s.	d.		£.	s.	d.	
Questions.	3d	127	at	0	$7\frac{1}{2}$	per yard.	3	19	$4\frac{1}{2}$	Answers.
	4th	159	--	0	8	———	5	6	0	
	5th	173	--	0	9	———	6	9	9	
	6th	241	--	0	10	———	10	0	10	
	7th	249	--	7	6	———	93	7	6	
	8th	357	--	12	6	———	223	2	6	
	9th	345	--	13	4	———	230	0	0	
	10th	323	--	14	0	———	226	2	0	
	11th	287	--	15	0	———	215	5	0	
	12th	253	--	16	8	———	210	16	8	

CASE 26.

When the price is any even number of shillings, if it be required to know what quantity of any thing may be bought for so much money;---Annex a cypher to the money, and divide it by half of the price, and the quotient will be the quantity to be purchased.

EXAMPLES.

1. How many yards of Cloth, at 18*s.* *per* yard, may I have for £345?

Half the price = 9)3450 = *Money with a cypher annexed.*

$383\frac{1}{3}$ *Yards, Answer.*

			s.		£.		Yds.	
Questions.	2d	How many *yds*, at	2	*per yd.* for	427?	———	4270	Answers.
	3d	———	4	———	312	———	1560	
	4th	———	6	———	917	———	$3056\frac{2}{3}$	
	5th	———	8	———	195	———	$487\frac{1}{2}$	
	6th	———	10	———	247	———	494	
	7th	———	12	———	439	———	$731\frac{2}{3}$	

CASE 27.

To find the discount of any Invoice, or Bill of Parcels, at any rate per Cent.---Multiply the pounds in the Invoice by the amount of the discount of 1 pound, at the rate *per Cent.* and take parts for the shillings and pence; then add them together, and the sum will be the discount required.

N. B. The discount for 1 pound at any rate *per Cent.* is in the 3d. Practice-Table.

EXAMPLES.

1. WHAT is the discount of an Invoice, amounting to £65 13*s.* 4*d.* at £$7\frac{1}{2}$ *per Cent.*?

OPERA-

OPERATION.	£.	s.	d.
THE discount of £65 at £5 *per Cent.* is 65*s.* or	3	5	0
THE discount of £65 at £$2\frac{1}{2}$ *per Cent.* is - -	1	12	6
THE discount of 10*s.* being half a pound, at £$7\frac{1}{2}$ *per Cent.* is - - - - - - -	0	0	9
THE discount of 3*s.* 4*d.* being $\frac{1}{6}$ of a pound at £$7\frac{1}{2}$ *per Cent.* is - - - - - -	0	0	3
The sum is,	£4	18	6 *Ans.*

2. WHAT is the discount of a Bill of Parcels, amounting to £8 17*s.* 8*d.* at £$2\frac{1}{2}$ *per Cent.*?

	£.	s.	d.
THE discount of £8 is - - -	0	4	0
Ditto —— of 10*s.* is - - -	0	0	3
Ditto —— of 6*s.* 8*d.* is - -	0	0	2
Ditto —— of 1*s.* is - - -	0	0	$0\frac{1}{4}$
Answer	£0	4	$5\frac{1}{4}$

N. B. WHEN the rate *per Cent.* is any even part of £100, it may be performed by dividing the amount by *that even* part.

3. WHAT is the discount of an invoice amounting to £57 13*s.* 9*d.* at £$12\frac{1}{2}$ *per Cent.*?

		£.	s.	d.
$12\frac{1}{2}$	$\frac{1}{8}$	57	13	9
		£7	4	$2\frac{1}{2}$ *Answer.*

CASE 28.

To find the value of goods sold by particular quantities, viz. I. By the Score. II. Round Timber. III. By 5 Score to the Hundred. IV. By 112 to the Hundred. V. By 6 Score to the Hundred. VI. By the great Gross. VII. By the 1000.

I. *To find the value of goods sold by the Score.*

THE price of one is given, to find the price of one Score.

IF the given price be shillings and pence, or only pence,——*Divide* the given price, in pence, by 12: the quotient will be the answer in pounds, and the remainder will be so many times 1/8.

EXAMPLES.

1. AT 9*d.* each; what is that *per* Score?

12)9*d.*(,75 = £— 15 0 *Answ.*

Or by inverting the question.

1 Score = 20 = 1/8
9
———
15/0

2. AT 4/9 each; what is that *per* Score?

4/9
12
———
12)57*d.*
———
£4 15 *Answer.*

It

It may be remarked, that when the price is shillings and pence, the answer will be just so many pounds as there are shillings, and so many times 1/8 as there are pence : if farthings are given ; for $\frac{1}{4}$*d.* reckon 5*d.* for $\frac{1}{2}$*d.* 10*d.* and for $\frac{3}{4}$*d.* 1/3.

TABLE *of aliquot parts.* 20 *the* Integer.

2	is	$\frac{1}{10}$	6	is	$\frac{3}{10}$	12	is	$\frac{6}{10}$	16	is	$\frac{8}{10}$
4	—	$\frac{1}{5}$	8	—	$\frac{4}{10}$	14	—	$\frac{7}{10}$	18	—	$\frac{9}{10}$
5	—	$\frac{1}{4}$	10	—	$\frac{1}{2}$	15	—	$\frac{3}{4}$			

3. WHAT cost 7 ; at 2/9 *per* Score ?

		s. *d.*
5	$\frac{1}{4}$	2 9
2	$\frac{1}{10}$	-- $8\frac{1}{4}$
—		-- $3\frac{1}{4}$
7	=	-- $11\frac{1}{2}$

4. WHAT cost 17 ; at 19/19 *per* Score ?

		s. *d.*
10		19 10
5	$\frac{1}{2}$	9 11
2	$\frac{1}{4}$	4 $11\frac{1}{2}$
	$\frac{1}{10}$	1 $11\frac{3}{4}$
—		
17	=	16 $10\frac{1}{4}$

II. *Round Timber.*

FORTY feet make a Load or Ton of Round Timber.

IF the given price of a foot be shillings,

RULE.

MULTIPLY the given price by 2, and the product will be the answer in pounds.

5. WHAT cost a ton at 3/ *per* foot ? 3*s.* × 2 = £6 *Ans.*

6. WHAT cost a ton at 9/ *per* foot ? 9*s.* × 2 = £18 *Ans.*

IF the given price of 1 foot be pence only, or shillings and pence, ——*Divide* the given price, in pence, by 6 ; the quotient will be the answer in pounds, and the remainder will be so many times 3/4.

7. WHAT cost 40 feet, at 17*d.* *per* foot ?

6)17
£2 16 8 *Answ.*

8. AT 1/9 *per* foot, what cost a ton ?

6)21
£3 10 *Answer.*

IF the given price of a foot be farthings only, or pence and farthings——*Divide* the given price, in farthings, by 6, then divide *that quotient* by 4, and *this last quotient* will be the answer.

9. AT $\frac{3}{4}$*d.* *per* foot, what cost a ton ?

6)3
4)-- 10
£-- 2 6 *Answ.*

10. AT $13\frac{1}{4}$ *per* foot, what cost a ton ?

$13\frac{1}{4}$
4
6)53
4)8 16 8
£2 4 2 *Answ.*

Or,

Or,

Suppose every shilling in the price to be so many times £2 : every penny to be so many times 3/4 : and every farthing to be 10*d.*

11. What cost 40 feet at $\frac{3}{4}d.$ *per* foot?

$\frac{3}{4}d. \times 10 = £$-- | 2 6 *Answ.*

12. What cost 40 at $15\frac{1}{2}d.$ *per* foot?

s.	d.		£	s.	d.
1	--	× 2 =	£2	--	--
3	4	× 3 =	--	10	--
	$\frac{1}{2}$	× 10 =	--	1	8
			£2	11	8 *Answ.*

III. *To find the value of goods sold by 5 Score to the Hundred.*

1. If the given price be pounds and shillings, or shillings only.

RULE.

Multiply the given price, in shillings, by 5, and the quotient will be the answer in pounds.

13. At 19*s per* yard, what cost 100 yards?

```
 19s
  5
----
£95 Answ.
```

14. At £4 13*s per Cwt.* what cost 100 *Cwt.* or 5 tons?

```
  4  13
  20
 ----
  93
   5
 ----
£465 Answ.
```

2. If the given price of 1 be pence only, or shillings and pence.

RULE.

Multiply the given price, in pence, by 5; then divide that product by 12; the quotient will be pounds, and the remainder so many times 1/8.

15. If 1 yard cost 9*d*; what cost 100 yards?

```
    9
    5
   --
12)45
  ----
 £3 15 Answ.
```

16. What cost 100 bushels, at 35*s* 4*d per* bushel?

```
     s.  d.
     35  4
     12
    ----
    424
      5
    ----
12)2120
   -----
£176 13 4 Answ.
```

Or,

```
            35s. 4d.
| 4d. | 1/3 |  5
           ----
            175
              1 13 4
           --------
           £176 13 4
```

Here 5 is divided by $\frac{1}{3}$.

3. If

3. If the given price of 1 be shillings and pence;---Multiply the price by 5, and the product, under the place of shillings, will be the answer in pounds, and the product under the place of pence, will be so many times 1*s.* 8*d.*

17. At 2/5 *per* bushel; what cost 100 bushels?

	s.	*d.*	
	2	5	
		5	
	12	1	
Answ.	£12	1	8

18. At 25/3 *per* ton; what cost 100 tons?

s.	*d.*	
25	3	1/8×3=5/
	5	
126	3	
£126	5	*Answer.*

4. To find the price of *one*, at so much *per* hundred of 5 score.

GENERAL RULE.

Multiply the given price by 12; divide the product by 5, and the quotient will be the answer in pence.

But if the price be pounds only;

RULE.

Divide the given price by 5, and the quotient will be the answer in shillings.

19. If 100 *yds.* cost £65; what cost 1 *yd.*?

5)65

13*s.* *Answer.*

20. If 100 *yds.* cost £2 18/4; what is that *per yard*?

2	18	4
		12
5)35	--	--
		7*d.* *Answer.*

21. If 100 yards cost £11 7*s.* 9*d.* what cost 1 yard?

£11	7	9
		12
5)136	13	--
12)27	6	7
	2*s.*	$3\frac{1}{4}$*d.* *Answer.*

In dividing 27 by 12 (in the 21st. question) the quotient is 2s. and the remainder 3d.---the 6 is $\frac{6}{10}$ of a penny = one farthing, and the 7 is of no account.

TABLE *of aliquot parts.* 100 *the* Integer.

5	is	$\frac{1}{20}$	25	is	$\frac{1}{4}$	50	is	$\frac{1}{2}$	75	is	$\frac{3}{4}$
10	—	$\frac{1}{10}$	30	—	$\frac{3}{10}$	60	—	$\frac{6}{10}$	80	—	$\frac{8}{10}$
20	—	$\frac{1}{5}$	40	—	$\frac{4}{10}$	70	—	$\frac{7}{10}$	90	—	$\frac{9}{10}$

22. At

22. At £3 7/6 *per* 100; what will 23 cost?

		£. s. d.	
20	$\frac{1}{5}$	3 7 6	
		- 13 6	Add.
2	$\frac{1}{10}$	- 1 4	
1	$\frac{1}{2}$	- -- 8	
23=			
		£- 15 6	*Answer.*

23. At £2 1/10 *per* 100; what cost 18?

		£. s. d.	
20	$\frac{1}{5}$	2 1 10	
		- 8 $4\frac{2}{5}$	Sub.
2	$\frac{1}{10}$	- - 10	
18			
		=£- 7 $6\frac{2}{5}$	*Answ.*

24. At £5 9/6 *per* 100; what cost 35?

		£. s. d.	
		5 9 6	
		3	
		10) 16 8 6	
5	$\frac{1}{6}$	1 12 10	Add.
		5 $5\frac{2}{3}$	
		£ 1 18 $3\frac{2}{3}$	*Answer.*

IV. *To find the value of goods sold by* 112 ℔. *the Cwt.*

The price of 1 ℔. is given to find the value of 1 *Cwt.*

RULE.

For a farthing, account 2/4 *per Cwt.* For a half-penny, 4/8. For three farthings, 7/; and for every penny, 9/4 *per Cwt.*

25. What cost 1 *Cwt.* at $3\frac{1}{2}d.$ *per* ℔.

	s. d.	
At 1*d.* *per* ℔. 1 *Cwt.* costs	- 9 4	
	3	
At 3*d.* - -	£1 8 -	Add.
At $\frac{1}{2}d.$ - -	- 4 8	
	£1 12 8	*Answ.*

26. At $8\frac{3}{4}d.$ *per* ℔. what cost 1 *Cwt.*?

	s. d.	
At 1*d.* *per* ℔. 1 *Cwt.* costs -	- 9 4	
	8	
At 8*d.* - -	£3 14 8	Add.
At $\frac{3}{4}$	- 7 -	
	£4 1 8	*Answ.*

V. *To find the value of goods sold by* 6 *score to the hundred.*

The price of 1 is given, to find the price of 1 hundred.

RULE.

Suppose every penny in the price to be so many pounds, and for the farthings, such a part of a pound, as they are of a penny; then, half of that sum will be the answer.

27. At

27. At $4\frac{1}{2}d.$ *per* yard, what cost 120 yards?

	£.	s.
2)	4	10
	£2	5 *Answ.*

28. At $16/9\frac{1}{4}$ *per* yard; what cost 120 yards?

$16/9\frac{1}{4}$
12
201

	£.	s.	
2)	201	5	
	£100	12	6 *Answer.*

To find the price of *one*, at so much *per* hundred of 6 score.

RULE.

Multiply the price by 2, then call the pounds so many pence, and the shillings, such a part of a penny, as they are of a pound, and you will have the answer.

29. If 120 *yds.* cost £3 12*s*; what cost 1 yard?

£.	s.
3	12
	2
7	4

Answ. $7\frac{1}{5}d.$

30. If 120 *yds.* cost £5 18/6; what cost 1?

£.	s.	d.
5	18	6
		2
11	17	-

Ans. $11\frac{3}{4}d. + \frac{2}{5}$ *of a farthing.*

TABLE *of aliquot parts.* 120 *the* Integer.

Also,

6	is	$\frac{1}{20}$	24	is	$\frac{1}{5}$	36	is	$\frac{3}{10}$	72	is	$\frac{3}{5}$	96	is	$\frac{4}{5}$
10	—	$\frac{1}{12}$	30	—	$\frac{1}{4}$	45	—	$\frac{3}{8}$	75	—	$\frac{5}{8}$	100	—	$\frac{5}{6}$
12	—	$\frac{1}{10}$	40	—	$\frac{1}{3}$	48	—	$\frac{2}{5}$	80	—	$\frac{2}{3}$	105	—	$\frac{7}{8}$
15	—	$\frac{1}{8}$	60	—	$\frac{1}{2}$	50	—	$\frac{5}{12}$	84	—	$\frac{7}{10}$	108	—	$\frac{9}{10}$
20	—	$\frac{1}{6}$				70	—	$\frac{7}{12}$	90	—	$\frac{3}{4}$	110	—	$\frac{11}{12}$

31. At £3 17/6 *per* hundred; what cost 14?

		£.	s.	d.	
12	$\frac{1}{10}$	3	17	6	
2	$\frac{1}{6}$	-	7	9	*Add.*
		-	1	$3\frac{1}{2}$	
14	=	£ -	9	$0\frac{1}{2}$	*Answ.*

32. At £2 13/6½ *per* hundred; what cost 49?

		£. s. d.
40	½	2 13 6½
8	⅕	1 6 9¼
1	⅛	- 5 4¼
		- - 8

49 = £1 12 9½ *Answ.*

33. At £1 19/3 *per* hundred; what cost 75;

£. s. d.
1 19 3
5
8)9 16 3
£1 4 6¼ *Answ.*

VI. *To find the value of Goods sold by the great gross.*

Note, 12 make 1 dozen, 12 dozen 1 small gross, 12 small gross 1 Great gross.

The price of 1 dozen being given, in pence, to find the price of a Great gross.

RULE.

Multiply the price of 1 dozen, in pence, by 3, then divide that product by 5, and the quotient will be the answer in pounds, &c.

For proof, do the contrary.

N. B. *If the price of* 1 *be given, the price of* 1 *small gross is found after the same manner.*

34. At 18*d. per* dozen; what cost 1 great gross?

3
5)54
£10 16 *Answ.*

35. At 4/3 *per* dozen; what cost 1 great gross.

4/3
12
51*d.*
3
5)153
Ans. £30 12

Or.
s. d.
4 3
12
2 11 -
12
£30 12 -

Or.
£. s.
144 = 7 4
4
| 3*d.* | ¼ | 28 16 } *Add.*
1 16 }
£30 12

TABLE *of aliquot parts.* 144 *the* Integer.

Also,

12	is	$\frac{1}{12}$	36	is	$\frac{1}{4}$	32	is	$\frac{2}{9}$	84	is	$\frac{7}{12}$	128	is	$\frac{8}{9}$
16	—	$\frac{1}{9}$	48	—	$\frac{1}{3}$	60	—	$\frac{5}{12}$	96	—	$\frac{2}{3}$	132	—	$\frac{11}{12}$
18	—	$\frac{1}{8}$	72	—	$\frac{1}{2}$	64	—	$\frac{4}{9}$	108	—	$\frac{3}{4}$			
24	—	$\frac{1}{6}$				80	—	$\frac{5}{9}$	120	—	$\frac{5}{6}$			

36. At

36. At £2 12/9 *per* Great grofs; what coft 49 dozen?

Doz.		£.	s.	d.	
36	$\frac{1}{4}$	2	12	9	
9	$\frac{1}{4}$	-	13	$2\frac{1}{4}$	} *Add.*
		-	3	$3\frac{1}{2}$	
45 =		£ -	16	$5\frac{3}{4}$	*Anfwer.*

37. What coft 117 dozen, at £9 13/7 *per* Great grofs?

Doz.		£.	s.	d.	
		9	13	7	
				3	
108	$\frac{3}{4}$	29	00	9	
9	$\frac{1}{12}$	7	5	$2\frac{1}{4}$	} *Add.*
			16	$1\frac{1}{2}$	
117 =		£8	1	$3\frac{3}{4}$	*Anfw.*

38. At £3 16/8 *per* Great grofs; what coft 7 Great grofs, and 96 dozen?

	£.	s.	d.	
	3	16	8	
			12	
	3)7	13	4	
Doz. 96 =	2	11	$1\frac{1}{4}$	
Top-line × 7 =	26	16	8	
	£29	7	$9\frac{1}{4}$	*Anfw.*

VII. *To find the value of goods fold by the thoufand.*

The price of 1 is given, to find the price of 1000.

Rule.

Multiply the given price, in pence, by 50, then divide the product by 12, and the quotient will be the anfwer in pounds, &c.

Or, As 1000*f are* £50; *take parts, for the pence, out of* 50.

39. At 6*d.* each; what coft 1000?

6
50
12)300
£ 25 *Anf.*

6*d.*	$\frac{1}{2}$	50
		25 *Anf.*

40. What coft 1000 at $2\frac{1}{4}$*d.* each.

2*d.*	$\frac{1}{6}$	50			
$\frac{1}{4}$	$\frac{1}{8}$	8	6	8	} *Add.*
		1	-	10	
		£9	7	6	*Anf.*

To find the price of *one*, at fo much *per* thoufand.

Rule.

Multiply the price by 12; divide the product by 50; then take the pounds for fo many pence, and the fhillings for fuch a part of a penny, as they are of a pound, which will be the anfwer.

41. At

41. At £5 4/2 *per* 1000; what cost 1?

	£.	s.	d.
	5	4	2
			12
50 { 5)	62	10	—
10)	12	10	
	£1	5	

Answer $1\frac{1}{4}d.$

42. At £354 3/4*d. per* 1000; what cost 1?

	£.	s.	d.
	354	3	4
			12
50 { 10)	4250	-	-
5)	425		
	85		

Answ. 7*s.* 1*d.*

Or,

		£.	s.	d.
100	$\frac{1}{10}$	354	3	4
10	$\frac{1}{10}$	35	8	4
1	$\frac{1}{10}$	3	10	10
Answ.		—	7	1

TABLE *of aliquot parts.* 1000 *the* Integer.

Also,

50 is $\frac{1}{20}$	200 is $\frac{1}{5}$	300 is $\frac{3}{10}$	700 is $\frac{7}{10}$
100 — $\frac{1}{10}$	250 — $\frac{1}{4}$	375 — $\frac{3}{8}$	750 — $\frac{3}{4}$
125 — $\frac{1}{8}$	500 — $\frac{1}{2}$	400 — $\frac{2}{5}$	800 — $\frac{4}{5}$
		600 — $\frac{3}{5}$	875 — $\frac{7}{8}$
		625 — $\frac{5}{8}$	900 — $\frac{9}{10}$

43. At £1 17/9 *per* 1000; what cost 115?

		£.	s.	d.	
100	$\frac{1}{10}$	1	17	9	
10	$\frac{1}{10}$	-	3	$9\frac{1}{4}$	} *Add.*
5	$\frac{1}{2}$	-	-	$4\frac{1}{2}$	
		-	-	$2\frac{1}{4}$	
115 =		£-	4	4	*Answer.*

44. At

44. AT £2 1s 8 *per* 1000; what cost 875?

£.	s.	d.
2	1	8
		7
8)14	11	8
£1	16	$5\frac{1}{2}$ *Answ.*

45. WHAT cost 33, at 24s 8 *per* 1000?

		£.	s.	d.	
50	$\frac{1}{20}$	1	4	8	
25	$\frac{1}{2}$	--	1	$2\frac{2}{4}$	
		--	--	$7\frac{1}{4}$	*Add.*
5	$\frac{1}{5}$	--	--	$1\frac{1}{4}$	
30	=	--	--	$8\frac{1}{2}$	*Add.*
3	$\frac{1}{10}$	--	--	$\frac{3}{4}$	
33	=	--	--	$9\frac{1}{4}$	*Answer.*

PRACTICE BY DECIMALS.

I. SINCE 2s is $\frac{1}{10}$ of £1, the decimal of 2s is ,1: wherefore any quantity being given at 2s *per* lb. yard, &c. the price is found in pounds and decimal parts of a pound, by separating the unit figure of the given quantity from the rest, for a decimal.

LET it be required to find the value of 356 yards at 2s *per* yard?

BY pointing off the unit figure 6 for a decimal, I find the amount to be £35,6, which is known to be equal to £35 12s. £35,6

II. CONSEQUENTLY, if the price be a multiple of 2s (viz. any even number of shillings) the amount at 2s being first found in pounds and decimal parts, as above, and that amount multiplied by the number which shews how often 2s is contained in the given price, the product will be the amount required in pounds and decimal parts of a pound.

WHAT cost 427 gallons of wine, at 8s *per* gallon?

£42,7 amount at 2s *per* gallon.
4

Answ. £170,8 *or* £170 16s.

THE examples in Case 5th may be worked in this manner.

LIKEWISE, If the price be pounds and even shillings.

754 yards at £1 8s.

75,4 amount at 2s.
14×2=28s.

3016
754

Answ. £1055,6=£1055 12s.

Or,

754
754×,4=301,6 } *Add.*

£1055,6

III.

III. *If the price be an aliquot part of 2s.*—Find the amount at 2s. and divide it by the denominator of the part, and the quotient will be the anſwer.

At 8*d.* *per* ℔. what coſt 976 ℔. ?

| 8*d.* | $\frac{1}{3}$ | 97,6

£32,533=£32 10 $8\frac{1}{2}$ *Anſw.*

IV. *If the price be an aliquant (that is, uneven) part*; Divide it into aliquot parts.

7235 Yards, at 7*d.*

| 4*d.* | $\frac{1}{6}$ | 723,5
| 3*d.* | $\frac{1}{3}$ | 120,583
90,437

211,02 = £211 0 $4\frac{3}{4}$

V. *If the price be pounds and ſhillings, or pounds, ſhillings and pence*; Reduce the ſhillings, &c. to the decimal of a pound, and multiply the quantity thereby, or the price by the quantity.

At £15 12 6 *per Cwt.*; what coſt 75 *Cwt.* ?

£15 12 6=£15,623
,75

78115
109361

1171,725

£1171 14 $6\frac{1}{2}$ *Anſw.*

VI. *If the quantity likewiſe be of divers denominations*; Reduce the leſs denominations to the decimal of that, whereof the price is given.

9℔. 10*oz.* of Silk, at £4 5 9=£4,218
9℔. 10*oz.*=9,625

21 435
8 574
257 22
3858 3

£41,262375 =£41 5 $2\frac{3}{4}$ *Anſw.*

Cases 12th. and 13th. may be wrought in this manner.

Or,

Or, You may take parts for the lower denominations.

8*oz.*	$\frac{1}{2}$	4,287
2*oz.*	$\frac{1}{4}$	9
		38,583
		2,1435
		,535875
		41,262375
		£41 5 $2\frac{3}{4}$

VII. When the price is any *odd* number of ſhillings: If it be required to know what quantity of any thing may be bought for any ſum of money, in pounds: Annex *two* cyphers to the money, and divide it by half the price.

Note. As half a ſhilling (or 6 pence) is ,5, therefore, to halve any odd number of ſhillings, is only to annex ,5 to half of the greateſt even number in the price.

1. How many *yds*. at 7/ *per yd*. may I have for £435 ?

Half=3,5)43500(1242$\frac{30}{35}$ *yds. anſ.*
35
85
70
150
140
100
70
30

2. How many pounds of Tea, at 5/ *per* ℔. for £37 ?

2,5)3700(148 *yds. Anſw.*
25
120
100
200
200

3. How many yards, at 9/ *per* yard, may I have for £540 ?

Anſw. 1200 *yards.*

Bill *of* Parcels.

Newbury-port, June 1ſt. 1787.

Mr. Timothy Huckſter

Bought of *Samuel Merchant.*

$25\frac{1}{2}$ ℔. Bohea-Tea, at 3/6 *per* ℔.
48 ℔. Cheeſe at /9 *per* ℔.
15 Pair of worſted Hoſe, at 5/8 *per* pair.
$4\frac{1}{2}$ Dozen Women's Gloves, at 36/6 *per* dozen.
19 Dozen Knives and Forks, at 5/9 *per* dozen.
9 Grindſtones at 15/9 *per* ſtone.
$\frac{1}{2}$ *Cwt.* Brown Sugar, at 51/ *per Cwt.*
31 ℔. Loaf Sugar, at $1/0\frac{1}{2}$ *per* ℔.

£34 3 $3\frac{1}{2}$

Received payment in full.

Samuel Merchant.

TARE

TARE and TRETT.

TARE and TRETT are practical rules for deducting certain allowances, which are made by Merchants and Tradesmen in selling their goods by weight.

TARE is an allowance, made to the buyer, for the weight of the box, barrel, or bag, &c. which contains the goods bought, and is either at so much *per* box, &c.—at so much *per Cwt.* or at so much in the gross weight.

TRETT is an allowance of 4 lb. in every 104 lb. for waste, dust, &c.

CLOFF is an allowance of 2 lb. upon every 3 *Cwt.*

GROSS-WEIGHT is the whole weight of any sort of goods, together with the box, barrel, or bag, &c. which contains them.

SUTTLE is, when part of the allowance is deducted from the gross.

NEAT-WEIGHT is what remains after all allowances are made.

CASE 1.†

When the Tare is at so much per *box, barrel or bag, &c.*—Multiply the number of boxes, barrels, &c. by the Tare, and subtract the product from the gross, and the remainder will be the neat weight required.

EXAMPLES.

1. IN 6 hogsheads of Sugar, each weighing 9 *Cwt.* 2 *qrs.* 10 lb. gross, tare 25 lb. *per* hogshead; how much neat?

	Cwt.	qr.	lb.	Cwt.	qr.	lb.	
25×6=	1	1	10	9	2	10	*Gross wt. of one hhd.*
						6	
				57	2	4	*Gross.*
				1	1	10	*Tare.*
Answer				56	0	22	*Neat.*

2. IN 5 bags of Cotton, marked with the gross weight as follows, tare 23 lb. *per* bag; what neat weight?

	Cwt.	qr.	lb.
A =	7	1	19
B =	3	3	27
C =	5	1	12
D =	6	0	15
E =	8	1	00

	Cwt.	qr.	lb.
Answ.	30	0	14 *neat.*

3. WHAT is the neat weight of 15 hogsheads of Tobacco, each 7 *Cwt.* 1 *qr.* 13 lb. Tare 100 lb. *per hhd.*?

	Cwt.	qr.	lb.
Answer	97	0	11

CASE 2.

† THIS, as well as every other case in this rule, is only an application of the rules of Proportion and Practice.

CASE 2.

When the Tare is at so much per Cwt.—Divide the groſs weight by the aliquot parts of a *Cwt.* Subtract the quotient from the groſs, and the remainder will be the neat weight.

EXAMPLES.

1. IN 129*Cwt.* 3*qrs.* 16℔. Groſs, Tare 14℔. *per Cwt.* what neat weight?

		Cwt.	qr.	lb.	
14℔	$\frac{1}{8}$	129	3	16	*Groſs.*
		16	0	$26\frac{1}{2}$	*Tare.*
Anſwer		113	2	$17\frac{1}{2}$	*Neat.*

2. IN 97*Cwt.* 1*qr.* 7℔. Groſs, Tare, 20℔. *per Cwt.* what neat weight?

lb.		Cwt.	qr.	lb.	
16	$\frac{1}{7}$	97	1	7	*Groſs.*
4	$\frac{1}{4}$	13	3	17	*Add.*
		3	1	25	
Subtract		17	1	14	*Tare.*
Anſw.		79	3	21	*Neat.*

3. WHAT is the neat weight of 9 barrels of Pot-aſh, each weighing 305℔. Groſs; Tare 12℔. *per Cwt.*?

Anſw. 2450℔. 14*oz.* 5*dr.*

4. WHAT is the value of the neat weight of 7 *hhds.* of Tobacco, at £5 7/6 *per Cwt.* each weighing 8*Cwt.* 3*qrs.* 10℔. Groſs, Tare 21℔. *per Cwt.*? *Anſw.* £270 4/$4\frac{1}{2}$ *reckoning the odd ounces.*

CASE 3.

When Trett is allowed with Tare;—Divide the Suttle weight by 26, and the quotient will be the Trett, which ſubtract from the Suttle, and the remainder will be the neat.

EXAMPLES.

1. IN 247*Cwt.* 2*qrs.* 15℔. groſs, tare 28 *per Cwt.* and trett 4℔. *per* 104℔. what neat weight?

lb.		Cwt.	qr.	lb.		
28	$\frac{1}{4}$	247	2	15		*Groſs.*
		61	3	17	12	*Tare, Subtract.*
4	$\frac{1}{26}$	185	2	25	4	*Suttle.*
		7	0	16	0	*Trett, Subtract.*
Anſw.		178	2	9	4	*Neat.*

2. WHAT is the neat weight of 4 *hhds.* of Tobacco, weighing as follow; the 1ſt. 5*Cwt.* 1*qr.* 12℔. groſs, tare 65℔. *per* hogſhead: ſecond, 3*Cwt.* 0*qr.* 19℔. groſs; tare 75℔: the 3d. 6*Cwt.* 3*qrs.*

grofs, tare 49 lb. and the 4th. 4*Cwt.* 2*qrs.* 9 lb. grofs; tare 35 lb. and allowing Trett to each as ufual? *Anfw.* 17*Cwt.* 0*qr.* 9 lb.+

CASE 4.

When Tare, Trett and Cloff are allowed;—Deduct the Tare and Trett as before, and divide the Suttle by 168, and the quotient will be the Cloff, which fubtract from the Suttle, and the remainder will be the neat.

EXAMPLES.

1. WHAT is the neat weight of a *hhd* of Tobacco, weighing 16*Cwt.* 2*qrs.* 20 lb. grofs, Tare 14 lb *per* Cwt. Trett 4 lb. *per* 104 and Cloff 2 lb. *per* 3 *Cwt.*?

	Cwt.	*qrs.*	*lb.*	*oz.*		
14 lb. is $\frac{1}{8}$)	16	2	20	0	*Grofs.*	
	2	0	9	8	*Tare.*	*Subtract.*
4 lb is $\frac{1}{26}$)	14	2	10	8		
	—	2	6	13	*Trett.*	*Subtract.*
2 lb. is $\frac{1}{168}$)	14	0	3	11	*Suttle.*	
	—	-	9	5	*Cloff.*	*Subtract.*
Anfwer	13	3	22	6	*Neat.*	

2. IF 9 *hhds.* of Tobacco, contain 85*Cwt.* 0*qr.* 2 lb. Tare 30 lb. *per hhd.* Trett and Cloff as ufual; what will the neat weight come to, at 6½*d.* *per* lb. after deducting for duties and other charges, £51 11*s.* 8*d.*? *Anfwer* £187 18*s.* 5*d.*

INVOLUTION,

Or TO RAISE POWERS.

A POWER is the product arifing from multiplying any given number into itfelf continually a certain number of times, thus:

$3\times3=9$ is the 2d. power, or fquare of 3. $=3^2$

$3\times3\times3=27$ is the 3d. power, or cube of 3. $=3^3$

$3\times3\times3\times3=81$ is the 4th. power, or the biquadrate of 3, &c. $=3^4$

THE number denoting the power is called the *Index*, or the *Exponent* of that Power; thus, the fourth Power of 3 is 81, or 3^4—the fecond Power of 5 is 25, or 5^2, &c.

$2\times2=4$, the Square of 2: $4\times4=16=$4th. Power of 2: $16\times16=256=$8th. Power of 2, &c.

RULE.

MULTIPLY the given number, Root, or firft power continually by itfelf, till the number of multiplications be 1 lefs than the index of the power to be found, and the laft product will be the power required.

Note. Whence, becaufe fractions are multiplied by taking the products of their numerators, and of their denominators, they will be involved by raifing each of their terms to the power required;—and if a mixed number be propofed, either reduce it to an improper fraction,

fraction, or reduce the vulgar fraction to a decimal, and proceed by the rule.

EXAMPLES.

1. WHAT is the 5th. power of 9?

$$
\begin{array}{r l}
9 & \\
9 & \\
\hline
81 & = \textit{2d. power.} \\
9 & \\
\hline
729 & = \textit{3d. power.} \\
9 & \\
\hline
6561 & = \textit{4th. power.} \\
9 & \\
\hline
59049 & = \textit{5th. power, or answer} = 9^5.
\end{array}
$$

2. WHAT is the fifth power of $\frac{3}{5}$? *Answer* $\frac{243}{3125}$.
3. WHAT is the 4th. power of ,045? *Answ.* ,000004100625.

Here we see that, in raising a fraction to a higher power, we decrease its value.

EVOLUTION,

Or the EXTRACTION *of* ROOTS.

THE ROOT is a number whose continual multiplication into itself produces the power, and is denominated the square, cube, biquadrate, or 2d. 3d. 4th. Root, &c. accordingly as it is, when raised to the 2d. 3d. 4th. &c. power, equal to that power. Thus 4 is the square root of 16, because $4\times4=16$, and 3 is the cube root of 27, because $3\times3\times3=27$; and so on.

ALTHOUGH there is no number of which we cannot find any power exactly, yet there are many numbers, of which precise roots can never be determined. But, by the help of decimals, we can approximate towards the root, to any assigned degree of exactness.

THE Roots, which approximate, are called *surd Roots*, and those, which are perfectly accurate, are called *rational Roots*.

ROOTS are sometimes denoted by writing the character $\surd$ before the power, with the index of the root over it; thus the 3d. root of 36 is expressed $\sqrt[3]{36}$, and the 2d. root of 36 is $\sqrt{36}$, the index 2 being omitted when the square root is designed.

IF the power be expressed by several numbers, with the sign + or − between them, a line is drawn from the top of the sign over all the parts of it; thus, the 3d. root of 47 + 22 is $\sqrt[3]{47+22}$, and the 2d. root of 59 − 17 is $\sqrt{59-17}$, &c.

SOMETIMES roots are designed like powers, with fractional indices; thus, the square root of 15 is $15^{\frac{1}{2}}$, the cube root of 21 is $21^{\frac{1}{3}}$, and 4th root of 37 − 20 is $\overline{37-20}^{\frac{1}{4}}$, &c.

A TABLE OF POWERS.

Roots, - - -	or 1ſt. Powers.	1	2	3	4	5	6	7	8	9
Squares, - - -	or 2d. Powers.	1	4	9	16	25	36	49	64	81
Cubes, - - -	or 3d. Powers.	1	8	27	64	125	216	343	512	729
Biquadrates, - -	or 4th. Powers.	1	16	81	256	625	1296	2401	4096	6561
Surſolids, - - -	or 5th. Powers.	1	32	243	1024	3125	7776	16807	32768	59049
Square Cubes, - -	or 6th. Powers.	1	64	729	4096	15625	46656	117649	262144	531441
Second Surſolids, -	or 7th. Powers.	1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrates Squared,	or 8th. Powers.	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cubes Cubed, - -	or 9th. Powers.	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Surſolids Squared, -	or 10th. Powers.	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	4386784401
Third Surſolids, -	or 11th. Powers.	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
Square Cubes Squared,	or 12th. Powers.	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481
Fourth Surſolids, -	or 13th. Powers.	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329
2d. Surſolids Squared,	or 14th. Powers.	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961
Surſolids Cubed, -	or 15th. Powers.	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649

The Extraction *of the* SQUARE ROOT.

Rule.

† 1. Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points shew the number of figures the root will consist of.

2. Find the greatest square number in the first, or left hand, period, place the root of it at the right hand of the given number, (after the manner of a quotient in division) for the first figure of the root, and the square number, under the period, and subtract it therefrom, and to the remainder bring down the next period for a dividend.

3. Place the double of the root, already found, on the left hand of the dividend for a divisor.

4. Seek how often the divisor is contained in the dividend, (except the right hand figure) and place the answer in the root for the second figure of it, and likewise on the right hand of the divisor: multiply the divisor with the figure last annexed by the figure last placed in the root, and subtract the product from the dividend: To the remainder join the next period for a new dividend.

5. Double the figures already found in the root, for a new divisor, (or, bring down your last divisor for a new one, doubling the right hand figure of it) and from these, find the next figure in the root as last directed, and continue the operation, in the same manner, till you have brought down all the periods.

Note 1. If, when the given power is pointed off as the power requires, the left hand period should be deficient, it must nevertheless stand as the first period.

Note 2. If there be decimals in the given number, it must be pointed both ways from the place of units:—If, when there are integers, the first period in the decimals be deficient, it may be completed

† In order to shew the reason of the rule, it will be proper to premise the following *Lemma.* The product of any two numbers can have, at most, but so many places of figures as are in both the factors, and at least but one less.

Demonstration. Take two numbers consisting of any number of places; but let them be the least possible of those places, *viz.* Unity with cyphers, as 100 and 10: then their product will be 1 with so many cyphers annexed as are in both the numbers, *viz.* 1000; but 1000 has one place less than 100 and 10 together have; And since 100 and 10 were taken the least possible, the product of any other two numbers, of the same number of places, will be greater than 1000; consequently, the product of any two numbers can have, at least, but one place less than both the factors.—

Again, take two numbers, of any number of places, which shall be the greatest possible of those places, as 99 and 9. Now, 99 x 9 is less than 99 x 10; but 99 x 10 (= 990) contains only so many places of figures as are in 99 and 9; therefore 99 x 9, or the product of any other two numbers, consisting of the same number of places, cannot have more places of figures, than are in both its factors.

Corollary 1. A square number cannot have more places of figures than double the places of the root, and at least but one less.

Corollary 2. A cube number cannot have more places of figures than triple the places of the root, and at least but two less.

pleted by annexing ſo many cyphers as the power requires: And the root muſt be made to conſiſt of ſo many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the given number are exhauſted, the operation may be continued at pleaſure by annexing cyphers.

EXAMPLES.

1. REQUIRED the ſquare root of 30138696025?

```
                         30138696025(173605 the Root.
                         1
                         -
1ſt. Diviſor =        27)201
                         189
                         ---
2d. Diviſor =        343)1238
                         1029
                         ----
3d. Diviſor =       3466)20969
                         20796
                         -----
4th. Diviſor =    347205)1736025
                         1736025
                         -------
```

2. REQUIRED the ſquare root of 575,5?

```
     575,50(23,98+ the Root.
     4
     -
  43)175
     129
     ---
 469)4650
     4221
     ----
4788)42900
     38304
     -----
      4596 Remainder.
```

3. WHAT is the ſquare root of 10342656? *Anſw.* 3216.
4. WHAT is the ſquare root of 964,5192360241? *Anſ.* 31,05671
5. WHAT is the ſquare root of 234,09? *Anſw.* 15,3.
6. WHAT is the ſquare root of ,0000316969? *Anſw.* ,00563.
7. WHAT is the ſquare root of ,45369? *Anſw.* ,213.

RULES *for the* SQUARE ROOT *of* VULGAR FRACTIONS *and* MIXED NUMBERS.

AFTER reducing the fraction to its loweſt terms, for this and all other roots; then,

1. EXTRACT

1. Extract the *Root* of the *numerator* for a *new numerator*, and the *root* of the *denominator* for a *new denominator*, which is the best method, provided the denominator be a complete power. But if it be not,—

2. Multiply the numerator and denominator together; and the root of this product being made the numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional part required: *——*Or*,

3. Reduce the vulgar fraction to a decimal, and extract its root.

4. Mixed numbers may either be reduced to improper fractions, and extracted by the first or second rule; or the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

Examples.

1. What is the square root of $\frac{144}{15129}$?

By Rule 1.

$\frac{144}{15129} = \frac{16}{1681}$

$\dot{1}6(4$ *Root of the numerator.*
16
—

$\dot{1}6\dot{8}1(41$ *Root of the denominator.*
16
—
81)81
81

Therefore $\frac{4}{41} =$ *the Root of the given fraction.*

By Rule 2.

$16 \times 1681 = 26896$ and $\sqrt{26896} = 164$. then

$\frac{164}{1681} = \frac{16}{164} = \frac{4}{41} = ,09756+$

By Rule 3.

$1681)16(,0095181439+$. and $\sqrt{,0095181439} = ,09756+$.

2. What is the square root of $\frac{2793}{8208}$? *Answ.* $\frac{7}{12}$.

3. What is the square root of $42\frac{1}{4}$? *Answ.* $6\frac{1}{2}$.

Note. In extracting the square or cube root of any surd number, there is always a remainder or fraction left, when the root is found: To find the value of which, the common method is, to annex *pairs* of cyphers to the resolvend, for the square, and *ternaries* of cyphers to that of the cube, which makes it tedious to discover the value of the remainder, especially in the cube, whereas this trouble might be saved if the true denominator could be discovered.

Now,

* That is, Suppose $a=7$, and $b=2$, the rule may be thus expressed: $\sqrt{\frac{a}{b}} = \frac{\sqrt{ab}}{b} = \frac{a}{\sqrt{ab}}$; or numerically thus, $\sqrt{\frac{7}{2}} = \frac{\sqrt{7 \times 2}}{2} = \frac{7}{\sqrt{7 \times 2}} = 1,87+$, and this rule will serve whether the root be *finite* or *infinite*.

Now, all numbers whatever, which are to be extracted by the square or cube, contain in themselves their own fractions, and their own denominators; but neither of them can be known 'till an operation has taken place, when they are separated, and the fractions plainly appear: for as in division the divisor is always the denominator to its own fraction, so likewise it is in the square and cube, each of their divisors being the denominators to their own particular fractions or numerators.

In the square, the quotient is always doubled for a new divisor; therefore when the work is completed, the root doubled is the true divisor, or denominator to its own fraction; as, if the root be 12, the denominator will be 24; to be placed under the remainder, which vulgar fraction, or its equivalent decimal must be annexed to the quotient, or root, to complete it.‡

If to the remainder either of the square or cube, cyphers be annexed, and divided by their respective denominators, the quotient will produce the decimals belonging to the root.

Application *and* Use *of the* SQUARE-ROOT.

Problem I. *To find a mean proportional between two numbers.*

Rule.—Multiply the given numbers together, and extract the square root of the product; which root will be the mean proportional sought.

Examples.

1. What is the mean proportional between 24 and 96?

$\sqrt{96 \times 24} = 48$ *Answer.*

Problem II. *To find the side of a* Square *equal in* Area *to any given* Superficies *whatever.*

Rule.—Find the *Area*, and the square root is the side of the square sought.

Examples.

1. If the Area of a circle be 184,125, what is the side of a square equal in area thereto? $\sqrt{184{,}125} = 13{,}569+$ *Answ.*

2. If the area of a triangle be 160; what is the side of a square equal in area thereto? $\sqrt{160} = 12{,}649+$ *Answer.*

Problem III. A certain General has an army of 5625 men; pray how many must he place in rank and file, to form them into a square? $\sqrt{5625} = 75$ *Answer.* †

Prob. IV.

‡ Although these denominators give a small matter too much in the square root, and too little in the cube, yet they will be sufficient in common use, and are much more expeditious than the operation with cyphers.

† If you would have the number of men be double, triple, or quadruple, &c. as many in rank as in file; extract the square root of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ &c. of the given number of men, and that will be the number of men in file, which double, triple, quadruple, &c. and the product will be the number in rank.

PROB. IV. Let 10952 men be so formed, as that the number in rank may be double the file.

$$\sqrt{\frac{10952}{2}} = 74 \text{ in file, and } 74 \times 2 = 148 \text{ in rank.}$$

PROB. V. If it be required to place 2016 men so as that there may be 56 in rank, and 36 in file, and to stand 4 feet distance in rank, and as much in file; How much ground do they stand on?

To answer this, or any of the kind, use the following proportion: —As unity : to the distance :: so is the number in rank less by one : to a fourth number;—next, do the same by the file, and multiply the two numbers together, found by the above proportion, and the product will be the answer. †

As 1 : 4 :: 56—1 : 220. And as 1 : 4 :: 36—1 : 140. Then, 220×140=30800 *square feet, the answer.*

PROB. VI. Suppose I would set out an orchard of 600 trees, so that the length shall be to the breadth as 3 to 2, and the distance of each tree, one from the other, 7 yards; How many trees must it be in length, and how many in breadth; and how many square yards of ground do they stand on?

To resolve any question of this nature; say, as the ratio in length : is to the ratio in breadth :: so is the number of trees : to a fourth number; whose square root is the number in breadth;——And as the ratio in breadth : is to the ratio in length :: so is the number of trees : to a fourth, whose root is the number in length.

As 3 : 2 :: 600 : 400. *And* $\sqrt{400} = 20 =$ *number in breadth.*

As 2 : 3 :: 600 : 900. *And* $\sqrt{900} = 30 =$ *number in length.*

As 1 : 7 :: 30—1 : 203. *And*, as 1 : 7 :: 20—1 : to 133. *And* 203×133=26999 *square yards, the answer.*

PROB. VII. Admit a leaden pipe $\frac{3}{4}$ inch diameter will fill a cistern in 3 hours; I demand the diameter of another pipe which will fill the same cistern in 1 hour?

RULE.‡ —As the given time is to the square of the given diameter, so is the required time : to the square of the required diameter.

$\frac{3}{4}$=,75; *and* ,75×,75=,5625: *then, As* 3*h.* : ,5625 :: 1*h.* : 1,6875 *inversely, and* $\sqrt{1,6875}$=1,3 *inch nearly, answer.*

PROB. VIII. If a Pipe, whose diameter is 1,5 inch, fill a cistern in 5 hours; in what time will a pipe, whose diameter is 3,5 inches, fill the same?

† THE above rule will be found useful in planting trees, having the distance of ground between each given.

‡ THE Areas of circles are to one another as the Squares of their Diameters.

THE area of a circle is the same as the superficial content.

THE Circumference and Periphery are the same thing.

THE Diameter of a circle is a line drawn through the centre from one side of the circumference to the other, and is the longest line that can be drawn across a circle.

THE centre of a circle is the point round which the circle is described.

Note. For the measuring of a circle and its parts, See Mensuration of Superficies, from Art. 12th to Art. 26th and the Note.

$1,5 \times 1,5 = 2,25$; *and* $3,5 \times 3,5 = 12,25$: *then, As* $2,25 : 5 :: 12,25 : ,91$ *hour, inversely,* $=54$ *min.* 36 *sec. answer.*

Prob. IX. If a pipe 6 inches bore, will be 4 hours in running off a certain quantity of water; in what time will 3 pipes, each 4 inches bore, be in discharging double the quantity?

$6 \times 6 = 36$. $4 \times 4 = 16$, *and* $16 \times 3 = 48$. $48 \times 2 = 96$; *then, As* $36 : 4h. :: 96 : 10\frac{2}{3}$ *hours, answer.*

Prob. X. Given the diameter of a circle to make another circle, which shall be 2, 3, 4, &c. times greater or less than the given circle.

Rule.— Square the given diameter, and if the required circle be greater, multiply the square of the diameter by the given proportion, and the root of the product will be the required diameter; —But if the required circle be less; divide the square of the diameter by the given proportion, and the root of the quotient will be the diameter required.

There is a circle whose diameter is 4 inches; I demand the diameter of a circle 3 times as large?

$4 \times 4 = 16$; *and* $16 \times 3 = 48$; *and* $\sqrt{48} = 6,928+$ *inches, answer.*

Prob. XI. To find the diameter of a circle, equal in area, to an ellipsis, (or oval) whose transverse and conjugate diameters are given. †

Rule.—Multiply the two diameters of the ellipsis together; and the square Root of that product will be the diameter of a circle, equal to the ellipsis.

Let the transverse diameter of an ellipsis be 48, and the Conjugate, 36; what is the diameter of an equal circle?

$48 \times 36 = 1728$, *and* $\sqrt{1728} = 41,569+$ *the answer.*

Note. The square of the hypothenuse, or the longest side of a right angled triangle, (by 47th. B. 1. *Euc.*) is equal to the sum of the squares of the other two sides; and consequently the difference of the squares of the hypothenuse and either of the other sides is the square of the remaining side.

Prob. XII. A line 36 yards long will exactly reach from the top of a fort to the opposite bank of a river, known to be 24 yards broad; the heighth of the wall is required?

$36 \times 36 = 1296$; *and* $24 \times 24 = 576$: *then* $1296 - 576 = 720$, *and* $\sqrt{720} = 26,83+$ *yards, the answer.*

Prob XIII. The heighth of a tree, growing in the centre of a circular island 44 feet in diameter, is 75 feet, and a line stretched from the top of it over to the hither edge of the water, is 256 feet; what is the breadth of the stream, provided the land on each side of the water be level?

256

† The transverse and conjugate are the longest and shortest diameters of an Ellipsis; they pass through the centre, and cross each other at right angles.

$256 \times 256 = 65536$; *and* $75 \times 75 = 5675$: *then* $65536 - 5675 = 59861$ *and* $\sqrt{59861} = 244,66+$ *and* $244,66 - \frac{44}{2} = 222,66$ *feet, the answer.*

PROB. XIV. Suppose a ladder 60 feet long be so planted as to reach a window 37 feet from the ground, on one side of the street, and without moving it at the foot, will reach a window 23 feet high on the other side; I demand the breadth of the street?

$60 \times 60 = 3600$. $37 \times 37 = 1369$. $23 \times 23 = 529$: *then*, $3600 - 1369 = 2231$, *and* $\sqrt{2231} = 47,23+$, *and* $3600 - 529 = 3071$, *and* $\sqrt{3071} = 55,41+$, *then* $47,23 + 55,41 = 102,64$ *feet, the answer.*

PROB. XV. Two ships sail from the same port; one goes due north 45 leagues, and the other due west 76 leagues: How far are they asunder? ‖

$45 \times 45 = 2025$. $76 \times 76 = 5776$. then, $5776 + 2025 = 7801$ and $\sqrt{7801} = 88,32$ *leagues, the Answer.*

EXTRACTION OF THE CUBE ROOT.

A CUBE is any number multiplied by its *square*. To extract the cube root, is to find a number which, being multiplied into its square, shall produce the given number.

FIRST METHOD.

RULE.

‡ 1. SEPARATE the given number into periods of three figures each, by putting a point over the unit-figure, and every third figure beyond the place of units.

2. FIND the greatest cube in the left hand period, and put its root in the quotient.

3. SUBTRACT the cube, thus found, from the said period, and to the remainder bring down the next period, and call this the *dividend.*

4. MULTIPLY the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the *divisor.*

5. SEEK how often the divisor may be had in the dividend, and place the result in the quotient.

6. MULTIPLY the triple square by the last quotient-figure and write the product under the dividend; multiply the square of the last

‖ THE square root may in the same manner be applied to Navigation: and, when deprived of other means of solving problems of that nature, the following proportion will serve to find the course.

As the sum of the hypothenuse (or distance) and half the greater leg (whether difference of latitude or departure) is to the less leg; so is 86, to the angle opposite the less leg.

‡ THE reason of pointing the given number, as directed in the rule, is obvious from *Caroll.* 2 to the *Lemma* made use of in demonstrating the square root.

laſt quotient-figure by the triple quotient, and place this product under the laſt; under all, ſet the cube of the laſt quotient-figure and call their ſum the *ſubtrahend*.

7. SUBTRACT the ſubtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and ſo on till the whole be finiſhed.

Note. The ſame rule muſt be obſerved for continuing the operation, and pointing for decimals, as in the ſquare root.

EXAMPLES.

1. REQUIRED the cube root of 436036824287?

436036824287(7583 *the Root.*	7x7x300= 14700=*1ſt Triple ſq.*
343	7x30 = 210=*1ſt Trip. quo.*
1ſt. Diviſ.= 14910)93036 = *1ſt. Dividend.*	14910=*1ſt Diviſor.*
73500	14700 x 5= 73500
5250	5x5x210= 5250
125	5x5x5 = 125
78875=*1ſt. Subtrahend.*	78875= *1ſt Subtrab.*
2d. Div.=1689750)14161824=*2d. Divid.*	75x75x300= 1687500=*2d Trip. ſq.*
13500000	75x30 = 2250=*2d Trip. quo.*
144000	1689750=*2d Diviſor.*
512	1687500x8=13500000
13644512 =*2d. Subtrab.*	2250x8x8= 144000
	8x8x8 = 512
3d. Div.=172391940)517312287=*3d. Divid.*	13644512=*2d. Subtrab.*
517107600	758x758x300=172369200=*3d Trip. ſq.*
204660	758x30 22740=*3d Trip. quo.*
27	
517312287=*3d Subtr.*	172391940=*3d Diviſor.*
·········	172369200x3=517107600
	22740x3x3 = 204660
	3x3x3 = 27
	517312287=*3d Subtrab.*

2. WHAT is the cube root of 34965783? *Anſw.* 327.
3. WHAT is the cube root of 84,604519? *Anſw.* 4,39.
4. WHAT is the cube root of ,008649? *Anſw.* ,2052+
5. WHAT is the cube root of $\frac{125}{343}$? *Anſw.* $\frac{5}{7}$.

To find the true denominator, to be placed under the remainder, after the operation is finiſhed.

IN the extraction of the cube root, the quotient is ſaid to be ſquared and tripled for a new diviſor; but is not really ſo, 'till the triple number of the quotient be added to it, therefore when the operation is finiſhed, it is but ſquaring the quotient, or root, then multiplying

multiplying it by 3, and to that number adding the triple number of the root, when it will become the divisor, or true denominator to its own fraction, which fraction must be annexed to the quotient, to complete the root.

SUPPOSE the root to be 12, when squared it will be 144, and multiplied by 3, it makes 432, to which add 36, the triple number of the root, and it produces 468 for a denominator. ‖

SECOND METHOD.

RULE.

1. HAVING pointed the given number into periods of three figures each, find the greatest cube in the left hand period, subtracting it therefrom and placing its root in the quotient; to the remainder bring down the next period and call it the *dividend*.

2. UNDER this dividend write the triple square of the root, so that units in the latter may stand under the place of hundreds in the former; and under the said triple square, write the triple root, removed one place to the right hand, and call the sum of these the *divisor*.

3. SEEK how often the divisor may be had in the dividend, exclusive of the place of units, and write the result in the quotient.

4. UNDER the divisor write the product of the triple square of the root by the last quotient-figure, setting down the unit's place of this line, under the place of tens in the divisor; under this line, write the product of the triple root by the square of the last quotient-figure, so as to be removed one place beyond the right hand figure of the former; and, under this line, removed one place forward to the right hand, write down the cube of the last quotient-figure, and call their sum the *Subtrahend*.

5. SUBTRACT the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on 'till the whole be finished.

EXAMPLE.

‖ IT may not be amiss to remark here, that the denominators, both of the square and cube, shew how many numbers they are denominators to, that is, what numbers are contained between any square or cube number and the next succeeding square or cube number, exclusive of both numbers, for a complete number, of either, leaves no fraction, when the root is extracted, and consequently has no use for a denominator, but all the numbers contained between them have occasion for it:—Suppose the square root to be 12, then its square is 144, and the denominator 24, which will be a denominator to all the succeeding numbers, 'till we come to the next square number, *viz.* 169, whose root is 13, with which it has nothing to do, for between the square numbers 144 and 169 are contained 24 numbers, excluding both the square numbers. It is the same in the cube; for, suppose the root to be 6, the cube number is 216, and its denominator 126 will be a denominator to all the succeeding numbers, 'till we come to the next cube number, *viz.* 343, whose root is 7, with which it has nothing to do, as ceasing then to be a denominator; for between the cube 343 and 216 are 126 numbers, excluding both cubes. And so it is with all other denominators, either in the square or cube.

E X A M P L E S.

REQUIRED the Cube Root of 16194277?

16194277(253 = *Root.*	
8	
8194	= *First dividend.*
12	= *Triple square of* 2.
6	= *Triple of* 2.
126	= *First divisor.*
60	= *Triple square of 2 multiplied by* 5.
150	= *Triple of 2 multiplied by the square of* 5.
125	= *Cube of* 5.
7625	= *First Subtrahend.*
569277	= *Second dividend.*
1875	= *Triple square of* 25.
75	= *Triple of* 25.
18825	= *Second divisor.*
5625	= *Triple square of 25 multiplied by* 3.
675	= *Triple of 25 multiplied by the square of* 3.
27	= *Cube of* 3.
569277	= *Second Subtrahend.*

First Method by APPROXIMATION.

R U L E.

1. FIND, by trial, a cube near to the given number, and call it the *supposed* cube.

2. THEN, as twice the *supposed* cube, added to the given number, is to twice the given number, added to the *supposed* cube, so is the root of the *supposed* cube, to the true root, or an approximation to it.

3. BY taking the cube of the root, thus found, for the *supposed* cube, and repeating the operation, the Root will be had to a greater degree of exactness.

E X A M P L E S.

IT is required to find the cube root of 54854153?

Let

Let 64000000 = *supposed Cube, whose root is* 400;

	Then 64000000	54854153	
	2	2	
	128000000	109708306	
	54854153	64000000	
As	182854153 :	173708306 ::	400
		400	

182854153)69483322400(379 = *Root nearly.*

Again, let 54439939 = *supposed Cube, whose root is* 379.

Then	54439939	54854153	
	2	2	
	108879878	109708306	
	54854153	54439939	
As	163734031 :	164148245 ::	379
		379	
		1477334205	
		1149037715	
		492444735	

163734031)62212184855(379,958793+ = *Root corrected.*

Second Method by APPROXIMATION.

Rule.

1. Divide the Resolvend by three times the assumed Root, and reserve the quotient.

2. Subtract one twelfth part of the square of the assumed root from the quotient.

3. Extract the square root of the remainder.

4. To this root add one half of the assumed root, and the sum will be the true root, or an approximation to it;—Take this approximation as the assumed root, and, by repeating the process, a root further approximated will be found, which operation may be further repeated, as often as necessary, and the root discovered to any assigned exactness.

Note. In order to find the value of the first assumed root, in this or any other power, divide the resolvend into periods by beginning at the place of units, and including in each period, so many figures as there are units in the exponent of the root: viz. 3 figures in the cube root;—4 for the Biquadrate, and so on; Then, by a table of powers, or otherwise, find a figure, which (being involved to the power whose exponent is the same with that of the required root) is

is the nearest to the value of the first period of the resolvend at the left hand, and to that figure annex so many cyphers as there are periods remaining in the integral part of the resolvend; this figure, with the cyphers annexed, will be the assumed root, and equal to r in the Theorem: And it is of no importance whether the figure thus chosen be, when involved, greater or less than the left-hand period, as the Theorem is the same in both cases.

1. WHAT is the cube-root of 436036824287?

7000 = *assumed root.*
3

21,000)436036824287(20763658,2994

Subtract 7000 × 7000 ÷ 12 = 4083333,3333

$\sqrt{16680314,9661} = 4084,15$

Add $\frac{1}{2}$ *the assumed root* = 3500

And it gives the approximated root = 7584,15

FOR the second operation, use the approximated root as the assumed one, and proceed as above.

Third Method by APPROXIMATION.

1. ASSUME the root in the usual way, then multiply the square of the assumed root by 3, and divide the resolvend by this product; to this quotient add $\frac{2}{3}$ of the assumed root, and the sum will be the true root, or an approximation to it.

2. FOR each succeeding operation let the last approximated root be the assumed root, and, proceeding in this manner, the root may be extracted to any assigned exactness.

1. WHAT is the cube root of 7?

LET the assumed root be 2. Then $2 \times 2 \times 3 = 12$ the divisor.

12)7,0(,583 to this add $\frac{2}{3}$ of 2 = 1,333 &c. that is ,583 + 1,333 = 1,916 approximated root.

Now assume 1,916 for the root, then, by the second process, the root is $\frac{7}{3 \times 1,916^2} + \frac{2}{3} \times 1,916 = 1,9126$ &c.

2. WHAT is the cube root of 9?—Let 2 be the assumed root, as before——Then $\frac{9}{12} + \frac{2}{3} \times 2 = 2,08$ the approximated root. Now assume 2,08——Then $\frac{9}{3 \times 2,08^2} + \frac{2}{3} \times 2,08 = 2,08008$ &c.

3. WHAT is the cube root of 282?——Let 6 be the assumed root, then $6 \times 6 \times 3 = 108$)282(2,611 &c. and $2,611 + \frac{2}{3}$ of $6 = 6,611$ approximated root. Now assume 6,611, and it will be $6,611 \times 6,611 \times 3 = 131,116$)282(2,1507 &c. and $2,1507 + \frac{2}{3}$ of $6,611 = 6,558$ a further approximated root.

4. WHAT

4. WHAT is the cube root of 1728 ?——Here the assumed root is 10. Then $10\times10\times3=300)1728(5,76$—And $5,76+\frac{2}{3}$ of $10=12,426$.———Now assume 12,426, then $12,426\times12,426\times3=463,06)1728(3,732$, and $3,732+\frac{2}{3}$ of $12,426=12,016$ a further approximated root, and so on.

APPLICATION *and* USE *of the* CUBE ROOT.

1. To find *two* mean proportionals between any two given numbers.

RULE.—1. Divide the *greater* by the *less*, and extract the Cube root of the quotient.—

2. MULTIPLY the root, so found, by the least of the given numbers, and the product will be the *least*.

3. MULTIPLY this product by the same root, and it will give the greatest.

EXAMPLES.

1. WHAT are the two mean Proportionals between 6 and 750 ?

$750\div6=125$, and $\sqrt[3]{125}=5$. then $5\times6=30$=least, and $30\times5=150$=greatest.——*Ans.* 30 and 150.

Proof. As 6 : 30 :: 150 : 750.

2. WHAT are the two mean proportionals between 56 and 21096 ?

Ans. 336 *and* 2016.

Note. THE solid contents of similar figures are in proportion to each other, as the cubes of their similar sides or diameters.

3. IF a bullet 6 Inches diameter weigh 32 ℔. what will a bullet of the same metal weigh, whose diameter is 3 Inches ?

$6\times6\times6=216$. $3\times3\times3=27$. As 216 : 32 ℔ :: 27 : 4 ℔. *The Ans.*

4. IF a Globe of Silver of 3 Inches diameter, be worth £45 what is the value of another Globe, of a foot diameter ?

$3\times3\times3=27$. $12\times12\times12=1728$. As 27 : 45 :: 1728 : £2880 *Ans.*

THE side of a Cube being given, to find the side of that Cube which shall be double, triple, &c. in quantity, to the given Cube.

RULE. Cube your given side, and multiply it by the given proportion between the given and required Cube, and the Cube Root of the product will be the side sought.

5. IF a Cube of Silver, whose side is 4 Inches, be worth £50 ; I demand the side of a Cube of the like Silver, whose value shall be 4 times as much ?

$4\times4\times4\times64$ and $64\times4=256$. $\sqrt[3]{256}=6,349+$ *Inches, Answer.*

6. THERE is a cubical vessel, whose side is 2 feet, I demand the side of a vessel, which shall contain three times as much ?

$2\times2\times2=8$. and $8\times3=24$. $\sqrt[3]{24}\times2,884=2$ feet, $10\frac{3}{5}$ *Inches, Ans.*

7. THE diameter of a bushel-measure being $18\frac{1}{2}$ Inches, and the heighth 8 inches ; I demand the side of a cubic box, which shall contain that quantity ? *Ans.* 12,908 *Inches.*

 8. SUPPOSE

8. SUPPOSE a ship of 500 tons has 89 feet keel, 36 feet beam, and is 16 feet deep in the hold; what are the dimensions of a ship of 200 tons, of the same mould and shape?——

$89 \times 89 \times 89 = 704969 =$ cubed keel.

As 500 : 200 :: 704969 : 281987,6 *Cube of the required keel.*

$\sqrt[3]{281987{,}6} = 65{,}57$ *feet the required keel.*

As 89 : 65,57 :: 36 : 26.522 = $26\frac{1}{2}$ *feet, beam, nearly.*

As 89 : 65,57 :: 16 : 11,7 *feet, depth of the hold.*

9. FROM the proof of any cable to find the strength of any other.

RULE.—The strength of Cables, and consequently the weights of their Anchors, are as the cubes of their peripheries.

IF a cable, 12 inches about, require an anchor of 18 *Cwt.* of what weight must an anchor be, for a 15 inch Cable?

As $12 \times 12 \times 12$: 18 *Cwt.* :: $15 \times 15 \times 15$: 35,15625 *Cwt.* *Answer.*

10. IF a 15 Inch Cable require an Anchor 35,15625 *Cwt.* :: what must the circumference of a cable be, for an anchor of 18 *Cwt*?

As 35,15625 : $15 \times 15 \times 15$ :: 18 : 1728. and $\sqrt[3]{1728} = 12$ *Ans.*

EXTRACTION OF THE BIQUADRATE ROOT.

RULE.

EXTRACT the Square Root of the Resolvend, and then, the Square Root of that Root, and you will have the Biquadrate Root.

WHAT is the Biquadrate root of 20736?

20736(144	144(12 *Root required.*
1	1
24)107	22)44
96	44
284)1136	
1136	

Two methods of extracting the Biquadrate Root by approximation, according to the two general Theorems *for extracting the Roots of all Powers, in Pages* 213 *and* 215.

RULE I.

1. DIVIDE the resolvend by *six* times the square of the assumed root, and from the quotient subtract $\frac{1}{18}$ part of the square of the assumed root.

2. EXTRACT the Square Root of the remainder.

3. ADD $\frac{2}{3}$ of the assumed root to the Square Root, and the sum will be the true root, or an approximation to it.

4. FOR

4. For every succeeding operation (either in this or the following method) proceed in the same manner, as in the first, each time using the last approximated root for the assumed root.

The Biquadrate Root of 20736 is required.

Here 10 *is the assumed Root.*

10×10×6=600)20736(34,56

Subtract. 10×10÷18= 5,5555

√29,0044=5,38

Add ⅔ *of* 10 =6,66

Approximated Root 12,04, *To be made the assumed root for the next operation.*

RULE 2.

Divide the Resolvend by *four* times the Cube of the assumed Root:- To the quotient add *three fourths* of the assumed root, and the sum will be the true root, or an approximation to it.

Let the Biquadrate of 20736 be required, as before?

The assumed Root is 10.

10×10×10×4=4000)20736(5,184

Add ¾ *of* 10 =7,5

Approximated Root 12,684, *To be made the assumed root for the next operation.*

EXTRACTION OF THE SURSOLID ROOT, BY APPROXIMATION.

A particular RULE.*

1. Divide the Resolvend by *five* times the assumed Root, and to the quotient add *one twentieth* part of the *fourth* power of the same root.

2. From the Square Root of this sum subtract *one fourth* part of the square of the assumed root.

3. To the Square Root of the remainder add *one half* of the assumed Root, and the sum is the root required, or an approximation to it.

Note. This Rule will give the Root true to *five* places, at the least, (and generally to eight or nine places) at the first process.

Required

* $r \pm e = \sqrt{\sqrt{\frac{G}{5r} + \frac{r^4}{20}} : - \frac{rr}{4} :} + \frac{r}{2}$

REQUIRED the Surſolid Root of 281950621875?

200 = *Aſſumed Root.*

5

1000) 281950621,875 *Quotient.*

Add 200×200×200×200÷20) =80000000

√361950621,875=19025 *nearly.*

Subtract 200×200÷4=10000

√ 9025=95

Add half the Aſſumed Root =100

∴ *Root* 195 *required.*

A GENERAL RULE FOR EXTRACTING THE ROOTS OF ALL POWERS.

RULE.

‡ 1. PREPARE the given number, for extraction, by pointing off from the unit's place, as the required root directs.

2. FIND the firſt figure of the root by trial, or by inſpection into the Table of Powers, and ſubtract it from the left hand period.

3. To the remainder bring down the firſt figure in the next period, and call it the *dividend.*

4. INVOLVE the root to the next inferior power to that which is given, and multiply it by the number, denoting the given power, for a *diviſor.*

5. FIND how many times the diviſor may be had in the dividend, and the quotient will be another figure of the root.

6. INVOLVE the whole Root to the given Power, and ſubtract it from the *given number*, as before.

7. BRING down the firſt figure of the next period to the remainder for a new dividend, to which find a new diviſor, as before, and, in like manner, proceed till the whole be finiſhed.

EXAMPLES.

‡ THE extracting of roots of very high powers will, by this rule, be a tedious operation:—The following method, when practicable, will be much more convenient.

WHEN the index of the power, whoſe root is to be extracted, is a compoſite number, take any two or more *indices*, whoſe product is equal to the given *index*, and extract out of the given number a root anſwering to another of the indices, and ſo on, to the laſt.

THUS, the fourth root = ſquare root of the ſquare root;—the ſixth root = ſquare root of the cube root:—the eighth root = ſquare root of the fourth root:—the ninth root = the cube root of the cube root:—the tenth root = ſquare root of the fifth root;—the twelfth root = cube root of the fourth, &c.

EXAMPLES.

1. WHAT is the Cube root of 20346417 ?

	20346417(273	$2\times2\times2=8$ *Root of the 1st. period, or 1st. Subtr.*
$2^3 =$	8 = *1st. Subtrah.*	$2\times2=4$ (*=next inferior Power,*) *and,*
		4×3 (*the index of the given pow.*)$=12$ *1st divis.*
$2^2\times3 =$	12)123 = *Dividend.*	$27\times27\times27=19683=$ *2d. Subtrabend.*
		$27\times27=729$ (*next inferior power,*) *and,*
$27^3 =$	19683 = *2d. Subtr.*	729×3 (*=index of the given power.*)$=2187=$ *2d. divisor.*
$27^2\times3=2187)$	6634 = *2d. Divid.*	$273\times273\times273=20346417=$ *3d. Subtrabend.*
$273^3 =$	20346417 = *3d. Subt.*	

2. WHAT is the biquadrate root of 34827998976 ? *Anʃ.* 432.

3. EXTRACT the Surʃolid, or fifth Root of 281950621875 ?
Anʃwer 195.

4. EXTRACT the ʃquare cubed, or ʃixth root of 1178420166015625 ?
Anʃwer 325.

A GENERAL RULE FOR EXTRACTING ROOTS, BY APPROXIMATION.

RULE.†

1. SUBTRACT *one* from the exponent of the root required, and multiply half of the remainder by the ʃame exponent, and further multiply

† THE general Theorem for the extraction of all Roots, by Approximation, from whence the rule was taken, and the Theorems deducible from it, as high as the twelfth power. Let G = Reʃolvend whoʃe root is to be extracted. $r \pm e$ = Root required; r being aʃʃumed, as near the true root, and m = exponent of the power—then the equation will ʃtand thus.

$$r \pm e = \sqrt{\frac{G}{m\times\frac{\overline{m-1}}{2}r^{m-2}} - \frac{m\times\overline{m-3}+2}{m\times\overline{m-1}\;3}rr} : + \frac{m-2}{m-1}r. \text{ Hence,}$$

Theorem for the Cube Root $\quad r \pm e = \sqrt{\frac{G}{3r} - \frac{rr}{12}} + \frac{r}{2}$

For the Biquadrate - - - $\sqrt{\frac{G}{6rr} - \frac{rr}{18}} + \frac{2r}{3}$

For the Surʃolid - - - $\sqrt{\frac{G}{10r^3} - \frac{3rr}{80}} + \frac{3r}{4}$

For the ʃquared cube root $\sqrt{\frac{G}{15r^4} - \frac{2rr}{75}} + \frac{4r}{5}$

For the ʃecond ʃurʃolid $\sqrt{\frac{G}{21r^5} - \frac{5rr}{252}} + \frac{5r}{6}$

For

multiply this product by that power of the assumed root, whose exponent is *two* less than the exponent of the root required, and with this last product divide the resolvend and reserve the quotient.

2. FROM this quotient is to be subtracted the following number, viz. the square of the assumed root multiplied by a fraction whose numerator is found by subtracting *three* from the exponent of the root, multiplying the remainder by that exponent, and adding *two* to the product; and the denominator is found by subtracting *one* from that exponent and multiplying the cube of the remainder by the same exponent.

3. AFTER this subtraction is made, extract the square root of the remainder.

4. MULTIPLY the assumed root by a fraction whose numerator is the remainder after *two* is subtracted from the exponent, and the denominator is the remainder after *one* is subtracted from the exponent: Add this product to that square root and the sum is the root required, or an approximation to it.

EXAMPLE.

WHAT is the square cubed root of 117842016601$\dot{5}$6$\dot{2}$$\dot{5}$?

Let the assumed root $= 300$.

Exponent of the required root is 6; *then* $\frac{\overline{6-1}}{2} \times 6 = 15$.

$300^4 = 8100000000$ *and this, multiplied by* $15 = 121500000000$.

$391018798828125 \div 121500000000 = 9698{,}9314$, *from this*

Subtract $\frac{\overline{6-3} \times 6 + 2}{\overline{6-1} \times \overline{6-1} \times \overline{6-1} \times 6} \times 90000 = \underline{2400}$

And $\sqrt{7298{,}9314} = 85{,}43$

Add $\frac{\overline{6-2}}{\overline{6-1}} \times 300 = \underline{240}$

Approximated root $= 325{,}43$

For the 2d. operation, let $325{,}43 =$ *assumed root.*

Another

For the squared Biquadrate $\sqrt{\frac{G}{28r^6} - \frac{21rr}{1372}} + \frac{6r}{7}$

For the cubed Cube - - - $\sqrt{\frac{G}{36r^7} - \frac{7rr}{578}} + \frac{7r}{8}$

For the squared Sursolid - - $\sqrt{\frac{G}{45r^8} - \frac{4rr}{405}} + \frac{8r}{9}$

For the third Sursolid - - - $\sqrt{\frac{G}{55r^9} - \frac{9rr}{1100}} + \frac{9r}{10}$

For the squared square-Cube - - $\sqrt{\frac{G}{66r^{10}} - \frac{5rr}{726}} + \frac{10r}{11}$ &c.

Another METHOD *by* APPROXIMATION. *

RULE.

1. HAVING assumed the root in the usual way, involve it to that power denoted by the exponent less 1.
2. MULTIPLY this power by the exponent.
3. DIVIDE the resolvend by this product, and reserve the quotient.
4. DIVIDE the exponent of the given power, less 1, by the exponent, and multiply the assumed root by the quotient.
5. ADD this product to the reserved quotient, and the sum will be the true root, or an approximation.
6. FOR every succeeding operation, let the root, last found, be the assumed root.

EXAMPLE.

WHAT is the square cubed root of 1178420166015625?

The Exponent is 6. *Let the assumed Root be* 300.

Then $300^5 \times 6 = 1458000000000$.

1458000000000)1178420166015625(80,824

$$\textit{Add}\ \frac{5}{6}\times 300 = \underline{\ \ 250\ \ }$$

330,824 = *Approximated Root.*

For the next operation, let 330,824 *be the assumed root.*

OF

* A rational formula for extracting the root of any pure power by approximation.

LET the Resolvend be called G, and let $r \pm e$ be the required root, r being assumed in the usual way.

LET $G^{\frac{1}{m}}$ be required: then $r \pm e = \dfrac{G}{mr^{m-1}} + \dfrac{m-1}{m}r$ the general Theorem.

Hence, For the Cube Root $r \pm e = \dfrac{G}{3r^2} + \dfrac{2}{3}r.$

For the Biquadrate $\dfrac{G}{4r^3} + \dfrac{3}{4}r.$

For the Surfolid $\dfrac{G}{5r^4} + \dfrac{4}{5}r.$

For Cube cubed $\dfrac{G}{6r^5} + \dfrac{5}{6}r.$

For the seventh Root $\dfrac{G}{7r^6} + \dfrac{6}{7}r.$

For the eighth $\dfrac{G}{8r^7} + \dfrac{7}{8}r.$

For the ninth $\dfrac{G}{9r^8} + \dfrac{8}{9}r.$

For the tenth $\dfrac{G}{10r^9} + \dfrac{9}{10}r.$

For the eleventh $\dfrac{G}{11r^{10}} + \dfrac{10}{11}r.$

For the twelfth $\dfrac{G}{12r^{11}} + \dfrac{11}{12}r.$ &c.

OF PROPORTION IN GENERAL.

NUMBERS are compared together to difcover the relations they have to each other.

THERE muft be two numbers to form a comparifon: the number, which is compared, being written firft, is called the *antecedent*; and that, to which it is compared, the *confequent*.

NUMBERS are compared with each other two different ways: the one comparifon confiders the *difference* of the two numbers, and is called Arithmetical Relation, the difference being fometimes named the Arithmetical Ratio; and the other confiders their *quotient* which is termed Geometrical Relation, and the quotient, the Geometrical Ratio.—Thus, of the numbers 12 and 4; the difference, or Arithmetical Ratio, is $12-4=8$; and the Geometrical Ratio is $\frac{12}{4}=3$. *

IF two, or more, couplets of numbers have equal ratios, or differences, the equality is termed proportion; and their terms fimilarly pofited, that is, either all the greater, or all the lefs taken as antecedents, and the reft as confequents, are called proportionals. So the two couplets 2, 4, and 6, 8, taken thus, 2, 4, 6, 8, or thus, 4, 2, 8 6, are arithmetical proportionals; and the two couplets, 2, 4, and 8, 16, taken thus, 2, 4, 8, 16, or thus, 4, 2, 16, 8, are geometrical proportionals. ‡

PROPORTION is diftinguifhed into continual and difcontinual. If, of feveral couplets of proportionals, written down in a feries, the difference or ratio of each confequent, and the antecedent of the next following couplet, be the fame as the common difference or ratio of the couplets, the proportion is faid to be continual, and the numbers themfelves, a feries of continual proportionals, or an arithmetical or geometrical proportion. So 2, 4, 6, 8 form an arithmetical proportion; for $4-2=6-4=8-6=2$; and 2, 4, 8, 16, a geometrical proportion; for $\frac{4}{2}=\frac{8}{4}=\frac{16}{8}=2$.

BUT

* RATIOS are, here, always confidered as the refult of the greater term of comparifon diminifhed, or divided, by the lefs; not regarding which of them be the antecedent.

‡ To denote numbers as being geometrically proportional, the couplets are feparated by a double colon, and a colon is written between the terms of each couplet; we may, alfo, denote arithmetical proportionals by feparating the couplets by a double colon, and writing a colon turned horizontally between the terms of each couplet. So the above arithmeticals may be written thus, 2 ·· 4 : : 6 ·· 8, and 4 ·· 2 : : 8 ·· 6; where the firft antecedent is lefs or greater than its confequent by juft fo much as the fecond antecedent is lefs or greater than its confequent: And the geometricals thus, 2 : 4 : : 8 : 16 : 8, and 4 : 2 : : 16 : 8; where the firft antecedent is contained in, or contains its confequent, juft fo often, as the fecond is contained in, or contains its confequent.

FOUR numbers are faid to be *reciprocally* or *inverfely* proportional, when the fourth is lefs than the fecond, by as many times, as the third is greater than the firft, or when the firft is to the third, as the fourth to the fecond, and *vice verfa*. Thus 2, 9, 6 and three are reciprocal proportionals.

Note. It is common to read the geometricals 2 : 4 : : 8 : 16; thus, 2 is to 4 as 8 to 16, or, As 2 to 4 fo is 8 to 16.

But if the difference or ratio of the consequent of one couplet, and the antecedent of the next couplet be not the same as the common difference or ratio of the couplets, the proportion is said to be discontinual. So 4, 2, 8, 6 are in discontinued arithmetical proportion; for $4-2=8-6=2=$common difference of the couplets, $8-2=6=$difference of the consequent of one couplet and the antecedent of the next; also, 4, 2, 16, 8 are in discontinued geometrical proportion: for $\frac{4}{2}=\frac{16}{8}=2=$ common ratio of the couplets, and $\frac{16}{2}=8=$ ratio of the consequent of one couplet and the antecedent of the next.

ARITHMETICAL PROPORTION.

Theorem I.

If any four quantities $a, b, c, d,$ (2. 4. 6. 8.) be in Arithmetical

Proportion,

Harmonical Proportion is that, which is between those numbers which assign the lengths of musical intervals, or the lengths of strings sounding musical notes; and of *three numbers* it is, when *the first is to the third, as the difference between the first and second is to the difference between the second and third*, as the numbers 3, 4, 6.—Thus if the lengths of strings be as these numbers, they will sound an octave 3 to 6, a fifth 2 to 3, and a fourth 3 to 4.

Again, between four numbers, when *the first is to the fourth, as the difference between the first and second is to the difference between the third and fourth*, as in the numbers 5, 6, 8, 10: For strings of such lengths will sound an octave 5 to 10; a sixth greater 6 to 10; a third greater 8 to 10; a third less 5 to 6; a sixth less 5 to 8; and a fourth 6 to 8.

A Series of numbers in harmonical proportion is, reciprocally, as another series in arithmetical proportion.

As $\left\{\begin{array}{l}\text{Harmonical} \quad 10 \cdot\cdot 12 \cdot\cdot 15 \cdot\cdot 20 \cdot\cdot 30 \cdot\cdot 60 \\ \text{Arithmetical} \quad 6 \cdot\cdot 5 \cdot\cdot 4 \cdot\cdot 3 \cdot\cdot 2 \cdot\cdot 1\end{array}\right\}$ for here $10:12::5:6$; and $12:15::4:5$, and so of all the rest. Whence those series have an obvious relation to, and dependance on, each other.

1. Let a, b, c be the three numbers in musical proportion: then, because we have $a:c::a-b:b-c$; therefore, $ab-ac=ac-bc$; whence, if any two of the three be given, the other may be found by the following Theorems.

I. $\frac{ab}{2a-b}=c$. II. $\frac{2ac}{2a-b}=b$. III. $\frac{cb}{2c-b}=a$.

For Example. Suppose you would find a musical mean proportional between the *monochord* $50=a$, and the *octave* $25=c$; then, by Theor. II. $\frac{2ac}{a+c}=b=\frac{2500}{75}=33,3$, which is the length of that chord, called a *fifth*.

2. If there be four numbers in Musical Proportion, as a, b, c, d; then, since it is $a:d::a-b:c-d$, we have $ac-ad=ad-db$. From which equation we have the following Theorems.

I. $\frac{db}{2d-c}=a$. II. $\frac{a}{d}\times\overline{2d-c}=b$. III. $\frac{2ad-db}{a}=c$. IV. $\frac{ac}{2a-b}=d$.

Hence, when any three of those numbers are given, the fourth may be found.

Thus, let 10, 8, 6 be given to find a fourth harmonical proportion.

$\frac{a\times c}{2a-b}=\frac{10\times 6}{20-8}=\frac{60}{12}=5$, *the octave*.

This Harmonical Theory may be carried much farther——See *Martin's Newtonian Philosophy*, Vol. II. Page 123.

Proportion, † the sum of the two means is equal to the sum of the two extremes. ||

And if any three quantities, *a*, *b*, *c*, (2, 4, 6) be in Arithmetical Proportion, the double of the mean is equal to the sum of the extremes.

THEOREM 2.

In any continued arithmetical proportion (1, 3, 5, 7, 9, 11) the sum of the two extremes, and that of every other two terms, equally distant from them, are equal: thus $1+11=3+9=5+7$.¶

When the number of terms is odd, as in the proportion 3. 8. 13. 18. 23, then, the sum of the two extremes being double to the mean, or middle term, the sum of any other two terms, equally remote from the extremes, must likewise be double to the mean.

THEOREM 3.

In any continued arithmetical proportion, a, $a+b$, $a+2b$, $a+3b$, $a+4b$, &c. (4, 4+2, 4+4, 4+6, 4+8, &c.) the last or greatest term is equal to the first or least more the common difference of the terms drawn into the number of all the terms after the first, or into the whole number of the terms, less one. §

THEOREM 4.

The sum of any rank, or series, of quantities in continued arithmetical proportion (1. 3. 5. 7. 9. 11) is equal to the sum of the two extremes multiplied into half the number of terms.*

ARITH-

† Although, in the comparison of quantities according to their differences, the term *proportion* is used; yet the word, *progression*, is frequently substituted in its room, and is indeed more proper; the former from being, in the common acceptation of it, synonimous with ratio, which is only used in the other kind of comparison.

|| For since $b-a$ $(4-2)=d-c$ $(8-6)$, therefore, $b+c$ $(4+6)=a+d$ $(2+8.)$

¶ Since, by the nature of progressionals, the second term exceeds the first by just so much as its corresponding term, the last but one, wants of the last, it is evident that when these corresponding terms are added, the excess of the one will make good the defect of the other, and so their sum be exactly the same with that of the two extremes, and in the same manner it will appear that the sum of any two other corresponding terms must be equal to that of the two extremes.

§ For since each term, after the first, exceeds that preceding it by the common difference, it is plain that the last must exceed the first by so many times the common difference as there are terms after the first; and therefore must be equal to the first, and the common difference repeated that number of times.

* For, because (by the second Theorem) the sum of the two extremes, and that of every other two terms, equally remote from them, are equal, the whole series, consisting of half so many such equal sums as there are terms, will therefore be equal to the sum of the two extremes, repeated half as many times as there are terms.

The same thing also holds, when the number of terms is odd; as in the series 4, 8, 12, 16, 20; for then, the mean, or middle term, being equal to half the sum of any two terms, equally distant from it on contrary sides, it is obvious that the value of the whole series is the same as if every term thereof were equal to the mean, and therefore is equal to the mean (or half the sum of the two extremes) multiplied by the whole number of terms; or to the sum of the extremes multiplied by half the number of terms.

The

ARITHMETICAL PROGRESSION.

Any rank of numbers, more than two, increasing by a common excess, or decreasing by a common difference, is said to be in Arithmetical Progression.

If the succeeding terms of a progression exceed each other, it is called an ascending series or progression; if the contrary, a descending series.

So { 0. 1. 2. 4. 6. 8, &c. *is an ascending arithmetical series.*
   { 1. 2. 4. 8. 16. 32, &c. *is an ascending geometrical series.*

And { 8. 6. 4. 2. 1. 0, &c. *is a descending arithmetical series.*
    { 32. 16. 8. 4. 2. 1, &c. *is a descending geometrical series.*

The numbers which form the series, are called the *terms* of the Progression.

Note.—The first and last terms of a Progression are called the extremes, and the other terms the means.

Any three of the five following things being given, the other two may be easily found.

1. The *first* Term.
2. The *last* Term.
3. The *number of* Terms.
4. The *common difference.*
5. The *sum* of all the Terms.

Problem I.

The first term, the last term, and the number of terms being given, to find the common difference.

Rule.*

Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference sought.

Example.

The sum of any number of terms (x) of the arithmetical series of odd numbers 1, 3, 5, 7, 9, &c. is equal to the square (x^2) of that number.

For, 0+1 *or the sum of* 1 *term* $= 1^2$ *or* 1
1+3 *or the sum of* 2 *terms* $= 2^2$ *or* 4
4+5 *or the sum of* 3 *terms* $= 3^2$ *or* 9
9+7 *or the sum of* 4 *terms* $= 4^2$ *or* 16
16+9 *or the sum of* 5 *terms* $= 5^2$ *or* 25, *&c.*

Whence, it is plain that, let x be any number whatever, the sum of x terms will be x^2.

Example.

The first term, the ratio, and number of terms given, to find the sum of the series.

A Gentleman travelled 29 days, the first day he went but 1 mile, and increased every day's travel 2 miles; how far did he travel? 29 x 29 = 841 *miles, the answer.*

* The difference of the first and last terms evidently shews the increase of the first term by all the subsequent additions, till it becomes equal to the last; and as the number of those additions was one less than the number of terms, and the increase, by every addition, equal, it is plain that the total increase, divided by the number of additions, must give the difference of every one separately; whence, the rule is manifest.

EXAMPLES.

1. THE Extremes are 3 and 39, and the number of terms is 19; what is the common difference?

$$\left.\begin{array}{r} 39 \\ +\ 3 \end{array}\right\} \textit{Extremes.}$$

Divide by the number of terms less $1 = 19 - 1 = 18)36(2$ *Answer.*

$$36$$

Or, $\dfrac{39-3}{19-1} = 2.$

2. A Man had 10 Sons, whose several ages differed alike; the youngest was 3 years old, and the eldest 48; what was the common difference of their ages?

Answer $\dfrac{48-3}{10-1} = 5.$

3. A Man is to travel from Boston to a certain place in 9 days, and to go but 5 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 37 miles; required the daily increase?

$\dfrac{37-5}{9-1} = 4$ *The answer.*

PROBLEM 2.

The first term, the last term, and the number of terms being given, to find the sum of all the terms.

RULE.†—Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

EXAMPLES.

1. THE extremes of an arithmetical series are 3 and 39, and the number of terms 19; required the sum of the series?

39

† SUPPOSE another series of the same kind with the given one be placed under it in an inverse order; then will the sum of every two corresponding terms be the same as that of the first and last; consequently, any one of those sums, multiplied by the number of terms, must give the whole sum of the two series.

Let 1. 2. 3. 4. 5. 6. 7. 8 *be the given series.*
And 8. 7. 6. 5. 4. 3. 2. 1 *the same inverted.*
Then, $9+9+9+9+9+9+9+9=9\times8=72$, *and*

$$1+2+3+4+5+6+7+8=\frac{72}{2}=36.$$

$$\begin{array}{r} 39 \\ +\ 3 \end{array}\Big\}\ \textit{Extremes,}$$

$$\begin{array}{rr} \textit{Sum} = & 42 \\ \textit{Number of terms} = & \times\ 19 \\ \hline & 378 \\ & 42 \\ \hline & 2)798 \\ \hline & 399 \end{array}$$ *the Answer.*

Or, $\frac{\overline{39+3}\times 19}{2} = 399.$

2. It is required to find how many ſtrokes the hammer of a clock would ſtrike in a week, or 168 hours, provided it increaſed 1 at each hour?

$$\frac{\overline{168+1}\times 168}{2} = 14196 \textit{ the Anſwer.}$$

3. Suppose a number of ſtones were laid a yard diſtant from each other for the ſpace of a mile, and the firſt, a yard from a baſket; what length of ground will that man travel over, who gathers them up ſingly, returning with them one by one to the baſket?

$$\frac{\overline{3520+2}\times 1760}{2} = 3099360 \textit{ yards} = 1761 \textit{ miles, the Anſwer.}$$

N. B. In this queſtion, there being 1760 yards in a mile, and the man returning with each ſtone to the baſket, his travel will be doubled; therefore the firſt term will be 2, and the laſt 1760×2, and the number of terms 1760.

4. A Man bought 25 yards of Linen in Arithmetical Progreſſion, for the 4th yard he gave 12 ſhillings, and for the laſt yard £3 15*s.* what did the whole amount to, and what did it average *per* yard?

$\frac{75-12}{22-1} = 3$ *the common difference by which the firſt term is found to be* 3.

Then $\frac{\overline{75+3}\times 25}{2} =$ £48 15*s.* *& the average price is* £1 19*s.* *per yd.*

5. Required the ſum of the firſt 1000 numbers in their natural order?

$$\frac{\overline{1000+1}\times 1000}{2} = 500500 \textit{ Anſw.}$$

Problem 3.

Given the extremes and the common difference, to find the number of terms.

Rule.

RULE.‡—Divide the difference of the extremes by the common difference, and the quotient increased by 1 will be the number of terms required.

EXAMPLES.

1. THE extremes are 3 and 39, and the common difference 2; what is the number of terms?

$$\left.\begin{array}{r} 39 \\ -\ 3 \end{array}\right\} \textit{Extremes.}$$

$$\begin{array}{rr} \textit{Common difference} = & 2)36 \\ \textit{Quotient} = & 18 \\ \textit{Add} & 1 \\ & 19 \ \textit{the Answer.} \end{array}$$

$$Or\ \frac{39-3}{2}+1=19.$$

2. A Man, going a journey, travelled the first day 7 miles, the last day 51 miles, and each day increased his journey by 4 miles; How many days did he travel, and how far?

$$\frac{51-7}{4}+1=12 \textit{ days, and } \frac{51+7\times12}{2}=348 \textit{ miles, Answer.}$$

PROBLEM 4.

The extremes and common difference given, to find the sum of all the Series.

RULE.—Multiply the sum of the extremes by their difference increased by the common difference, and the product divided by twice the common difference, will give the sum.

EXAMPLES.

1. IF the extremes are 3 and 39, and the common difference 2; what is the sum of the series?

$39+3=42$ *Sum of the extremes.*
$39-3=36=$ *Difference of extremes.*
$36+2=38=$ *difference of extremes increased by the common difference,*

42

‡ BY the first Problem, the difference of the extremes, divided by the number of terms less 1, gave the common difference: consequently, the same divided by the common difference, must give the number of terms less 1; hence, this quotient, augmented by 1, must be the answer to the question,

$$\begin{array}{r} 42 \\ \times\ 38 \\ \hline 336 \\ 126\ \ \\ \hline \end{array}$$

Twice the common difference $= 4)1596$

399 *the Answer.*

$$Or\ \frac{\overline{39+3}\times\overline{39-3+2}}{2\times 2} = 399.$$

2. A owes B a certain sum, to be discharged in a year, by paying $6d.$ the first week, $18d.$ the second, and thus to increase every weekly payment by a shilling, 'till the last payment be £2 11*s.* 6*d.*; what is the debt?

$$\frac{\overline{51,5,+5}\times\overline{51,5-,5}+1}{1\times 2} = £67\ 12s.\ \textit{the Answer.}$$

PROBLEM 5.

The Extremes and the sum of the Series given, to find the common difference.

RULE. Divide the product of the sum and difference of the extremes, by the difference of twice the sum of the series, and the sum of the extremes, and the quotient will be the common difference.

EXAMPLES.

1. Let the extremes be 3 and 39, and the sum 399: What is the common difference?

Sum of the extremes $= 39 + 3 = \quad 42$

Diff. of the extremes $= 39 - 3 = \times\ 36$

$$\begin{array}{r} 252 \\ 126\ \ \\ \hline \end{array}$$

$$\overline{399\times 2} - 42 = 756)1512(2\ \textit{the Answer.}$$
$$1512$$

$$Or,\ \frac{\overline{39+3}\times\overline{39-3}}{\overline{399\times 2}-\overline{39+3}} = 2.$$

2. A owes B, £67 12*s*, to be discharged in a year, by weekly payments; the first payment to be 6*d.* and the last, £2 11*s.* 6*d.*; what is the common difference of the payments, and what will each payment be?

$$\frac{\overline{51,5+,5}\times\overline{51,5-,5}}{\overline{1352\times 2}-\overline{51,5+,5}} = 1s.\ \textit{and}\ 6d. + 1s. = 1/6 = 2d.\ \textit{payment},\ 1/6.$$

$+1s. = 2s.\ 6d. = 3d.$ *payment, &c.*

PROBLEM 6.

PROBLEM 6.

The Extremes and sum of the Series given, to find the number of terms.

RULE.—Twice the sum of the Series, divided by the sum of the Extremes, will give the number of Terms.

EXAMPLES.

1. LET the extremes be 3 and 39, and the sum of the series 399; what is the number of terms?

$$\begin{array}{rl} \textit{Sum of the Series} = & 399 \\ & \times\ 2 \\ \hline \textit{Sum of Extr.} = 39+3=42) & 798(19 \textit{ the Answer.} \\ & 42 \\ \hline & 378 \\ & 378 \\ \hline \end{array}$$

$$Or\ \frac{399\times 2}{39+3} = 19.$$

2. A owes B, £67 12*s.* to be paid weekly in Arithmetical Progression, the first payment to be 6*d.* and the last 51*s.* 6*d.* How many payments will there be, and how long will he be in discharging the debt?

$$\frac{1352\times 2}{51{,}5+{,}5} = 52 \textit{ Payments, and as many weeks, Answer.}$$

PROBLEM 7.

The first Term, the common difference, and sum of the Series given, to find the number of terms.

RULE.—To the square of the difference of twice the first term and the common difference, add the rectangle (or product) of the sum and the common difference multiplied by 8, and extract the square root of the sum, from which root take twice the first term less the common difference; divide the remainder by twice the common difference, and the quotient will be the number of terms.

EXAMPLE.

1. IF the first Term be 3, the common difference 2, and the sum of the Series 399; required the number of terms?

$3\times 2=6=$ *Twice the sum of the first Term.*

$6-2=4=$ *Difference of twice the first term and the common difference.*

$4\times 4=16=$ *Square of the said difference.*

$399\times 2\times 8=6384=$ *Rectangle of the sum and common diff. mult. by 8.*

$6384+16=6400=$ *Sum of the said eight-fold rectangle and the square of the aforesaid difference.*

$\sqrt{6400}=80=$ *Square Root of the last mentioned sum.*

80—

$80-4=76=$ *difference of the said Root and twice the first term less the common difference.*

$\frac{76}{4}=19$ *The number of terms.*

$$Or, \frac{\sqrt{\overline{3\times2-2}|^2+399\times2\times8}-3\times2-2}{2\times2}=19.$$

PROBLEM 8.

The first Term, the common difference, and the sum of the series given, to find the last term.

Rule. To the square of the difference of twice the first Term and the common difference, add the rectangle of the sum and the common difference, and extract the square root of their sum, from which root take the common difference; and the remainder, divided by 2, will be the last term.

EXAMPLE.

If the first Term be 3, the common difference 2, and the sum of the series 399: What is the last Term?

$3\times2=6.\ 6-2=4.\ 4\times4=16.\ 399\times2\times8=6384.\ 6384+16=6400.$
$\sqrt{6400}=80.\ 80-2=78$, and $78\div2=39$ *the Answer.*

$$Or, \frac{\sqrt{\overline{3\times2-2}|^2+399\times2\times8}-2}{2}=39.$$

PROBLEM 9.

The first Term, the common difference, and the number of Terms given, to find the last Term.

Rule.—The number of Terms less 1, multiplied by the common difference, and the first Term added to the product, will give the last Term.

EXAMPLES.

1. If the first Term be 3, the common difference 2, and the number of Terms 19; what is the last Term?

Number of Terms =	19
	− 1
Number of Terms less 1 =	18
Common difference =	× 2
	36
First Term =	× 3
	39 *the Answer*

$$Or, \overline{19-1}\times2+3=39.$$

2. A owes B a certain sum to be paid in Arithmetical Progression, the first payment is 6*d.* the number of payments 52, and the com-

 mon

mon difference of the payments is 12*d*; what is the last payment? $\overline{52-1}\times 12+6=618d.=$ £2 11*s.* 6*d. the Answer.*

PROBLEM 10.

The first Term, common difference, and number of Terms given, to find the sum of the series.

RULE.—To the first Term add the product of the number of terms less 1 by half the common difference, and their sum, multiplied by the number of Terms, will give the sum of the progression.

EXAMPLES.

1. IF the first Term be 3, the common difference 2, and number of Terms 19; what is the sum of the series?

First Term $= 3$

Add the product of the number of Terms less 1 *by* $\frac{1}{2}$ *com. diff.* $=\overline{19-1}\times 1=18$

Their sum 21

Multiply by the number of terms $= 19$

189

21

Or, $19\times\overline{3+\overline{19-1}\times 1}=399$ *The Answer* $= 399$

2. SIXTEEN persons gave charity to a poor man; the first gave 5*d.* and the second 9*d.* and so on in arithmetical progression, I demand what sum the last person gave, and how much the poor man received in all?

Answer $\overline{16-1}\times 4+5=65d.=5s.\ 5d.$ *the last gave.*

And $16\times\overline{5+\overline{16-1}\times\frac{4}{2}}=560d.=$ £2 6/8 *the whole sum.*

PROBLEM 11.

Given the first Term, the number of Terms, and the sum of the series, to find the common difference.

RULE. The double of the sum, less the rectangle of the first Term and number of Terms less 1, will give the common difference.

EXAMPLE.

IF the first Term be 3, the number of Terms 19, and the sum 399; what is the common difference?

Sum of the series $= 399$

Subtract the Product of the first Term and number of Terms $= 3\times 19= 57$

Remainder $= 342$

Multiplied by 2

Divide by the product of the number of Terms and number of Terms less 1 $\}=19\times 18=342\ )684($ 2 *the Ans.*

684

...

Or, $\dfrac{2\times\overline{399-3\times 19}}{\overline{19-1}\times 19}=2$

PROBLEM 12.

Given the first Term, number of Terms, and the sum of the series, to find the last Term.

RULE.

RULE. Divide twice the sum by the number of Terms; from the quotient take the first Term, and the remainder will be the last.

EXAMPLES.

1. IF the first Term be 3, the number of Terms 19, and the sum 399; what is the last Term?

Sum of the Terms = 399
Multiplied by 2

Divide by the number of Terms = 19)798

Quotient = 42
Subtract the first Term = 3

The Answer = 39

Or, $\frac{399 \times 2}{19} - 3 = 39$.

2. A Merchant being indebted to 12 Creditors £2460, ordered his Clerk to pay the first £40 and the rest increasing in arithmetical progression; I demand the difference of the payments, and the last payment?

Ans. $\frac{\overline{2 \times 2460 - \overline{40 \times 12}}}{\overline{12-1} \times 12} = £30 = diff.$ & $\frac{2460 \times 2}{12} - 40 = £370$ *last paym.*

PROBLEM 13.

The common difference, the last term, and sum of the progression given, to find the first term.

RULE.—From the square of twice the last term plus the common difference, take 8 times the rectangle of the sum and common difference, and extract the square root of the remainder, which (root) either add to, or subtract from the common difference, (as the case may require) and half the sum or difference will be the first term.

EXAMPLES.

1. IF the common difference be 2, the last term 39, and the sum of the terms 399; required the first term?

Last Term 39
Multiplied by 2

Product = 78
Add the common difference = 2

80
Multiplied by 80

From the square of twice the last term plus the common difference = 6400
Take 8 times the rectangle of the sum and common diff. = 399 × 2 × 8 = 6384

Remainder = 16
Square Root of 16 = 4
Sum of the common difference and the Square Root of 16 = 2 + 4 = 6
And half the sum = $\frac{6}{2}$ = 3 *the Ans.*

Or,

Or, $\dfrac{2+\sqrt{\overline{39\times2+2}|^2-399\times2\times8}}{2}=3.$

2. A Merchant being indebted to ſeveral perſons £1080, he ordered his Clerk to pay the greateſt creditor £140: the greateſt but one £132 and ſo on, to decreaſe in Arithmetical Progreſſion; what did the leaſt Creditor receive?

Anſwer, $\dfrac{10-\sqrt{\overline{142\times2+10}|^2-1080\times10\times8}}{2}=£2.$

PROBLEM 14.

Given the common difference, the laſt Term and ſum of the ſeries, to find the number of Terms.

RULE.—From the ſquare of twice the laſt Term plus the common difference take 8 times the rectangle of the ſum and common difference, and extract the Square Root of the remainder, which (root) either ſubtract from, or add to, twice the laſt Term plus the common difference (as the caſe may require) and the remainder or ſum, divided by twice the common difference, will give the number of Terms.

EXAMPLES.

1. IF the common difference be 2, the laſt term 39, and the ſum of the Terms 399: I demand the number of Terms.

Laſt term 39
Multiply by 2

78
Add the com. diff. = 2

80
80

Square of twice the laſt term plus the com. diff. = 6400
Sub. 8 times the rectan. of the ſum & com. diff. = $399\times2\times8$ = 6384

16

Square Root of 16 = 4
Sum of twice the laſt term plus the com. diff. = $39\times2+2$ = 80
Sum of twice the laſt term and com. diff. minus The ſquare root of $16=80-4$ = 76

Which remainder divided by twice the com. diff. $=\dfrac{76}{4}=19$ *Anſ.*

Or, $\dfrac{39\times2+2-\sqrt{\overline{39\times2+2}|^2-399\times2\times8}}{2\times2}=19$

2. A Merchant being indebted to ſeveral perſons £1080, he ordered his Clerk to pay the greateſt Creditor £142: the greateſt but one £132, and ſo on, to decreaſe in Arithmetical Progreſſion: How many creditors had he?

Anſwer, $\dfrac{142\times2+10+\sqrt{\overline{142\times2+10}|^2-1080\times10\times8}}{10\times2}=15$ *Credit.*

PROBLEM 15.

Given the last Term, the number of Terms, and the sum of the Terms, to find the first Term.

RULE. Divide twice the ſum by the number of Terms; from the quotient ſubtract the laſt Term, and the remainder will be the firſt.

EXAMPLES.

1. If the laſt Term be 39, the number of Terms 19, and the ſum of the ſeries 399; what is the firſt Term?

Sum of the ſeries = 399
Multiply by 2

Divide by the laſt Term = 19)798

Quotient = 42
From the quotient take the laſt Term = 39
Remainder = 3 *the Anſwer.*

Or, $\frac{399 \times 2}{19} - 39 = 3.$

2. A man had 10 ſons, whoſe ſeveral ages differed alike; the eldeſt was 48 years old, and the ſum of all their ages was 255; what was the age of the youngeſt?

Anſwer $\frac{255 \times 2}{10} - 48 = 3$ *years.*

PROBLEM 16.

Given the laſt Term, the number of Terms, and the ſum of the Series, to find the common difference.

RULE. Double the rectangle of the number of Terms and the laſt Term minus the ſum of the ſeries; divide the product by the rectangle of the number of Terms and the number of Terms minus 1, and the Quotient will be the common difference.

EXAMPLES.

1. If the laſt Term be 39, the number of Terms 19, and the ſum of the ſeries 399; what is the common difference?

Number of Terms = 19
Multiply by the laſt Term = 39

171
57

Rectangle of the number of Terms and the laſt Term = 19 × 39 = 741

Subtract the ſum of the ſeries = 399

Remainder = 342
Multiply by 2

Divide by the rectangle of the number of Terms, and number of Terms minus 1 = 19 × 18 = 342)684(2 *the Anſwer*
684

Or,

Or, $\dfrac{2 \times \overline{19 \times 39} - 399}{\overline{19 - 1} \times 19} = 2.$

2. SIXTEEN persons gave charity to a poor man in such proportion as to form an Arithmetical Series: The last gave 5*s*. 5*d*. and the whole sum amounted to £2 6*s* 8*d*; what did each give less than the other from the last down to the first?

Answer, $\dfrac{2 \times \overline{16 \times 65} - 560}{\overline{16 - 1} \times 16} = 4d.$

PROBLEM 17.

The common difference, number of Terms, and the last Term given, to find the first Term.

RULE. From the last Term subtract the Product of the Terms less 1 by the common difference, and the remainder will be the first Term.

1. IF the common difference be 2, the number of Terms 19, and the last Term 39; what is the first?

EXAMPLES.

Last Term $= 39$

Subtract the number of Terms less 1 multiplied by the common difference $\Big\} = \overline{19 - 1} \times 2 = 36$

Remains 3 *the Answ.*

Or, $39 - \overline{19 - 1} \times 2 = 3.$

2. A man travelled 6 days, each day going 4 miles further than on the preceding day, 'till the last day's Journey was 40 miles; How far did he ride the first day?

Answer. $40 - \overline{6 - 1} \times 4 = 20$ *miles.*

PROBLEM 18.

The common difference, the number of Terms and last Term given, to find the sum of the series.

RULE. From the last Term take the number of Terms, minus 1, multiplied by half the common difference, and the Remainder multiplied by the number of Terms will give the sum.

1. IF the common difference be 2, number of Terms 19, and the last Term 39; what is the sum of the series?

EXAMPLES.

Last Term $= 39$

Subtract the number of Terms, less 1, multiplied by ½ *the common difference,* $\Big\} = \overline{19 - 1} \times 1 = 18$

Remainder $= 21$

Multiply by the number of Terms $= 19$

189
21

The Answer — 399

Or

Or, $19 \times \overline{39 - \overline{19 - 1} \times 1} = 399$

2. A Man performed a Journey in 6 days, and, each day, travelled 4 miles further than on the preceding day, till his last day's travel was 40 miles: How far did he travel in the whole?

Answer, $6 \times \overline{40 - \overline{6 - 1} \times \frac{4}{2}} = 180$ *miles.*

PROBLEM 19.

The sum of the Terms, the number of Terms, and the common difference given, to find the first Term.

RULE.—Divide the Sum by the number of Terms; from the Quotient take half the Product of the number of Terms, minus unity, by the common difference, and the remainder will be the first Term.

1. If the sum of the series be 399, the number of Terms 19, and the common difference 2; what is the first Term?

Number of Terms $= 19)\overline{399} =$ *Sum*

Quotient $= 21$

Subtract $\frac{1}{2}$ *the Product of the number of Terms, less* 1, *by the common difference,* $\Big\} = \frac{\overline{19 - 1} \times 2}{2} = 18$

The Answer 3

Or, $\frac{399}{19} - \frac{2 \times \overline{19 - 1}}{2} = 3.$

2. A Man travelled 180 miles in 6 days; he increased his Journey, each day, by 4 miles: How far did he travel the first day?

$\frac{180}{6} - \frac{4 \times \overline{6 - 1}}{2} = 20$ *miles, Answer.*

PROBLEM 20.

The sum of the Terms, number of Terms, and the common difference given, to find the last Term.

RULE.—Divide the sum of the series by the number of Terms; to the Quotient add half the product of the number of Terms minus unity by the common difference, and the sum will be the last Term.

EXAMPLES.

1. If the sum of the series be 399, the number of Terms 19, and the common difference 2; what is the last Term?

Divide by the number of Terms $= 19)\overline{399}$ *sum*

Quotient $= 21$

Add $\frac{1}{2}$ *the Product of the number of Terms, less* 1, *by the common difference* $\Big\} = \frac{\overline{19 - 1} \times 2}{2} = 18$

The Answer $= 39$

Or, $\frac{399}{19} + \frac{2 \times \overline{19 - 1}}{2} = 39$

2. A

2. A Person bought a farm for £510, to be paid monthly in Arithmetical Progression, and to be completed in a year, each payment

The following Table contains a summary of the whole doctrine of Arithmetical Progression.

Cases of Arithmetical Progression.			
Case	*Given*	*Required*	*Solution.*
1.	*aln*	*d*	$\frac{l-a}{n-1}$
		s	$\frac{\overline{a+l}\times n}{2}$
2.	*ald*	*n*	$\frac{l-a}{d}+1$
		s	$\frac{\overline{l+a}\times\overline{l-a+d}}{2d}$
3.	*als*	*d*	$\frac{\overline{l+a}\times\overline{l-a}}{2s-l+a}$
		n	$\frac{2s}{a+l}$
4.	*ads*	*n*	$\frac{\sqrt{\overline{2a-d}\vert^2+8ds}-\overline{2a-d}}{2d}$
		l	$\frac{\sqrt{\overline{2a-d}\vert^2+8ds}-d}{2}$
5.	*adn*	*l*	$\overline{n-1}\times d+a$
		s	$n\times a+\overline{n-1\times\frac{d}{2}}$

Case

ment to exceed that preceding by £5; what were the first and last Payments?

Answer, $\frac{510}{12} - \frac{5 \times \overline{12-1}}{2} =$ £15 *the first payment, and*

$\frac{510}{12} + \frac{5 \times \overline{12-1}}{2} =$ £70 *the last payment.*

GEOME-

Case	*Given*	*Required*	*Solution.*
6.	*ans*	*d*	$\frac{2 \times \overline{s-an}}{\overline{n-1} \times n}$
		l	$\frac{2s}{n} - a$
7.	*lds*	*a*	$\frac{d \pm \sqrt{\overline{2l+d}^2 - 8ds}}{2}$
		n	$\frac{2l+d \mp \sqrt{\overline{2l+d}^2 - 8ds}}{2d}$
8.	*lns*	*a*	$\frac{2s}{n} - l$
		d	$\frac{2 \times \overline{nl-s}}{\overline{n-1} \times n}$
9.	*lnd*	*a*	$l - \overline{n-1} \times d$
		s	$n \times \overline{l - \overline{n-1} \times \frac{d}{2}}$
10.	*dns*	*a*	$\frac{s}{n} - \frac{d \times \overline{n-1}}{2}$
		l	$\frac{s}{n} + \frac{d \times \overline{n-1}}{2}$

Here $\left\{ \begin{array}{l} a = \textit{first term} \\ l = \textit{last term} \\ n = \textit{number of terms} \\ d = \textit{common difference} \\ s = \textit{sum of all the terms} \end{array} \right.$

GEOMETRICAL PROPORTION.

THEOREM 1.

If four quantities, $a.\ b.\ c.\ d.$ (2. 6. 4. 12) be in geometrical proportion, the product of the two means, bc (6×4) will be equal to that of the two extremes, ad (2×12) whether they are continued, or discontinued,¶ and, if three quantities, $a.\ b.\ c.$ (2. 4. 8) the square of the mean is equal to the Product of the two extremes.

THEOREM 2.

If four quantities, $a.\ b.\ c.\ d.$ (2. 6. 4. 12) are such, that the product of two of them, ad (2×12) is equal to the Product of the other two, bc (6×4), then are those quantities proportional. †

THEOREM 3.

If four quantities, $a, b, c, d,$ (2. 6. 4. 12) are proportional, the rectangle of the means, divided by either extreme, will give the other extreme.*

THEOREM 4.

The Products of the corresponding Terms of two geometrical proportions are also proportional.

That is, if $a : b :: c : d$ ($2 : 6 :: 4 : 12$), and $e : f :: g : h$ ($2 : 4 :: 5 : 10$), then will $ae : bf :: cg : dh$ ($2 \times 2 : 6 \times 4 :: 4 \times 5 : 12 \times 10$.)‡

Theorem

¶ For since the ratio of a to b (2 to 6), or the part, which a is of b (2 is of 6) is expressed by $\frac{a}{b}$ $\left(\frac{2}{6}\right)$ and the ratio of c to d (4 to 12), in like manner, by $\frac{c}{d}$ $\left(\frac{4}{12}\right)$; & since, by supposition, the two ratios are equal, let them both be multiplied by bd, (6×12) and the products $\frac{a}{b} \times bd$ $\left(\frac{2}{6} \times 6 \times 12\right)$ and $\frac{c}{d} \times bd$ $\left(\frac{4}{12} \times 6 \times 12\right)$ will likewise be equal; that is, $\frac{abd}{b} = \frac{cbd}{d}$ or $ad = cb$ $\left(\frac{2 \times 6 \times 12}{6} = \frac{4 \times 6 \times 12}{12}\right.$, or, $2 \times 12 = 6 \times 4$.)

† For since, by supposition, the products ad (2×12) and bc (6×4) are equal, let both be divided by bd (6×12) and the quotients $\frac{ad}{bd}$ $\left(\frac{a}{b}\right)$ and $\frac{bc}{bd}$ $\left(\frac{c}{d}\right)$ $\frac{2 \times 12}{6 \times 12}$ $\left(\frac{2}{6}\right)$ $\frac{6 \times 4}{6 \times 12}$ $\left(\frac{4}{12}\right)$ will also be equal; and therefore $a : b :: c : d$.

‡ For, by the second Theorem, $ad = bc$ ($2 \times 12 = 6 \times 4$) whence dividing both sides of the equation by a (2) we have $d = \frac{bc}{a}$ $\left(12 = \frac{6 \times 4}{2}\right)$, Hence, if the two means and one extreme be given, the other extreme may be found.

* For $\frac{a}{b} = \frac{c}{d}$ $\left(\frac{2}{6} = \frac{4}{12},\right)$ and $\frac{e}{f} = \frac{g}{h}$ $\left(\frac{2}{4} = \frac{5}{10}\right)$ by supposition; whence, $\frac{a}{b} \times \frac{e}{f} = \frac{c}{d} \times \frac{g}{h}$ $\left(\frac{2}{6} \times \frac{2}{4} = \frac{4}{12} \times \frac{5}{10},\right)$ by equal multiplication; and consequently $\frac{ae}{bf} = \frac{cg}{dh}$ $\left(\frac{2 \times 2}{6 \times 4} = \frac{4 \times 5}{12 \times 10}\right)$ that is, $ae : bf :: cg : dh$ ($2 \times 2 : 6 \times 4 :: 4 \times 5 : 12 \times 10$); Hence it follows, that if any quantities be proportional, their squares, cubes, &c. will likewise be proportional.

THEOREM 5.

IF four quantities, a, b, c, d, (2, 6, 4, 12) are directly proportional.

Then,	1. *Directly,*	$a : b :: c : d$	(2 : 6 :: 4 : 12)
	2. *Inversely,*	$b : a :: d : c$	(6 : 2 :: 12 : 4)
	3. *Alternately,*	$a : c :: b : d$	(2 : 4 :: 6 : 12)
	4. *Compoundedly,*	$a : a+b :: c : c+d$	(2 : 8 :: 4 : 16)
	5. *Dividedly,*	$a : b-a :: c : d-c$	(2 : 4 :: 4 : 8)
	6. *Mixtly,*	$b+a : b-a :: d+c : d-c$	(8 : 4 :: 16 : 8)
	7. *By Multiplication,*	$ra : rb :: c : d$	($2r$: $6r$ :: 4 : 12)
	8. *By Division,*	$\frac{a}{r} : \frac{b}{r} :: c : d$	($\frac{2}{r} : \frac{6}{r}$ :: 4 : 12)

BECAUSE the product of the means, in each case, is equal to that of the extremes, and therefore the quantities are proportional by Theorem 2.

THEOREM 6.

IF three numbers, $a.\ b.\ c.$ (2, 4, 8) be in continued proportion, the square of the first will be to *that* of the second, as the first number to the third; that is, $a^2 : b^2 :: a : c$ $(2\times2 : 4\times4 :: 2 : 8.)$ ‡

THEOREM 7.

IN any continued Geometrical Proportion (1. 3. 9. 27. 81. &c.) The Product of the two extremes, and *that* of every other two Terms, equally distant from them, are equal. †

THEOREM 8.

THE sum of any number of quantities, in continued geometrical proportion, is equal to the difference of the rectangle of the second and last Terms and the square of the first, divided by the difference of the first and second Terms. §

GEOME-

‡ FOR since $a : b :: b : c$ $(2 : 4 :: 4 : 8)$ thence will $ac = bb$ $(2\times8 = 4\times4)$ by Theorem 1; and therefore $aac = abb$ $(2\times2\times8 = 2\times4\times4)$ by equal multiplication; consequently, $a^2 : b^2 :: a : c$ $(2\times2 : 4\times4 :: 2 : 8)$ by Theorem 2.

IN like manner it may be proved that, of four quantities continually proportional, the *cube* of the first is to *that* of the second, as the first quantity to the fourth.

† FOR the ratio of the first term to the second being the same as *that* of the last but one to the last, these four terms are in proportion; and therefore by Theorem 1, the rectangle of the extremes is equal to *that* of their two adjacent terms: and, after the same manner, it will appear that the rectangle of the third and last but two is equal to *that* of their two adjacent terms, the second and last but one, and so of the rest; whence the truth of the proposition is manifest.

§ FOR, let the first term of the proportion be denoted by a, the common ratio by r, the number of terms by n, and the sum of the whole series by s: then, it is plain that the second term will be expressed by $a \times r$, or, ar; the third by $ar \times r$, or ar^2; the fourth by $ar^2 \times r$, or, ar^3; and the nth, or last, term by ar^{n-1}; and therefore the proportion will stand thus, $a + ar + ar^2 + ar^3 \text{----} + ar^{n-2} + ar^{n-1} = s$; which equation multiplied by r, gives $ar + ar^2 + ar^3 + ar^4 \ldots\ldots + ar^{n-1} + ar^n = rs$; from the first equation

GEOMETRICAL PROGRESSION.

A GEOMETRICAL PROGRESSION is, when a *Rank*, or *Series*, of numbers increaſes, or decreaſes, by the continual multiplication, or diviſion, of ſome equal number.

PROBLEM I.

Given one of the Extremes, the Ratio, and the number of the Terms of a Geometrical Series, to find the other extreme.

RULE. Multiply, or divide, (as the caſe may require) the given extreme by ſuch power of the Ratio, whoſe exponent ‡ is equal to the *number* of Terms leſs 1, and the product, or quotient, will be the other extreme.

EXAMPLES.

equation being ſubtracted, there will remain $-a + ar^n = rs - s$: whence, $s = \left\{\frac{ar^n - a}{r - 1} = \frac{r \times ar^{n-1} - a}{r - 1}\right\} = \frac{ar \times ar^{n-1} - aa}{ar - a}$: (or, take any ſeries of numbers whatever, as 2. 6. 18. 54. 162. 486. and their ſum will be $2+6+18+54+162+486 = 728$: this equation multiplied by the ratio, will ſtand thus, $6+18+54+162+486+1458 = 2184$; now it is plain that the ſum of the ſecond ſeries will be ſo many times the firſt, as is expreſſed by the ratio ;—Subtract the firſt ſeries from the ſecond, and it will give $1458-2=2184-728$, which is evidently ſo many times the ſum of the firſt ſeries, as is expreſſed by the ratio leſs 1 ; whence $\frac{1458-2}{3-1} = \frac{2916-4}{6-2}$, as was to be demonſtrated.)

‡ As the *laſt term*, ar any term near the laſt, is very tedious to be found by continual multiplication, it will often be neceſſary, in order to aſcertain them, to have a ſeries of numbers in Arithmetical Proportion, called *Indices*, or *Exponents*, beginning either with a cypher, or an unit, whoſe common difference is *one*.

WHEN the *firſt* term of the ſeries and the *ratio* are *equal*, the *indices* muſt begin with an unit, and, in this caſe, the product of any two terms is equal to that term, ſignified by the *ſum* of their *indices*.

Thus, { 1. 2. 3. 4. 5. 6, *&c. Indices, or arithmetical ſeries.*
{ 2. 4. 8. 16. 32. 64, *&c. Geometrical ſeries (leading terms.)*

Now, $6+6 = 12$ = *the index of the twelfth term, and*

$64 \times 64 = 4096$ = *the twelfth term.*

BUT, when the *firſt* term of the ſeries and the *ratio* are *different*, the *indices* muſt begin with a cypher, and the ſum of the *indices*, made choice of, muſt be *one leſs* than the *number of terms*, given in the queſtion ; becauſe 1 in the *indices* ſtands over the *ſecond term*, and 2, in the *indices*, over the *third* term, &c. And, in this caſe, the *product* of any *two* terms, divided by the *firſt*, is equal to that term *beyond* the firſt, ſignified by the *ſum* of their *indices*.

Thus, { 0. 1. 2. 3. 4. 5. 6, *&c. Indices.*
{ 1. 3. 9. 27. 81. 243. 729, *&c. Geometrical ſeries.*

Here, $6 + 5 =$ 11 *the Index of the 12th. term.*

$729 \times 243 = 177147$ *the 12th. term, becauſe the firſt term of the ſeries and the ratio are different, by which mean a cypher ſtands over the firſt term.*

THUS, by the help of theſe *indices*, and a few of the firſt terms in any geometrical ſeries, any term, whoſe diſtance from the firſt term is aſſigned, though it were ever ſo remote, may be obtained without producing all the terms.

Note. If the *ratio* of any geometrical ſeries be *double*, the *difference* of the *greateſt* and *leaſt* terms is equal to the ſum of all the terms, except the greateſt ;—If the *ratio* be *triple*, the *difference* is *double* the ſum of all but the greateſt ;—If the *ratio* be *quadruple*, the difference is *triple* the ſum of all but the greateſt, &c.

IN any geometrical ſeries decreaſing, and continued *ad infinitum*, *half* the *greateſt* term is equal to the *ſum* of all the remaining terms, *ad infinitum*.

EXAMPLES.

1. If the firſt Term be 4, the ratio 4, and the number of Terms 9; what is the laſt Term?

1. 2. 3. 4. + 4 = 8
4. 16. 64. 256. × 256 = 65536 = *Power of the Ratio, whoſe exponent is leſs by* 1, *than the number of Terms.*

65536 = *8th power of the Ratio.*
Multiply by 4 = *firſt Term.*

262144 = *laſt Term.*

Or, $4 \times 4^8 = 262144$ = *the Anſwer.*

2. If the laſt Term be 196608, the Ratio 4, and the number of Terms 9, what is the firſt Term?

Laſt Term.
8th. Power of the Ratio 4^8 = 65536)262144(4 = *the firſt Term.*

Or, $\frac{262144}{4^8} = 4$ *the firſt Term.*

Again, *Given the firſt Term, and the Ratio, to find any other Term aſſigned.*

RULE I.

When the Indices begin with an unit.

1. Write down a few of the leading Terms of the ſeries, and place their Indices over them.

2. Add together ſuch Indices, whoſe ſum ſhall make up the entire index to the Term required.

3. Multiply the Terms of the geometrical ſeries, belonging to thoſe Indices, together, and the product will be the Term ſought.

1. If the firſt Term be 2, and the ratio 2; what is the 13th Term?

1. 2. 3. 4. 5 + 5 + 3 = 13
2. 4. 8. 16. 32 × 32 × 8 = 8192 *Anſ.* *Or*, $2 \times 2^{12} = 8192$.

2. A Merchant wanting to purchaſe a cargo of horſes for the Weſt-Indies, a Jockey told him he would take all the trouble and expence, upon himſelf, of collecting and purchaſing 30 horſes for the voyage, if he would give him what the laſt horſe would come to by doubling the whole number by a half-penny, that is, two farthings for the firſt, four for the ſecond, eight for the third, &c. to which the merchant, thinking he had made a very good bargain, readily agreed: Pray, what did the laſt horſe come to, and what did the horſes, one with another, coſt the Merchant?

[*30th, or laſt Term,*
1. 2. 3. 4. 5. 6 + 6 = *12th* 12 + 12 + 6 =
2. 4. 8. 16. 32. 64 × 64 = 4096 *and* 4096 × 4096 × 64 =
1073741824 *qrs.* = £1118481 1*s.* 4*d.* *and their average price was* £37282 14*s.* $0\frac{1}{2}$*d.* *a-piece.*

RULE

Rule 2.

When the Indices begin with a Cypher.

1. Write down a few of the leading Terms of the ſeries, as before, and place their Indices over them.

2. Add together the moſt convenient indices to make an Index, leſs by 1 than the number expreſſing the place of the Term ſought.

3. Multiply the Terms of the geometrical ſeries together, belonging to thoſe Indices, and make the product a dividend.

4. Raise the firſt Term to a Power, whoſe Index is one leſs than the *number* of Terms multiplied, and make the reſult a diviſor, by which divide the dividend, and the quotient will be that Term *beyond the firſt*, ſignified by the ſum of thoſe Indices, or the Term ſought.

3. If the firſt Term be 5, and the Ratio 3; what is the 7th Term?

0. 1. 2. $3+2+1=$ $6=$ *Index to the 6th term beyond the 1ſt. or 7th.*
5. 15. 45. $135\times45\times15=91125=$ *Dividend.*

The number of Terms, multiplied, is 3 (viz. $135\times45\times15$), *and* $3-1=2$ *is the power to which the Term* 5 *is to be raiſed; but the* 2*d Power of* 5 *is* $5\times5=25$, *and therefore* $91125\div25=3645$ *the* 7*th Term required.*

Problem 2.

Given the firſt Term, the Ratio, and number of Terms, to find the ſum of the ſeries.

Rule.—Raiſe the Ratio to a power, whoſe Index ſhall be equal to the number of Terms, from which ſubtract 1; divide the remainder by the Ratio, leſs 1, and the quotient, multiplied by the firſt Term, will give the ſum of the ſeries.

Examples.

1. If the firſt Term be 5, the Ratio 3, and the number of Terms 7; what is the ſum of the ſeries?

Ratio $=3\times3\times3\times3\times3\times3\times3=2187=$ *7th power of the Ratio.*
Subtract 1

Divide by the Ratio leſs $1=3-1=2)2186$

Quotient $=1093$
Multiply by the firſt Term $=5$

Sum of the ſeries $=5465$

$$\text{Or } \frac{3^7-1}{3-1}\times5=5465 \text{ the Anſwer.}$$

2. A Shop-keeper ſold 13 yards of Cloth, on the following Terms, viz. 2*d* for the firſt yard, 4*d* for the ſecond, 8*d* for the third, &c. I demand the price of the Cloth.

$$\frac{2^{13}-1}{2-1}\times2=16382d.=£68\ 5s.\ 2d. \text{ Anſw.}$$

3. A

3. A Gentleman, whoſe daughter was married on a new year's day, gave her a guinea, promiſing to triple it on the firſt day of each month in the year; Pray what did her portion amount to?

$$\frac{3^{12}-1}{3-1} \times 1 = 265720 \textit{ Guineas, Anſwer.}$$

4. What debt can be diſcharged in a year, by paying 1 ſhilling the firſt month, 10*ſ*. the ſecond, and ſo on, each month in a tenfold proportion?

$10^{12}-1 \div 10-1 \times 1 = 111111111111$*ſ*.$=$£55555555555 11*ſ*. *Anſ.*

5. A Man threſhed wheat 9 days for a farmer, and agreed to receive but 8 wheat corns for the firſt day's work, 64 for the ſecond, and ſo on, in an eight-fold Proportion; I demand what his 9 days' labour amounted to, rating the wheat at 5*ſ*. per buſhel?†

Anſwer, $\frac{8^9-1}{8-1} \times 8 = 153391688$ *Corns. Amount*=£78 0*s*. 4½*d*.

6. An ignorant Fop wanting to purchaſe an elegant houſe; a facetious Gentleman told him he had one which he would ſell him on theſe moderate Terms, viz. that he ſhould give him a penny for the firſt door, 2*d*. for the ſecond, 4*d*. for the third, and ſo on, doubling at every door, which were 36 in all: It is a bargain, cried the ſimpleton, and here is a Guinea to bind it; Pray, what would the houſe have coſt him?

Anſwer, $\frac{2^{36}-1}{2-1} \times 1 = 68719476735$*d*.=£286331153 1*ſ*. 3*d*.

7. A young fellow, well ſkilled in numbers, agreed with a rich farmer to ſerve him 10 years, without any other reward, but the produce of one wheat corn for the firſt year, and that produce to be ſowed from year to year, till the end of the time, allowing the increaſe but in a tenfold proportion; what is the ſum of the whole produce, and what will it amount to, at 5*s*. *per* buſhel?

Anſwer, $\frac{10^{10}-1}{10-1} \times 10 = 11111111110$ *Corns. Amount*=£5651 8/0$\frac{9}{16}$

8. Suppose one farthing had been put out, at 6 *per cent. per annum*, compound Intereſt,‡ at the commencement of the Chriſtian Æra: what would it have amounted to in 1784 years; and ſuppoſe the amount to be in ſtandard gold, allowing a cubic Inch to be worth £53 2*s*. 8*d*. How large would the maſs have been?

Anſ. $\frac{2^{150}-1}{2-1} \times 1 =$ £148671634656874820943571455150989076706 5361 11*s* 3¾.

=279808597221212304159795712329335942107 66 *Cubic Inches of Gold.*

As 355 : 113 :: 360 x 69,5 : 7964 *Earth's Diameter.* 360 x 69,5 x 7964 x 1327,33 =264482820122 *Cubic miles in the Globe* = =672733373088547413688320000 *Cubic Inches in the Globe. Then* 27980859722121230415979571232933594210766 ÷ 672733373088547413688320000 =41593089984028 8,8 *which, however incredible it may appear to ſome, is more than four*

† *Note*, 7680 wheat or barley corns are ſuppoſed to make a pint.

‡ Any ſum, at £6 *per cent. per annum* compound intereſt, will double in eleven years

*four hundred and fifteen millions of millions, nine hundred and thirty thousand, eight hundred and ninety nine million, eight hundred and forty thousand, two hundred and eighty-eight times larger than the globe we inhabit.**

PROBLEM 3.

The first Term, the last Term (or the Extremes) and the Ratio given, to find the sum of the series.

RULE. Divide the difference of the Extremes by the Ratio less by 1: Add the greater extreme to the quotient, and the result will be the sum of all the Terms.

OR, Multiply the greatest Term by the Ratio, from the product subtract the least Term; then divide the remainder by the Ratio, less by 1, and the quotient will be the sum of all the Terms.

OR, When all the Terms are given, then, from the Product of the *second* and *last* Terms subtract the *square* of the *first* Term; this remainder being divided by the *second* Term *less* the *first*, will give the sum of the series.

EXAMPLES.

1. IF the Series be 2. 6. 18. 54. 162. 486. 1458. 4374; what is its sum total?

First method

From the greatest Term = 4374
Subtract the least = 2

Divide by the Ratio, less 1 = 3 — 1 = 2)4372 *diff. of Extremes.*

Quotient = 2186
Add the greater extreme = 4374

6560

$$Or,\ \frac{4374-2}{3-1}+4374=6560\ \textit{Answer.}$$

Second Method.

Greatest term = 4374
Multiply by the Ratio = 3

Product = 13122
Subtract the least term = 2

Divide by the Ratio, less by 1 × 3 — 1 = 2)13120

Answer 6560

$$Or\ \frac{\overline{4374\times 3}-2}{3-1}=6560.$$

Third

years and three hundred and twenty-five days, or 11,889 years, or 11,89 is near enough, then, if you divide 1784 by 11,89, it will give the number of Terms in this case equal to 150;—The Ratio will be 2, and the first Term 1.

* To find the solid content of a globe.—See Art. 34th. of Mensuration of Solids. *Note* that ,523598 is two thirds of ,785398 the Area of a Circle, whose diameter is 1.

Third Method.

Greatest term = 4374
Multiply by the second term = 6

Product = 26244
Subtract the square of the first Term = 2 × 2 = 4

Divide the remainder by the 2d term less the 1st =6—2=4)26240

Answer, 6560

Or, $\frac{\overline{4374\times6-4}}{6-2}=6560.$

2. A man travelled 6 days, the first day he went 4 miles, and each day doubling his day's travel, his last day's ride was 128 miles; How far did he go in the whole?

$\frac{128-4}{2-1}+128=252$ *miles, Answer.*

3. A Gentleman, dying, left 5 Sons, to whom he bequeathed his Estate as follows, viz. to his youngest Son £1000; to the eldest £5062 10*s*. and ordered that each Son should exceed the next younger by the equal Ratio of $1\frac{1}{2}$; what did the several legacies amount to?

$\frac{5062,5-1000}{1,5-1}+5062,5=$ £13187 10*s*. *Answer.*

PROBLEM 4.

Given the Extremes and Ratio, to find the number of Terms.

RULE. Divide the greatest Term by the least; find what Power of the Ratio is equal to the quotient, then, add 1 to the Index of that Power, and the sum will be the number of Terms.

Or, Subtract the Logarithm ‡ of the least Term from *that* of the greatest; divide the remainder by the Logarithm of the Ratio, and add 1 to the quotient.

EXAMPLES.

1. IF the least Term be 2, the greatest Term 4374, and the Ratio 3, what is the number of Terms?

Divide by the least term = 2)4374 = *greatest term.*

3×3×3×3×3×3×3 = *Quotient* 2187 = 7*th power, then* 7+1=8 *the Answer.*

 Or,

‡ LOGARITHMS are artificial numbers, the addition of which answers to multiplication of whole numbers, and subtraction, to division.

Or,

From the logarithm of the greatest term = 3.64088
Subtract the logarithm of the least term = 0.30103

Divide the remainder by the logarithm of the ratio } = ,47712)3.33985(7+1= 8, *Ans.*
3 33984
———
1

2. A Gentleman travelled 252 miles, the first day he rode 4 miles; the last 128, and each day's Journey was double to the preceding one; How many days was he in performing the Journey?

PROBLEM 5.

Given the least Term, the Ratio, and the sum of the series, to find the last Term.

RULE. Multiply the sum of the series by the Ratio, less 1, to that Product add the first Term, and the result divided by the Ratio, will give the last Term.

EXAMPLES.

1. IF the first Term be 2, the Ratio 3, and the sum of the series 6560; what is the last Term?

Sum of the Series = 6560
Multiply by the Ratio less 1 = 2

Product = 13120
Add the least term = 2

Divide their sum by the Ratio = 3)13122

4374 *Ans.*

Or, $\frac{\overline{3-1}\times 6560+2}{3}$ = 4374 *Answer.*

2. A Gentleman performed a Journey of 252 miles; the first day he rode 4 miles, and each day after the first, twice so far as the day before; How far did he ride the last day?

$\frac{\overline{2-1}\times 252+4}{2}$ = 128 *miles, Answer.*

PROBLEM 6.

Given the least Term, the Ratio, and the sum of the series, to find the number of Terms.

RULE.

To the product of the sum of the series and the Ratio minus 1, add the first Term, which sum, divided by the first Term, will give that power of the Ratio signified by the number of Terms.

OR, From the Logarithm of the sum of the series plus the first Term, multiplied by the Ratio minus unity, take the Logarithm of the first Term, the remainder, divided by the Logarithm of the Ratio, will give the number of Terms.

EXAMPLE

EXAMPLE.

If the least Term be 2, the Ratio 3, and the sum of the series 80; what is the number of Terms?

Sum= 80
Multiply by the Ratio less 1=3—1= 2

160
Add the first Term= 2

Divide by the first Term= 2)162

81 *which, found in the Table of Powers, is the 4th power of the Ratio, therefore the number of Terms is 4.*

By Logarithms.

Sum= 80
Add the first Term= 2

82
Multiply by the Ratio less 1=3—1= 2

Logarithm of 164= 2.21484
Subtract the Logarithm of the first Term= .30103

Divide by the Logarithm of the Ratio= .47712)1.91381(4 *Answ.*
190848

533

PROBLEM 7.

Given the Extremes, and the sum of the series, to find the Ratio.

RULE. From the sum of the series subtract the least Term; divide the remainder by the sum of the series minus the greatest Term, and the quotient will be the Ratio.

EXAMPLES.

1. If the least Term be 2, the greatest Term 4374, and the sum of the series 6560; what is the Ratio?

Sum of the series = 6560
Subtract the least Term= 2

Divide the rem. by the sum of the series, minus the greatest Term } =6560—4374=2186)6558(3 *Answ.*
6558

2. A

2. A Debt of £252 was paid in Geometrical Progression, the first payment was £4 and the last £128; In what Ratio did the payments exceed each other.

Answer $\frac{252-4}{252-128}=2$, *viz. a double Ratio.*

PROBLEM 8.

Given the Extremes, and the sum of the series, to find the number of Terms.

RULE. 1. From the Logarithm of the last Term subtract the Logarithm of the first, and make the remainder a dividend.

2. SUBTRACT the Logarithm of the sum minus the last Term from the Logarithm of the sum minus the first Term, and make the Remainder a Divisor.

3. DIVIDE the Dividend by the divisor, and the quotient plus 1, will be equal to the number of Terms.

EXAMPLE.

IF the least Term be 2, the greatest Term 4374, and the sum of the series 6560; what is the number of Terms?

From the Logarithm of the greatest Term =	3.64088
Take the Logarithm of the least Term =	0.30103
Dividend =	3.33985
From the Logarithm of the sum minus the first Term =	3.81677
Take the Logarithm of the sum minus the last Term =	3.33965
Divisor =	.47712

Then, .47712)3.33985(7+1=8 *The Answer.*
3 33984

1

Or, $\frac{L.4374-L.2}{L.6558-L2186}+1=8.$

PROBLEM 9.

The first Term, the number of Terms, and the last Term given, to find the Ratio.

RULE. Divide the greater Extreme by the less, and extract such Root of the Quotient, whose Index is equal to the number of terms, less 1. Or find the quotient in the Table of Powers, the root of which is the answer.

EXAMPLES.

1. GIVEN the Extremes 2 and 4374, and the number of Terms 8; what is the Ratio?

Divide

Divide by the least Term $= 2)\overline{4374} =$ *greatest Term.*

$\sqrt[7]{2187} = 3$

Or, $\left|\frac{4374}{2}\right|^{\frac{1}{8-1}} = 3$, *The Answer.*

PROBLEM 10.

The extremes and number of Terms given, to find the sum of the Series.

RULE. 1. Subtract the least Term from the greatest, and make the difference a dividend.

2. DIVIDE the greatest Term by the least, and extract such Root of the Quotient, whose Index is equal to the number of Terms less 1; take 1 from the said Root, and make the remainder a Divisor.—(Or find the Quotient in the Table of Powers, which will shew the root, from which subtract 1).

3. DIVIDE the dividend by the divisor, and the greatest Term, added to the quotient, will give the sum of the Series.

EXAMPLE.

GIVEN the Extremes 2 and 4374, and the number of Terms 8; what is the sum of the series?

From the greatest Term = 4374
Take the least = 2

Make this remainder a dividend 4372

Divide the greatest Term by the least $2)\overline{4374}$

And extract the 7th Root of the Quotient, $\sqrt[7]{2187} = 3$, *Then*
$3 - 1 = 2)\overline{4372}$

Quotient = 2186
Add the greatest Term = 4374

Answer, 6560

$$\text{Or, } 4374 + \frac{4374-2}{\left|\frac{4374}{2}\right|^{\frac{1}{8-1}} - 1} = 6560$$

PROBLEM 11.

Given the Ratio, the number of Terms, and the greatest Term to find the least Term.

RULE. Divide the greatest term by such power of the Ratio, whose index is equal to the number of Terms, less 1, and the quotient will be the least Term.

EXAMPLE

EXAMPLE.

If the Ratio be 2, the number of terms 6, and the greatest term 128; what is the least;

Divide the last term by $2\times2\times2\times2\times2$ $=$ *5th. power of the Ratio* $\Big\}$ $= 32)128(4$ *the Answer.*
128

$$Or\ \frac{128}{2^{6-1}} = 4.$$

PROBLEM 12.

Given the Ratio, the number of Terms, and the greatest Term, to find the sum of the series.

RULE. 1. Divide the greatest Term by such Power of the Ratio, whose Index is equal to the number of Terms less 1; take the quotient from the last Term, and make the remainder a dividend.

2. DIVIDE the dividend by the Ratio less 1, and the Quotient, added to the greatest Term, will give the sum of the Series.

EXAMPLE.

If the Ratio be 4, the number of Terms 6, and the greatest Term 3072; what is the sum of the Series?

Divide the last term by the 5th Power of the Ratio $\Big\}$ $=4\times4\times4\times4\times4=1024)3072(3$
3072

From the last Term $= 3072$
Take the Quotient $= 3$

Divide by the Ratio less $1 = 4 - 1 = 3)3069$

Quotient $= 1023$
Add the greatest Term $= 3072$

Answer, $= 4095$

$$Or\ 3072 + \frac{3072 - \frac{3072}{4^{6-1}}}{4-1} = 4095.$$

PROBLEM 13.

Given the Ratio, the number of Terms, and the sum of the series, to find the least Term.

RULE. Divide the Ratio, less 1, by such Power, less 1, of the Ratio, whose Index is equal to the number of Terms, and the quotient, multiplied by the sum of the series, will give the least Term.

EXAMPLE.

EXAMPLE.

If the Ratio be 4, the number of Terms 6, and the ſum of the Series 4095; what is the leaſt Term?

$4\times4\times4\times4\times4\times4=4096$, *and* $4096-1=4095$, *then, the Ratio leſs* 1, *divided by* 4095, *is* $\frac{3}{4095}$, *and* $\frac{3}{4095}\times\frac{4095}{1}=\frac{12285}{4095}=3$ *the Anſwer.*

Or $\frac{4-1}{4^6-1}\times4095=3$.

PROBLEM 14.

Given the Ratio, the number of Terms, and the ſum of the Series, to find the greateſt Term.

RULE. 1. Subtract that Power of the Ratio, which is equal to the number of Terms leſs 1, from that Power of it, which is equal to the whole number of Terms.

2. DIVIDE the Remainder by that Power of the Ratio minus unity which is equal to the number of Terms, and the quotient, multiplied by the ſum of the ſeries, will give the greateſt Term.

EXAMPLE.

If the Ratio be 4, the number of Terms 6, and the ſum of the ſeries 4095; what is the greateſt Term?

From $4\times4\times4\times4\times4\times4=4^6=4096$
Subtract $4\times4\times4\times4\times4 \quad =4^5=1024$

Divide by $4^6-1=4095)3072=\frac{3072}{4095}$ *which, multiplied by the ſum, is* $\frac{3072}{4095}\times\frac{4095}{1}=\frac{12579840}{4095}=3072$ *the Anſwer.*

Or $\frac{4^6-4^{6-1}}{4^6-1}\times4095=3072$.

THE two laſt Problems may be ſolved by one ſhort operation thus; Divide the ſum by the ratio, and the remainder after the operation will be the leaſt Term; then take the quotient from the ſum of the ſeries, and the remainder will be the greateſt Term.

For the leaſt Term.

```
4)4095(1023 Quotient
  4
  ——
  09
   8
  ——
   15
   12
  ——
Anſwer 3
```

For the greateſt Term.

From the ſum = 4095
Subtract the Quotient = 1023

Anſwer, = 3072

PROBLEM 15.

PROBLEM 15.

Given the Ratio, the last Term, & the sum of the series, to find the first Term.

RULE.

FROM the sum of the series take the last Term, and multiply the remainder by the Ratio; then take this product from the sum of the series, and the remainder will be the first Term.

EXAMPLE.

IF the Ratio be 4, the last Term 3072, and the sum of the Series 4095; what is the first Term?

From the sum = 4095
Take the last Term = 3072

Remainder = 1023
Multiply by the Ratio = 4

Subtract 4092 *from the sum.*

And the remainder 3 *is the Answer.*

PROBLEM 16.

Given the Ratio, the last Term, and the sum of the series, to find the number of Terms.

RULE.

1. MULTIPLY the difference between the sum and the last Term by the Ratio, and note the product.
2. SUBTRACT this product from the sum, and note the remainder.
3. FROM the Logarithm of the last Term subtract the Logarithm of the remainder.
4. DIVIDE this last remainder by the Logarithm of the Ratio, and the quotient, plus unity, will give the number of Terms.

EXAMPLE.

IF the Ratio be 3, the last Term 54, and the sum of the Series 80; what is the number of Terms?

From the sum = 80
Take the last = 54

Remainder = 26
Multiply by the Ratio = 3

Product = 78
From the sum = 80
Take the product = 78

Remainder = 2
From the Logarithm of 54 = 1.73239
Take the Logarithm of the remainder = .30103

Divide by the Logarithm of the Ratio=.47712)1.43136(3+1=4 *Ans.*

PROBLEM

PROBLEM 17 and 18.

Given the number of Terms, the last Term, and the sum of the series, to find the first Term and the Ratio.

THE solution of these two Problems being very tedious by the Theorems, they may be solved by a very short operation; thus, Divide the sum of the series by the difference between the sum and the last Term; the quotient will give the Ratio, and the remainder, after the operation, the first Term. EXAMPLE.

The following Table exhibits a summary view of the doctrine of Geometrical Progression.

CASES *of* GEOMETRICAL PROGRESSION.

Case	*Given*	*Required*	*Solution.*
1.	*arn*	*l*	ar^{n-1}
		s	$\frac{r^n-1}{r-1} \times a$
2.	*arl*	*s*	$l+\frac{l-a}{r-1}$
		n	$\frac{L.l-L.a}{L.r}+1$
3.	*ars*	*l*	$\frac{\overline{r-1}\times s\times a}{r}$
		n	$\frac{L.\overline{\overline{r-1}\times s+a}-L.a}{L.r}$
4.	*als*	*r*	$\frac{s-a}{s-l}$
		n	$\frac{L.l-L.a}{L.\overline{s-a}-L.\overline{s-l}}+1$
5.	*ans*	*r*	$\frac{rs}{a}-r^{n-1}=\frac{s-a}{a}$
		l	$l\times\overline{s-l}\,\vert^{n-1}=a\times\overline{s-a}\,\vert^{n-1}$

EXAMPLE.

If the number of Terms be 4, the last Term 54, and the sum of the series 80; Required the first Term and the Ratio?

From the sum = 80
Take the last Term = 54

Divide by the difference = 26)80(3 *the Ratio.*
78

The first Term = 2

SIMPLE

Case	*Given*	*Required*	*Solution.*
6.	*anl*	*r*	$\left(\frac{l}{a}\right)^{\frac{1}{n-1}}$
		s	$l + \dfrac{l-a}{\left(\frac{l}{a}\right)^{\frac{1}{n-1}} - 1}$
7.	*rnl*	*a*	$\dfrac{l}{r^{n-1}}$
		s	$l + \dfrac{l - \frac{l}{r^{n-1}}}{r-1}$
8.	*rns*	*a*	$\dfrac{r-1}{r^{n}-1} \times s$
		l	$\dfrac{r^{n} - r^{n-1}}{r^{n}-1} \times s$
9.	*rls*	*a*	$s - r \times \overline{s-l}$
		n	$\dfrac{L.l - L.\overline{s - r \times \overline{s-l}}}{L.r} + 1$
10.	*nls*	*a*	$a \times \overline{s-a}\,\vert^{n-1} = l \times \overline{s-l}\,\vert^{n-1}$
		r	$r^{n} + \dfrac{s}{l-s}\, r^{n-1} = \dfrac{l}{l-s}$

Here {
a= *First or least term.*
l=*Last or greatest term.*
s=*Sum of all the terms.*
n=*Number of terms.*
r=*Ratio,*
L=*Logarithm.*

SIMPLE INTEREST.

Interest is a premium allowed by the borrower of any ſum of money to the lender, according to a certain rate *per Cent.* agreed on, which by law is ſtated at £6, that is, £6 for the uſe of £100 for one year, &c.

Principal is the money lent.

Rate is the ſum *per Cent.* agreed on.

Amount is the principal and intereſt added together.

Interest is of two ſorts, *ſimple* and *compound.*

Simple Intereſt is that, which is allowed for the principal lent only.

Note. The rules for Simple Intereſt ſerve alſo to calculate Commiſſion, Brokerage, Inſurance, purchaſing Stocks, or any thing elſe rated at ſo much *per Cent.*

General Rule.

1. Multiply the principal by the Rate, and divide the Product by 100 (or, which is the ſame, cut off the two right hand figures in the pounds, which muſt be reduced to the loweſt denomination, each time cutting off as at firſt) and the quotient will be the anſwer for one year.

2. For more years than one, multiply the Intereſt of one year by the given number of years, and the product will be the anſwer for that time.

3. If there be parts of a year, as months, or days, work for the months by the aliquot parts of a year, and for the days, by the Rule of Three Direct, or (which is ſufficiently exact for common uſe, allowing 30 days to the month, take aliquot parts of the ſame.

Note, when the Rate *per Cent. per Annum,* is

£.		
9	multiply the principal by	$\frac{3}{4}$
8		$\frac{2}{3}$
6		$\frac{1}{2}$
4		$\frac{1}{3}$
3		$\frac{1}{4}$
2		$\frac{1}{6}$

of the given number of months, cutting off, as before directed, and you will have the Intereſt for the given time.

Examples.

1. What is the Intereſt of £573 13 9½ for one year, at £6 *per Cent. per Annum?*

£.	s.	d.
573	13	9½
		6
34|42	2	9
20		
8|42		
12		
5|13		
4		
0|52		

Anſ. £34 8 5

2. What

2. WHAT is the Interest of £329 17 6$\frac{1}{2}$ for 3 years, 7 months, and 12 days, at £5 *per Cent. per Annum?*

£.	s.	d.
329	17	6$\frac{1}{2}$
		5
16|49	7	8$\frac{1}{2}$
20		
9|87		
12		
10|52		
4		
2|10		

Or thus:

		£.	s.	d.
£5	$\frac{20}{1}$	329	17	6$\frac{1}{2}$
		16	9	10$\frac{1}{2}$
				3
6 *months*	$\frac{1}{2}$	49	9	7$\frac{1}{2}$
1 *ditto*	$\frac{1}{6}$	8	4	11$\frac{1}{4}$
10 *days*	$\frac{1}{3}$	1	7	5$\frac{3}{4}$
2 *days*	$\frac{1}{5}$		9	1$\frac{3}{4}$
			1	9$\frac{3}{4}$
		£59	13	*Ans.*

Then

6 *months*	$\frac{1}{2}$	16	9	10$\frac{1}{2}$	*Interest of* 1 *year*
				3	
		49	9	7$\frac{1}{2}$	*ditto of* 3 *years*
1 *month*	$\frac{1}{6}$	8	4	11$\frac{1}{4}$	*ditto of* 6 *months*
10 *days*	$\frac{1}{3}$	1	7	5$\frac{3}{4}$	*ditto of* 1 *month*
2 *days*	$\frac{1}{5}$		9	1$\frac{3}{4}$	*ditto of* 10 *days*
			1	9$\frac{3}{4}$	*ditto of* 2 *days*
					Y. m. d.
		59	13		*ditto of* 3 7 12

3. WHAT is the Interest of £439 12 9$\frac{1}{2}$ at £6 *per Cent. per Annum*, for 16 months?

	£.	s.	d.	
	439	12	9$\frac{1}{2}$	
			8	= $\frac{1}{2}$ *the number of months*,
Ans.	£35|17	2	4	
	20			
s.	3|42			
	12			
d.	5|08			

4. WHAT

4. What is the Intereſt of £591 15 9¼ for 15 months, at £8 *per Cent. per Anuum?* 10=⅔ *the number of months.*

£59|17 17 8½
20
—
s. 3|57
12
—
d. 6|92
4
—
qrs. 3|70

5. What is the Intereſt of £347 7 5½ at £4 *per Cent. per Annum?* 5=⅓ *the number of months*

£17|36 17 3½
20
—
s. 7|37
12
—
d. 4|47
4
—
qrs. 1|90

6. What is the Intereſt of £517 15 4 for one month at £6 *per Cent. per Annum?* *Anſ.* £2 11 9¼

7. Of £457 12 8½ for 2 months at £6 *per Cent. per Annum?* *Anſ.* £4 11 6¼

8. Of £347 5 9 for 3 months at £6 *per Cent. per Annum?* *Anſ.* £5 4 2.

9. Of £397 19 for 4 months at £6 *per Cent. per Annum?* *Anſ.* £7 19 2.

10. £508 10 5½ for 5 months at £6 *per Cent. per Annum?* *Anſ.* £12 14 4¼

11. £719 19 4 for 6 months at £6 *per Cent. per Annum?* *Anſ.* £21 11 11¾

12. £396 5 10 for 7 months at £6 *per Cent, per Annum?* *Anſ.* £13 17 4¾

13. £517 11 11½ for 8 months at £6 *per Cent. per Annum?* *Anſ.* £20 14 0¾

14. £245 5 8¾ for 9 months at £6 *per Cent. per Annum?* *Anſ.* £11 2 9

15. £195 15 5½ for 10 months at £6 *per Cent. per Anmum?* *Anſ.* £9 15 9¼

16.

16. £148 12 6½ for 11 months at £6 *per Cent. per Annum?*
Ans. £8 3 5¾

17. £509 9 2 for 13 months at £6 *per Cent. per Annum?*
Ans. £33 2 3¼

18. £317 17 8½ for 14 months, at £6 *per Cent. per Annum?*
Ans. £22 5 — ¼

19. £443 10 3 for 15 months at £6 *per Cent. per Annum?*
Ans. £33 5 3.

20. £293 7 9 for 16 months at £6 *per Cent. per Annum?*
Ans. £23 9 5

21. £333 13 3¾ for 17 months, at £6 *per Cent. per Annum?*
Ans. £28 7 2¾

22. £517 6 6 for 18 months, at £6 *per Cent. per Annum?*
Ans. £46 11 2

23. £347 11 7½ for 19 months, at £6 *per Cent per Annum?*
Ans. £33 00 0½

24. £419 12 5 for 20 months, at £6 *per Cent. per Annum?*
Ans. £41 19 2¾

25. £537 13 5½ for 16 months, at £9 *per Cent. per Annum?*
Ans. £64 10 4¾

26. £197 19 1½ for 15 months, at £8 *per Cent. per Annum?*
Ans. £19 15 10¾

27. £217 10 4½ for 18 months, at £4 *per Cent. per Annum?*
Ans. £13 1 0¾

28. £327 15 9 for 16 months, at £3 *per Cent. per Annum?*
Ans. £13 2 2¾

29. £487 16 4½ for 30 months, at £2 *per Cent. per Annum?*
Ans. £24 7 9¾

30. £1 for 1 year, at 6 *per Cent. per Annum?*
Ans. 1*s.* 2*d.* 1$\frac{6}{10}$*qr.*

31. £517 12 8½ for 5 years, 11 months and 25 days, at £6. *per Cent. per Annum?* *Ans.* £186 19 9¼

A

A TABLE of RATIOS, from *one* pound, &c. to *ten* pounds.

Rate per Cent.	*Ratios.*	*Rate per Cent.*	*Ratios.*	*Rate per Cent.*	*Ratios.*
1	,01	4	,04	7	,07
$1\frac{1}{4}$	,0125	$4\frac{1}{4}$	,0425	$7\frac{1}{4}$	,0725
$1\frac{1}{2}$	,015	$4\frac{1}{2}$	,0 5	$7\frac{1}{2}$	,075
$1\frac{3}{4}$	,0175	$4\frac{3}{4}$	,0475	$7\frac{3}{4}$	,0775
2	,02	5	,05	8	,08
$2\frac{1}{4}$	,0225	$5\frac{1}{4}$	,0525	$8\frac{1}{4}$	,0825
$2\frac{1}{2}$	,025	$5\frac{1}{2}$	,055	$8\frac{1}{2}$	,085
$2\frac{3}{4}$	,0275	$5\frac{3}{4}$	,0575	$8\frac{3}{4}$	,0875
3	,03	6	,06	9	,09
$3\frac{1}{4}$	,0325	$6\frac{1}{4}$	,0625	$9\frac{1}{4}$	,092
$3\frac{1}{2}$	,035	$6\frac{1}{2}$	,065	$9\frac{1}{2}$	,095
$3\frac{3}{4}$	, 375	$6\frac{3}{4}$	,0675	$9\frac{3}{4}$	,0975
				10	1,

RATIO is the Simple Intereſt of £1 for 1 year, at the rate *per Cent.* agreed on.

A TABLE *for the ready finding the decimal parts of a year, equal to any number of* days, *or* quarters *of a* year.

Days	*Decimal parts*	*Days*	*Decimal parts*	*Days*	*Decimal parts.*
1	,00274	10	,027397	100	,273973
2	,005479	20	,054794	200	,547945
3	,008219	30	,082192	300	,821918
4	,010959	40	,109589	365	1,00 000
5	,013699	50	,136986	$\frac{1}{4}$ *of a year*=,25	
6	,016438	60	,164383	$\frac{1}{2}$ *of a year*=,5	
7	,019178	70	,191781	$\frac{3}{4}$ *of a year*=,75	
8	,021918	80	,219178		
9	,024657	90	,246575		

CASE I.‡

The Principal, Time, and Ratio are given, to find the Intereſt, and Amount.

RULE.—Multiply the Principal, Time and Ratio continually together, and the laſt product will be the Intereſt, Commiſſion, Brokage, &c. to which add the Principal, and the ſum will be the amount.

EXAMPLES.

1. REQUIRED the amount of £537 10s. at £6 *per Cent. per Annum*, for 5 years?

Principal

‡ The following Theorems will ſhew all the poſſible caſes of Simple Intereſt: where p=principal, t=time, r=ratio, and a=amount.

I. $ptr.+p=a$. II. $\frac{a}{tr+1}=p$. III. $\frac{a-p}{tp}=r$. IV. $\frac{a-p}{rp}=t$.

$$\begin{array}{rr} \textit{Principal} & 537,5 \\ \textit{Multiply by the Ratio} = & ,06 \\ \hline \textit{Product} & 32,250 \\ \textit{Multiply by the Time} = & 5 \\ \hline \textit{Interest} = & 161,250 \\ \textit{Add the Principal} = & 537,5 \\ \hline \textit{Amount} = & £698,75 \\ & 20 \\ \hline & 15,00 \end{array}$$

Answer £698 15*s.*

Or, $537,5 \times ,06 \times 5 + 537,5 =$ £698 15*s.*

2. WHAT is the simple Interest of £917 16*s.* at £5 *per Cent. per Annum*, for 7 years? *Answer*, £321 4*s.* 7*d.*

3. WHAT is the amount of £391 17*s.* *at* £4½ *per Cent. per Annum*, for 3¼ years? *Answer*, £449 2*s.* 11½*d.*

4. WHAT is the amount of £235 3*s.* 9*d.* at £5¼ *per Cent. per Annum*, from March 5th, 1784, to November 23d. 1784? *Answer*, £244 2*s.* 4¼*d.*

5. IF my Correspondent is to have £2½ *per Cent*; what will his Commission on £785 15*s.* amount to? *Answer*, £19 12*s* 10½*d.*

6. WHAT will be the Interest and amount of £445 10*s.* in 3 years and 129 days at £8½ *per Cent. per Annum?*

Answer. Interest, £126 19*s.* 8½*d. and the amount*=£572 9*s.* 8½*d.*

7. IF a Broker disposes of a Cargo for me, to the amount of £637 10*s.* on Commission at 1¼ *per Cent.* and procures me another Cargo of the value of £817 15*s.* on Commission at £1¾ *per Cent.* What will his Commission, on both Cargoes, amount to? *Answer*, £22 5*s.* 6¾*d.*

CASE 2.

The Amount, Time, and Ratio given, to find the Principal.

RULE. Multiply the Ratio by the Time; add unity to the Product for a Divisor, by which sum divide the amount, and the quotient will be the Principal.

EXAMPLES.

1. WHAT Principal will amount to £1045 14*s.* in 7 years, at £6 *per Cent. per Annum?*

$$\begin{array}{rl} \textit{Ratio} = & ,06 \\ \textit{Multiply by the Time} = & 7 \\ \hline \textit{Product} = & ,42 \\ \textit{Add} & 1 \\ \hline \textit{Divisor} = & 1,42)1045,7(736,4084+ \end{array}$$

$=$ £736 8*s.* 2*d.*

Or,

Or, $\frac{1045,7}{,06\times7+1}=$ £736 8s. 2d. *Answer.*

2. WHAT Principal will amount to £3810 in 6 years, at £4½ *per Cent. per Annum?* *Answer,* £3000.

3. WHAT Principal will amount to £666, 9s 0¼d in 3½ years, at £5¼ *per Cent. per Annum?* *Answer,* £563.

4. WHAT Principal will amount to £335 7s. 3d. in 3 years and 97 days, at £9½ *per Cent. per Annum?* *Answer,* £247.

CASE 3.

The Amount, Principal, and Time given, to find the Ratio.

RULE. Subtract the Principal from the amount; divide the Remainder by the Product of the Time and Principal, and the Quotient will be the *Ratio.*

EXAMPLES.

1. AT what Rate *per Cent.* will £543 amount to £705 18s. in 5 years?

From the Amount = 705,9
Take the Principal = 543

Divide by 543×5=2715)162,90(,06
162 90

Or $\frac{705,9-543}{543\times5}$ =,06=£6 *the Answer.*

2. AT what Rate *per Cent.* will £391 17s. amount to £449 3s. 1¼,84qr. in 3¼ years? *Answer,* £4½.

3. AT what Rate *per Cent.* will £413 12s 6d amount to £546 3s. 8d. in 4¾ years? *Answer,* £6¾.

4. AT what Rate *per Cent.* will £3000 amount to £3810 in 6 years? *Answer,* £4½.

CASE 4.

The Amount, Principal, and Rate per Cent. given, to find the Time.

RULE. Subtract the Principal from the Amount; divide the Remainder by the Product of the Ratio and Principal; and the Quotient will be the Time.

EXAMPLES.

1. IN what Time will £543 amount to £705 18s. at £6 *per Cent. per Annum?*

From the Amount = 705,9
Take the Principal = 543

Divide by 543×,06 = 32,58)162,9(5 *years, Answ.*
162 9

2. In what time will £3000 amount to £3810 at $4\frac{1}{2}$ *per Cent. per Annum?* *Answer*, 6 *years.*

3 In what time will £391 17*s.* amount to £449 3/$1\frac{3}{4}$,84*qr.* at £$4\frac{1}{2}$ *per Cent. per Annum?* *Answer*, $3\frac{1}{4}$ *years.*

To find the Interest of any sum, at 6 *per Cent. per Annum*, for any number of months.

Rule. If the months be an even number, multiply the principal by half that number: and if the months be uneven, halve the even months, to which annex $\frac{5}{10}$; thus, the half of 19 is 9,5 and multiply the Principal as before, cutting off two figures more at the right hand, than there are decimals in both factors, which reduce to farthings, each time cutting off as at first

4. What is the Interest of £345 16/6 for 9 years and 11 months, at 6 *per Cent. per Annum?*

```
                    Y.  M.
                    9   11
                   12
                  ------
                  2)119 months.
                  ------
        345,825    59,5 = ½ No. of months
           59,5
        -------
        1729125
       3112425
      1729125
      ---------
£205,765875 = £205  15  3¾ Ans.
  Principal = £345  16  6
              -----------
     Amount = £551  11  9¾
```

A Table *of decimal parts for every day in the twelfth part of a year, which consists of* $365\frac{1}{4}$ *days.*

Days	dec. pts.	Days	dec. pts.	Days	dec. pts.	Days	dec. pts.	Days	dec. pts.
1	,033	7	,23	13	,427	19	,624	25	,821
2	,066	8	,263	14	,4 6	20	,657	26	,854
3	,098	9	,296	15	,493	21	,69	27	,887
4	,131	10	,328	16	,526	22	,723	28	,92
5	,164	11	,361	17	,558	23	,756	29	,953
6	,197	12	,394	18	,591	24	,788	30	,986

To find the Interest of any sum, either for months, or months and days, at 6 *per Cent. per Annum.*

Rule.

Multiply the Principal by the number of months (or months and parts, answering to the given number of days in the Table) and

and cut off one figure at the right hand of the product more than is required by the rule in decimals, and the product will be the Interest for the given time in shillings and decimal parts of a shilling.

1. WHAT is the Interest of £100 for a year?

Principal = 100
Multiply by the months = 12

Answer, s. 120|0=£6.

Note, This table may also be used for the *parts of a year*; in Compound Interest, after having worked for whole years.

2. WHAT is the Interest of £250 10/ for 19 months and 7 days?

Principal = £250,5
Time = 19,23

7515
5010
22545
2505

Ans. £481,7115
=£24 1/9 7/10

Another method of calculating Interest for months, at £6. per Cent. per Annum.

RULE.

IF the Principal consist of pounds only, cut off the unit-figure, and, as it then stands, it will be the Interest for one month in shillings and decimal parts:—If it consist of pounds, shillings, &c. reduce the shillings, &c. to decimals, which, with the unit-figure of the pounds, will be decimal parts of a shilling.

EXAMPLES.

1. WHAT is the Interest of £175 for 5 months?

£175=17,5*shil.*=Interest for 1 month.

17,5
Multiply by the time= 5

2|0|87,5

Ans.=£4 7/6

2. WHAT is the Interest of £255 16/. for 7 months.

Shill.
£255 16=25,58 *Int. for* 1 *month*.
7

2,0|179,06

£8 19/0½ *Ans*.

To find the Interest of any Sum, at £6 *per Cent. per Annum*, in Dollars.

RULE.

DIVIDE the given Sum by 5, and the Quotient will be the Interest for a year in dollars and decimal parts of a dollar. Or multiply the Principal by 2, and the Product, (having the unit-figure of the pounds cut off) will be the Interest, as before.

EXAMPLES.

EXAMPLES.

1. WHAT is the Intereſt, in dollars, of £175 for a year?

£175
2

Anſ. 35,0 *Dollars,*

Or, 5)175

35 *Dollars as before.*

2. WHAT is the Intereſt of £255 16ſ. in dollars for a year?

5)255,8

51,16 *dol. as before.*

Or £255 16=255,8
2

Anſ. 51,16 *Dollars.*
6

96=51 1/6 *dollars,*

Rules for calculating Intereſt for days.

RULE 1.

MULTIPLY the given Principal by the given number of days, and that Product by the Rate on the pound: divide the laſt Product by 365 (the number of days in a year) and it will give the Intereſt.

EXAMPLE.

WHAT is the Intereſt of £360 10ſ for 175 days, at 6 *per Cent.*?

$$\frac{360,5 \times 175 \times ,06}{365} = £10,37 = £10\ 7/4\tfrac{3}{4}\ \textit{Anſw.}$$

RULE 2.

MULTIPLY the given principal by the given number of days, and divide the Product by 6083, for £6 *per Cent*; (the number of days in which any Sum will double, at that Rate) the Quotient will give the Anſwer.

EXAMPLE.

WHAT is the Intereſt of £327 10ſ at 6 *per Cent. per Annum* for 210 days?

$$\frac{327,5 \times 210}{6083} = £11,3 = £11\ 6ſ\ \textit{Anſw.}$$

Rule for making a Diviſor for any Rate per Cent.

MULTIPLY 365 by 100, and divide the product by the rate.

Thus, for 6 *per Cent.* $\frac{365 \times 100}{6} = 6083$ *diviſor.*

For 5 *per Cent.* $\frac{365 \times 100}{5} = 7300$ *diviſor, &c.*

PERHAPS

PERHAPS the most convenient way to calculate at 6 *per Cent.* is first to do it for 5, and then add one-fifth of the quotient to itself; because by cutting off the two cyphers in the divisor, you have to divide only by 73.

HENCE, when Interest is to be calculated on cash accounts, accounts current, or any other accounts, where partial payments are made, or partial debts are contracted; multiply the several balances into the days they are at interest, and the sum of these products, divided as above, will give the interest at £5 or £6 *per Cent.* and for any other rate, make the proper addition or deduction; or find a divisor, as before directed.

EXAMPLE.

ON the 1st of January I lent £450 10 6, which I received back in the following partial payments, viz. on the 14th of January, £57 11 9; on the 7th of February, £39 3 10; on the 19th of March, £63 5 2, on the 4th of April, £45; on the 26th of April, £19 12 6; on the 12th of May, £100, on the 10th of June, £60 7 3; and on the first of August, £65 10; what Interest is due at 6 *per Cent.*?

Dates.				£.	s.	d.	Days.	Products. £.	s.	d.
January	1	Lent on demand - - -		450	10	6	14	5607	7	0
	14	Received in part - -		57	11	9				
			Balance	392	18	9	24	9430	10	
February	7	Received in part - - -		39	3	10				
			Bal.	353	14	11	40	14149	16	8
March	19	Received in part - - -		63	5	2				
			Bal.	290	9	9	16	4647	16	
April	4	Received in part - - -		45	0	0				
			Bal.	245	9	9	22	5400	14	6
Ditto	26	Received in part - - -		19	12	6				
			Bal.	225	17	3	16	3613	16	
May	12	Received in part - - -		100	0	0				
			Bal.	125	17	3	29	3650	0	3
June	10	Received in part - - -		60	7	3				
			Bal.	65	10	0	62	4061	0	0
August	1	Received in full of the principal		65	10	0		50560	13	5

```
                          (5)
73,00)505,60  13  5(6  18  6¼  Int. at 5 per Cent.
      438              1   7  8½
      6760          £8   6  2¾  ditto at 6 per Cent.
        20
     )1352,13
      73
      622
      584
       3813
         12
      )457,61
       438
       1961
          4
      )7844
       73
       544
```

I have given Peter Trusty a Cash Credit for £1000, in consequence of which, on the 12th of May, I paid his bill for £250; ditto 27th, paid his draught for £280; June 21st, he gave me a bill on the Massachusetts Bank at sight for £290; July 17th, he paid me *per Receipt* £70; August 20th, he drew for £750 at sight; ditto 31st, he paid me *per Receipt*, £500; Sept. 15th he drew at sight for £135; and 3d of October for £175; October 29th, he paid me *per Receipt* £250; and November 3d. £125; November 12th, he drew at sight for £375, and ditto 18th, for £125; January 1st he paid me *per Receipt* £290, and 20th ditto £210.—On the 1st of March following he demands a settlement; what is then due to me, Interest at 6 *per Cent.*?

Dates.				£.	*Days.*	*Products.*
May	12	Paid his Bill - -		250	15	3750
Ditto	27	Paid his Draught -		280		
			Bal.	530	5	2650
June	1	Received in part - -		290		
			Bal.	240	46	11040
July	17	Received in part - -		70		
			Bal.	170	34	5780
August	20	Paid - - - - -		750		
			Bal.	920	11	10120
Ditto	31	Received in part - -		500		
			Bal.	420	15	6300
September	15	Paid - - - - -		135		
			Bal.	555	18	9990
October	3	Paid - - - - -		175		
			Bal.	730	26	18980
Ditto	29	Received in part - -		250		
			Bal.	480	5	2400 — 71010
November	3	Received in part - -		125		
			Carried over *Bal.*	355		

Dates,

Dates.				£.	Days.	Products.
						71010 brought over.
			Bal.	355	9	3195
November	12	Paid - - - - - -		375		
			Bal.	730	6	4380
Ditto	18	Paid - - - - - -		125		
			Bal.	855	62	53010
January	1	Received in part - - -		290		
			Bal.	565	19	10735
Ditto	20	Received in part - - -		210		
March	1		Bal.	355	40	14200
						156530

Then, 73,00)1565,30

5)21 8 10 *Interest at £5 per Cent. per Annum.*
4 5 10

25 14 8 *Int. at £6 per Cent. on the above account.*
355 *Balance of the account.*

£380 14 8 *Total Balance in my favor.*

When cash credits are given, a balance should be made upon every transaction, which should be multiplied into the days the first leisure minute, then, when the time of settlement comes, you will only have to add up the products, and divide as above, and the account will be finished.

A owes B the following sums, with the Interest on them, at 6 *per Cent. per Annum*, as follows; viz. £60 for 7 months, £150 for 15 months, £75 10*s*. for 9 months, £145 15*s*. for 27 months, and £397 12*s*. for 45½ months: What is the amount of Principal and Interest?

£.		Months.		
60	×	7	=	420
150	×	15	=	2250
75,5	×	9	=	679,5
144,75	×	27	=	3935,25
397,6	×	45,5	=	18090,8

£
200)25375,55(126,877 *Interest.*
828,85 *Principal.*

£955,727 *Amount, Answer.*

Note. I divide by 200, the number of months, in which any sum will double at 6 *per Cent. per Annum*, and it gives the Interest.

When partial payments are made upon Notes, Bonds, &c. at any interval *greater* than a year, the Interest is calculated in a progressive manner, by adding the Interest to the principal at the time of the first payment, and from the sum deducting the payment, &c.

ANNUITIES,

ANNUITIES, *or* PENSIONS *in* ARREARS, *at* SIMPLE INTEREST.

AN Annuity is a sum of money, payable every year, half year, or quarter, for a certain number of years, or forever.

WHEN the Debtor keeps the annuity in his own hands, beyond the time of payment, it is said to be in *Arrears*.

THE sum of all the annuities, for the Time they have been forborne, together with the Interest due upon each, is called the *Amount*.

IF an annuity is to be bought off, or paid all at once, at the beginning of the first year, the price, which ought to be given for it, is called the *Present worth*.

CASE I.

When the Annuity, Time, and Rate of Interest are given, to find the Amount.

RULE. †

1. FIND the sum of the natural series of numbers 1, 2, 3, 4, &c. to the number of years less one.

2. MULTIPLY this sum by one year's Interest of the annuity, and the product will be the whole interest due upon the annuity.

3. To this Product add the Product of the annuity and time, and the sum will be the amount sought.

EXAMPLES.

1. WHAT is the amount of an annuity of £90 for 6 years, allowing simple interest, at 6 *per Cent. per Annum?*

$1+2+3+4+5=15=3\times5=$ *Sum of the number of years less one.*

	£.	s.	
	5	8	= 1 *year's interest of the annuity.*
		3	
	16	4	
		5	
	81		= *Whole Interest due.*
Add	540		$=90\times6=$ *Annuity multiplied by the time.*
	621		= *Amount required.*

2. IF

† WHATEVER the time may be, there is due upon the first year's Annuity so many years' interest as the whole number of years less one, and gradually one less upon every succeeding year to the last but one, and none upon the last; therefore, in the whole, there is due so many years' interest of the annuity, as the sum of the series 1, 2, 3, 4, &c. to the number of years less one; consequently one year's interest, multiplied by this sum, must be the whole interest due; to which, if all the annuities be added, the sum is plainly the amount.

LET $r=$ ratio, $n=$ annuity, $t=$ time, and $a=$ amount:—Then will the following Theorems give the solution of all the different cases.

I.

2. If a house be let upon a lease for 8 years, at £36 *per Annum*, I demand the amount for that time, at 5 *per Cent. per Annum?*

3. If a salary of £120, payable every half year, remain unpaid 7 years; what would it amount to in that time, at £6 *per Cent. per Annum?* * — — *Ans.* £1003 16*s.*

4. If a salary of £120, payable every quarter, remain unpaid 7 years; what would it amount to in that time, at £6 *per Cent. per Annum?* — — *Ans.* £1010 2*s.*

Present Worth *of* ANNUITIES *at* Simple Interest.

Case I.

The Annuity, Rate, and Time given, to find the present worth.

Rule.‡

Find the present worth of each year, by itself, discounting from the time it falls due, and the sum of all these will be the present worth required.

L l

Examples.

I. $\frac{2a-2tn}{t^2n-tn}=a.$ II. $\frac{2a}{t^2r+2t-tr}=n.$ III. $\frac{2a-2tn}{t^2n-tn}=r.$

IV. $\left(\frac{2a}{rn}+\frac{d^2}{4}\right)^{\frac{1}{2}}-\frac{d}{2}=t.$ *In the last Theorem,* $d=\frac{2n-rn}{rn}$ and in Theorem first, if a sum cannot be found equal to the amount, the problem is impossible in whole years.

* In case 1, when the annuities, &c. are to be paid half yearly, or quarterly; then for half yearly payments, take $\frac{1}{2}$ of the Ratio, $\frac{1}{2}$ the annuity, &c. and twice the number of years.

For quarterly payments, take $\frac{1}{4}$ of the ratio, $\frac{1}{4}$ of the annuity, &c. and 4 times the number of years, and work as before.

‡ If we grant the condition of allowing Simple Interest to be consistent, the reason of this rule will be evident from the nature of discount; for all the Annuities may be considered separately as so many single and independent debts, due after 1, 2, 3, &c. years; so that the present worth of each being found, their sum must be the present worth of the whole.

But the purchasing of Annuities by Simple Interest is unjust and absurd, which may easily be made to appear by one instance only.—The Price of an Annuity of £100, to continue 30 years, discounting at 6 *per Cent.* will amount to £2000 nearly, the Interest of which for one year only exceeds the Annuity: Would it not therefore be highly absurd to give a sum, which would yield me nearly £120 yearly, forever, for an Annuity of £100, to continue only 30 years?

Let p=present worth, and the other letters as before.

Then $\begin{cases} n\times\frac{1}{1+r}+\frac{1}{1+2r}+\frac{1}{1+3r} \text{ \&c. to } \frac{1}{1+tr}=p \\ p\div\frac{1}{1+r}+\frac{1}{1+2r}+\frac{1}{1+3r} \text{ \&c. to } \frac{1}{1+tr}=n \end{cases}$

Most

EXAMPLES.

1. WHAT is the present worth of £400 *per Annum*, to continue 6 years, at 6 *per Cent. per Annum*?

$$\left.\begin{array}{r}106\\112\\118\\124\\130\\136\end{array}\right\} : 100 :: 400 : \left\{\begin{array}{l}377,35849 = \textit{Present worth of one year.}\\357,14285 = \text{—} \quad \text{—} \quad \textit{2d. year.}\\338,98305 = \text{—} \quad \text{—} \quad \textit{3d. year.}\\322,58064 = \text{—} \quad \text{—} \quad \textit{4th. year.}\\307,6923 = \text{—} \quad \text{—} \quad \textit{5th. year.}\\294,11764 = \text{—} \quad \text{—} \quad \textit{6th. year.}\end{array}\right.$$

1997,87497 = £1997 17*s.* 5¾*d.* = *pres. worth.*

2. WHAT is the yearly rent of a house of £36, to continue 8 years, worth in ready money, at 5 *per Cent. per Annum*?

Ans. £237 3*s.* 9½*d.*

DISCOUNT

Is an allowance made for the payment of any sum of money, before it becomes due, and is the difference between that sum, due some time hence, and its present worth.

THE *present worth* of any sum, or debt, due some time hence, is such a sum, as, if put to Interest, would in that time and at the rate *per Cent.* for which the discount is to be made, amount to the sum or debt, then due.

RULE.*

As the *Amount* of £100, for the given rate and time, is to £100: So is the given sum or debt, to the present worth.

SUBTRACT

Most Authors give the following Theorems, viz.

I. $\dfrac{t^2r - tr + 2t}{2tr + 2} \times n = p$. *II.* $\dfrac{tr + 1}{t^2r - tr + 2t} \times 2p = n$.

III. $\dfrac{\overline{2nt - p} \times 2}{2pt - nt^2 - nt} = r$. *IV.* $\dfrac{2}{r} - \dfrac{2p}{n} - 1 = x$. then, $\sqrt{\dfrac{2p}{nr} + \dfrac{x^2}{4}} - \dfrac{x}{2} = t$.

Note. The same thing is to be observed in this Case as in the first Case of Annuities in Arrears, respecting half-yearly and quarterly payments.

I have shewn the method of computing Annuities by simple Interest, more for the gratification of the curious than for real utility, it being not only customary; but also most equitable to allow compound Interest.

IT may be observed that the Theorem $\dfrac{t^2r - tr + 2t}{2tr + 2}$ gives the answer to the first question = £2028, which therefore appears to be erroneous.

* THAT an allowance ought to be made for paying money before it becomes due, which is supposed to bear no Interest till after it is due, is very reasonable: for if I keep the money in my own hands till the debt shall become due, it is plain I may make an advantage of it by putting it out to interest for that time; but if I pay it before it is due,

Subtract the present worth from the given sum, and the remainder will be the discount required.

Or

As the amount of £100, for the given rate and time, is to the Interest of £100 for that time: So is the given sum or debt to the discount required.

Examples.

1. **What** is the discount of £635 17*s.* due two years hence, at $5\frac{1}{2}$ *per Cent. per Annum?*

Int. of £100 *per Annum* } = 5 10
2 *years*,

11
Add 100 £. £. *s.*

As £111 : 11 :: 635 17
20 20

2220 12717
11

12717
12717

222|0) 13988|7 (£63 *discount.*
1332

668
666

27
20

222|0) 540 (0
12

222|0) 648|0 (2
444

204

Or,

£. £. £. *s.* £. *s.* *d.*
As 111 : 100 :: 635 17 : 572 16 $9\frac{1}{4}$
present worth.
£. £. £.
And 635 17 — 572 16 $9\frac{1}{4}$ = 63 0 $2\frac{3}{4}$
discount.

Ans. £63 0 $2\frac{3}{4}$*d.*

2. **What**

due, I give that benefit to another; therefore we have only to enquire what discount ought to be allowed. And here, many suppose that, since by not paying till it becomes due, they may employ it at interest, therefore by paying it before due, they shall lose that interest, and for that reason all such interest ought to be discounted; but the supposition is false; for they cannot be said to lose that Interest till the time arrives, when the debt becomes due; whereas we are to consider what would properly be lost, at present by paying the debt before it becomes due; this can, in point of equity, be no other than such a sum, which being put out at Interest till the debt shall become due, would amount to the Interest of the debt for the same time.—It is besides plain that the advantage

2. WHAT is the present worth of £350, payable in ½ a year, discounting at £6 *per Cent. per Annum?* *Ans.* £339 16 1¼

3. WHAT is the present worth of £65 due 15 months hence, at £6 *per Cent. per Annum?* *Ans.* 60 9 3½

4. WHAT is the discount on £97 10*s*. due January 22d. this being September 7th, reckoning Interest at £5 *per Cent?*

Ans. £1 16*s* 0 ¾*d*.

5. WHAT ready money will discharge a debt of £475 10*s*. due 5 months and 20 days hence, at 6 *per Cent.* *Ans.* £462 7 11½

6. BOUGHT a quantity of goods for £250 ready money, and sold them for £300, payable 9 months hence: what was the gain, in ready money, supposing discount to be made at £6 *per Cent?*

Ans. £37 1 7½

7. WHAT is the present worth of £275, payable as follows; viz ½ at 3 months, ⅓ at 6 months, and the rest at 9 months, supposing the discount to be made at £6 *per Cent?* *Ans.* £268 6*s* 6¾.

ABBREVIATIONS *in* DISCOUNT.

ANY Principal to be discounted for one year, at any of the following Rates, (or for any Rate and Time, whose Product is equal to any of the following Rates) being (multiplied by the multiplier, if any, and) divided by the corresponding Divisor, the Quotient will be the Discount.

	Rates.	
At	1¼	÷81 *(or by* 9 *and* 9*)*
	2	÷51
	2½	÷41
	4	÷26
	5	÷21 *(or by* 7 *and* 3*)*
	6	÷53 *and* × 3
	7½	÷43 *and* × 3
	8	÷27 *and* × 2 *(or* × 2, *and* ÷ 9 *and* 3
	8⅓	÷13
	10	÷11
	12	÷28 *and* × 3 *(or* × 3, *and* ÷ 7 *and* 4
	12½	÷9

8. HOW

vantage arising from discharging a debt due some time hence, by a present payment according to the principles above-mentioned, is exactly the same as employing the whole sum at Interest till the time, when the debt becomes due, arrives: For if the discount allowed for present payment be put out at Interest for that time, its *amount* will be the same as the Interest of the whole debt for the same time; thus the discount of £106, due one year hence, reckoning Interest at £6 *per Cent.* will be £6, and £6 put out to Interest at £6 *per Cent.* for one year will amount to £6 7*s*. 2¼*d*. which is exactly equal to the Interest of £106 for one year at £6 *per Cent.*

The truth of the rule for working is evident from the nature of Simple Interest; for since the debt may be considered as the *amount* of some Principal (called here the present worth) at a certain rate *per Cent.* and for the given time, that amount must be in the same proportion either to its Principal or Interest, as the amount of any other sum, at the same rate, and for the same time, to its Principal or Interest.

8. How much must I abate of £5394 10s. due 3 years hence at $2\frac{2}{3}$ *per Cent. per Annum?*

5394 10
2

$2\frac{2}{3}$
×3

∴ 27÷3=9) 10789 0

8, *therefore*, × 2, and ÷ 27 = 3) 1198 15 $6\frac{2}{3}$

£399 11 10 *Ans.*

9. What is the discount of £546 12/6, for $8\frac{1}{3}$ years at 1 *per Cent. per Annum*, (or for 1 year at $8\frac{1}{3}$ *per Cent. per Annum?*)

13) 546 12 6

£42 0 $11\frac{1}{2}$ *Ans.*

10. What is the discount of £125 at $\frac{1}{4}$ *per Cent. per Annum*, for 4 years, (or, at 4 *per Cent. per Annum*, for $1\frac{1}{4}$ year.

$1\frac{1}{4}$
×4

5, *therefore*, ÷ 21) 125

Ans. £5 19 $6\frac{1}{4}$

DISCOUNT BY DECIMALS.*

The Sum to be discounted, the Time and the Ratio given, to find the present worth.

Rule. Multiply the Ratio by the Time, add unity to the Product for a Divisor: by which sum divide the sum to be discounted, and the quotient will be the present worth.

Subtract the present worth from the principal or sum to be discounted, and the remainder will be the discount.

Or, as the amount of £1 for the given time, is to £1, so is the Interest of the debt for the said time, to the discount required.

Subtract the discount from the principal, and the remainder will be the present worth.

Examples

* As in Simple Interest, let a= amount of any debt, p= present worth, t= time, and r= ratio; then will the following Theorems exhibit all the cases in Discount at simple Interest.

I. $\frac{a}{tr+1}=p$. II. $ptr+p=a$. III. $\frac{a-p}{tp}=r$. IV. $\frac{a-p}{rp}=t$.

Note. When the ratio is ,06 *per Cent. per Annum*, and the given time is expressed in months, whether less or more than a year, if the debt be divided by 1 plus half as many hundredths of an unit, as there are months in the given time, the quotient will be the present worth.—Thus, for 1 month $\frac{a}{1,005}$, 2 months $\frac{a}{1,01}$, 3 months $\frac{a}{1,015}$, 36 months $\frac{a}{1,18}$, 42 months $\frac{a}{1,21}$, &c. &c.

EXAMPLES.

First method.

1. What is the present worth of £600, due 3 years hence at £6 *per Cent. per Annum?*

First method.

Ratio =,06
Multiply by the time = 3

Product =,18
Add 1

Divisor=1,18)600(508,4745 £ *present worth.*

$Or, \frac{600}{,06\times3+1}$=£508 9*s.* 5¾*d. Ans.*

Present worth=508,4745=£508 9/5¾*d.* which subtracted from the Principal, will give the discount=£91 10 6¼

Second method.

What is the discount of £600, due 3 years hence at £6 *per Cent. per Annum?*

Ratio =,06
Multiply by 3

,18
Add 1

Amount of £1, *for the given time* } =1,18

As 1,18:1::108
1
1,18)108,00(91,5254

And 600×,06×3=108= *Interest of the debt for the given time.* *Discount*=91,5254=£91 10*s.* 6*d.* which taken from the principal will leave the present worth=£508 9*s.* 6*d.*

2. What is the present worth of £312 10*s.* due 2 years hence at £4½ *per Cent. per Annum?* *Ans.* £286 13*s.* 11¼*d.*

3. What is the present worth of £1650 15*s.* 6*d.* at £6¾ *per Cent. per Annum?* due 18 months hence. *Ans.* 1499 0*s.* 0¼

4. What ready money will discharge a debt of £1354 8*s.* 0*d.* due 3 years, 3 months and 12 days hence, at £5⅞ *per Cent. per Annum?* *Ans.* £1135 17*s.* 3½*d*

BARTER

Is the exchanging of one Commodity for another, and teaches traders to proportion their quantities without loss.

CASE I.

When the quantity of one Commodity is given, with its value, or that of its Integer, that is of 1*lb.* 1 *Cwt.* 1 *yd. &c. as also the value of the Integer of some other commodity, to be given for it, to find the quantity of this; or, having the quantity thereof given, to find the rate of selling it.*

RULE.

RULE.

FIND the value of the given quantity by the concisest method, then find what quantity of the other, at the rate proposed, you may have for the same money; *Or*, if the quantity be given, find from thence the rate of selling it.

EXAMPLES.

1. How much Tea at 9/6 *per lb.* must be given in barter for 156 gallons of wine, at 12*s.* $3\frac{1}{2}$ *per* gallon?

$$\begin{array}{l|l|r}
3d. & \frac{1}{4} & 156 \textit{ gallons.} \\
\frac{1}{2} & \frac{1}{6} & 12 \\
\hline
 & & 1872 \\
 & & 39 \\
 & & 6 \quad 6 \\
\hline
\end{array}$$

$$9/6 = 114d. \qquad 1917 \quad 6$$

$$\begin{array}{r}
12 \\
\hline
23010
\end{array}$$

$$\begin{array}{llll}
 & d. & lb. & d. \\
\textit{As} & 114 : & 1 :: & 23010 \\
 & & & 1
\end{array}$$

$$\begin{array}{l}
114)23010(201 \\
\quad 228 \\
\hline
\quad\quad 210 \\
\quad\quad 114 \\
\hline
\quad\quad\ \ 96 \\
\quad\quad\ \ 16 \\
\hline
\quad\quad 576 \\
\quad\quad\ \ 96 \\
\hline
114)1536(13\frac{54}{114} \quad \textit{Ans. } 201 lb.\ 13\frac{54}{114}\ oz. \\
\quad\ \ 114 \\
\hline
\quad\quad 396 \\
\quad\quad 342 \\
\hline
\quad\quad\ \ 54
\end{array}$$

2. How much Cloth, at 15*s.* 8*d.* *per* Yard, must be given for 5 *Cwt.* 3 *qrs.* 19 *lb.* of Steel, at 5 Guineas *per Cwt.*

Ans. 50 *yds.* 2 *qrs.* 2 *n.*

3. SUPPOSE A has 350 yards of linen, at 1*s.* 4*d.* *per* yard, which he

he would truck with B for Sugar, at 25*s.* 6*d.* *per Cwt.* how much Sugar will the linen come to?

Cwt. qrs. lb.
Ans. 18 1 $5\frac{2}{3}$.

4. A has Broadcloths at £12 10*s.* *per* piece, and B has mace, at 8*s.* *per lb.* how many pounds of mace must B give A for 35 pieces of Cloth?

Ans. $1093\frac{3}{4}$ *lb.*

5. A has $7\frac{1}{2}$ *Cwt.* of Sugar, at 8*d.* *per lb.* for which B gave him $12\frac{1}{2}$ *Cwt.* of Flour; what was the Flour rated at *per lb.* *Ans.* $4\frac{3}{4}$*d.*

CASE 2.

If the quantities of two commodities be given, and the rate of selling them, to find, in case of inequality, how much of some other commodity must be given.

RULE.

FIND the separate values of the two given commodities; subtract the less from the greater, and the difference will be the amount of the third commodity, whose quantity and rate may be easily found.

1. Two Merchants barter; A has 30 *Cwt.* of Cheese at 23*s.* 6*d.* *per Cwt.* and B has 9 Pieces of Broadcloth, at £3 15*s.* *per* piece; which must receive money, and how much? *Ans. B must pay A* £1 10*s.*

2. A and B would barter, A has 150 Bushels of wheat at 5*s.* 9*d.* *per* Bushel, for which B gives 65 Bushels of barley, worth 2*s.* 10*d.* *per* Bushel, and the balance in Oats at 2*s.* 1*d.* *per* Bushel;—what quantity of Oats must A receive from B?

Ans. 325 $\frac{15}{25}$ *Bushels.*

CASE III. *Sometimes, in bartering, one Commodity is rated above the ready money price; then to find the bartering price of the other, say,* As the ready money price of the one, is to its bartering Price; so is *that* of the other, to *its* bartering price: Next, find the quantity required, according to either the bartering or ready-money price.

EXAMPLES.

1. A has Ribbands at 2*s.* *per* yard ready money; but in barter he will have 2*s.* 3*d.* B has broadcloth at 32*s.* 6*d.* *per* yard ready money:—At what rate must B value his Cloth *per* yard, to be equivalent to A's bartering Price, and how many yards of Ribband, at 2*s.* 3*d.* *per* yard, must then be given by A for 488 yards of B's Broadcloth?

Ans. B's Broadcloth, at £1 16*s.* $6\frac{1}{2}$ *per yard,* 7930 *yards of Ribband.*

2. A and B barter; A has 150 Gallons of Brandy at 7*s.* 3*d.* *per* Gallon ready money, but in barter he will have 8*s.* *per* Gallon: B has linen at 3*s.* 6*d.* *per* yard ready money; How must B sell his linen *per* yard in proportion to A's bartering-price and how many yards are equal to A's Brandy?

yds. qrs. n.
Ans. Barter price is 3*s.* $10\frac{1}{4}$*d.* *and he must give A* 310 2 3

3. P

3. P and Q barter. P has Irish linen, at 3*s*. 7*d*. *per* yard; but in barter will have 3*s*. 10*d*. Q delivers him broadcloth at £1 16*s*. 6*d*. *per* yard, worth only £1 13*s*. *per* yard:—Pray, which has the advantage in Barter, and how much linen does P give Q for 148 yards of broadcloth?

Yd. *Yds.* *s.* *Yd.*
As 1 : 36/6 :: 148 : 5402 *Price of the broadcloth.* *As* 3/1 : 1 :: 5402*s* : 1409$\frac{3}{23}$ *yds. of linen.*—*Q has the advantage; for as* 3/7 : 3/10 :: 33/ : 35/3 $\frac{27}{43}$ *his proportional price.*

4. A has 200 yards of Linen, at 1*s*. 6*d*. ready money *per* yard, which he barters with B, at 1*s*. 9*d*. *per* yard, taking Buttons at 7½*d*. *per* Grofs, which are worth but 6*d*:—How many Grofs of Buttons will pay for the Linen; who gets the beft bargain, and by how much both in the whole and *per Cent*?

Yd. *d.* *Yds.* *d.* *d.* *Grofs* *d.* *Grofs.* *Yd.* *d.* *Yds.* £.
As 1 : 21 :: 200 : 4200. *As* 7½ : 1 :: 4200 : 560. *As* 1 : 18 :: 200 : 15
Value of A's Linen.

gr. *d.* *gr.* £.
As 1 : 6 :: 560 : 14 *Value of B's goods.* *So that B gained* £1 *of A.*
£. £. £. £.
As 14 : 1 :: 100 : 7 2*s*. 10*d*. *per Cent.*

5. A has linen Cloth, at 1*s*. 8*d*. *per* yard ready money, in Barter 2*s*. B has 3610 yards of Ribband, at 1*s*. 2*d*. *per* yard ready money, and would have of A £70 in ready money, and the reft in Linen Cloth; what rate does the Ribband bear in Barter *per* yard, and how much Linen muft A give B?

B muft have but 1*s*. 2*d*. *per yard, for fo much Ribband as he receives money for*—
s. *d.* *s.* *s.* *d.* *s.* *d.*
As 1 8 : 2 :: 1 2 : 1 4¾ *the bartering price of the Ribband per yard.*
s. *d.* *Yd.* £.
As 1 2 : 1 :: 70 : 1200 *yards, which A muft have for* £70.—*Then,*
Yd. *s.* *d.* *Yds.* *Yds.* £. *s.* *d.*
As 1 : 1 4¾ :: 3610—1200 : 168 3 11½
s. *Yd.* £. *s.* *d.* *Yds.*
As 2 : 1 :: 168 3 11½ : 1681$\frac{24}{96}$ *the quantity of linen required.*

s. *d.*
Anf. The rate of the Ribband is 1 4¾ *per yard, and B muft receive* 1681$\frac{24}{96}$ *yards, and* £70 *in Cafh.*

LOSS AND GAIN

Is an excellent Rule by which Merchants and Traders difcover their Profit, or Lofs *per Cent.* or by the Grofs:—It alfo inftructs them to raife or fall the price of their goods, fo as to gain or lofe fo much *per Cent.* &c.

QUESTIONS in this Rule are performed by the Rule of Three.

CASE I.

To know what is gained or loft per Cent.

RULE. First see what the Gain or Loss is, by Subtraction; then, as the price it cost, is to the Gain or Loss: So is £100 to the Gain or Loss *per Cent.*

EXAMPLES.

1. IF I buy Serge at 5*s.* *per* yard, and sell it again, at 5*s.* 8*d.* *per* yard; what do I gain *per Cent.* or in laying out £100?

	s.	d.
Sold for	5	8
Cost	5	0
		8 *gain per yard.*

d. d. £.
As 60 : 8 :: 100
8
60)800(13
60
200
180
20
20
60)400(6
360
40
12
60)480(8
480
Ans. £13 6/8

N. B. The first questions in the several Cases, serve to elucidate each other.

2. IF I buy Serge at 5/8 *per* yard, and sell it again at 5/ *per* yard; what do I lose *per Cent.* or in laying out £100?

	s.	d.
Cost	5	8
Sold for	5	0
Loss per yd. 8*d.*		

d. d. £.
As 68 : 8 :: 100
8
68)800(11
68
120
68
52
20
68)1040(15
68
360
340
20
12
68)240(3
204
36
4
68)144(2
136
8 *Ans.* £11 15 3½.

3. If I buy a *Cwt.* of Tobacco for £9 6/8, and sell it again at 1/10 *per* ℔. do I gain or lose, and what *per Cent.*?

£. s. d.
℔ *Sold for* 10 5 4
112 *Cost* 9 6 8
—
£. s. 0 18 8 *gained in the gross.*
| 2*d.* | $\frac{1}{12}$ | 11 4 *Value at* 2*s per* ℔.
— 18 8 *Value at* 2*d. per* ℔.
10 5 4 *Value at* 1*s.* 10*d. per* ℔.
£. s. d. s. d. £. £.
As 9 6 8 : 18 8 :: 100 : 10 *Ans.* £10 *per Cent.*

4. A Draper bought 60 yards of Cloth at 28*s per* yard, and 38 yards of ditto at 14*s. per* yard, and sold them one with another, at 26*s. per* yard: Did he gain or lose, and what *per Cent.*?

yd. s. yds. £. yd. s. yds. £. s.
As 1 : 28 :: 60 : 84 *As* 1 : 26 :: 98 : 127 8
yd. s. yds. £. s. £. s.
As 1 : 14 :: 38 : 26 12 *Sold for* 127 8
Prime cost 110 12
£110 12
Gain in the Gross 16 16
£. s. £. s. £. £. s. d.
As 110 12 : 16 16 :: 100 : 15 3 9½ *per Cent.*

5. Bought Sugar at 6½*d. per* ℔. and sold it at £2 3/9 *per Cwt.* what was the Gain or loss *per Cent.*?

℔ ℔
As 1 : 6½*d* :: 112 : £3 0/8
Prime cost £3 0 8 *per Cwt.* £. s. d. s. d. £. £. s. d.
Sold at 2 3 9 *per Cwt.* *As* 2 3 9 : 16 11 :: 100 : 38 13 4 *Loss per Cent. Ans.*
Lost £0 16 11 *in the whole.*

6. At 2½*d.* in the shilling profit; how much *per Cent.*?

s. d. £. £. s. d.
As 1 : 2½ :: 100 : 20 16 8 *Ans.*

7. At 4*s.* 6*d.* in the pound profit; how much *per Cent*?

£. s. d. £. £. s.
As 1 : 4 6 :: 100 : 22 10 *Ans.*

8. If I buy Candles, at 1*s.* 6*d. per* ℔ and sell them again, at 2*s. per* ℔. and allow 3 months for payment; what do I gain *per Cent*?

d. d. £. £. s. d. Mo. £. Mo. £. s.
As 18 : 24 :: 100 : 133 6 8; *then by Discount.* *As* 12 : 6 :: 3 : 1 10
£. s. £. s. £. s. d. £. s. d.
Then, As 101 10 : 1 10 :: 133 6 8 : 1 19 4½, *which taken from* £133 6/8 *leaves* £131 7/3½, *therefore, Ans.* £31 7/3½.

9. If

9. If I buy Cloth at 6*s.* *per* yard, ready money, and sell it again at 6*s.* 8*d.* *per* yard on 3 months credit: what is gained *per Cent.*?

s. d. s. | s. d. £. £. s. d. | Mo. £. Mo. £. s.
6 8—6=8*d.* *As* 6 : 8 :: 100 : 11 2 $2\frac{1}{2}$. *As* 12 : 6 :: 3 : 1 10

£. s. £. s. £. s. d. s. d.
As 101 10 : 1 10 :: 11 2 $2\frac{1}{2}$: 3 $6\frac{3}{4}$; *therefore*

£. s. d. s. d. £. s. d.
11 2 $2\frac{1}{2}$—3 $6\frac{3}{4}$=10 8 $7\frac{3}{4}$ *Answer.*

10. If I buy Cloth at 13*s.* *per* yard, on 8 months credit, and sell it again at 12*s.* ready money, do I gain or lose and what *per Cent*?

Mo. £. Mo. £. £. s. £. s. d.
As 12 : 6 :: 8 : 4. *As* 104 : 13 :: 100 : 12 6 *So that* 13*s.* *on* 8 *months credit at* £6 *per Cent. is equal to* 12*s.* 6*d.* *ready money*; *then,*

	s.	d.			
Prime Cost	12	6	*ready money,*		s. d. d. £. £. s. d.
Sold at	12	0	*ready money,*		*As* 12 6 : 6 :: 100 : 7 13 $10\frac{12}{78}$.
Lost	0	6	*in the yard.*		*Ans. Lost* £7 13*s*/$10\frac{12}{78}$ *per Cent.*

11. If I buy Gloves at 7*s.* 4*d.* *per* pair; how long credit must I have, to gain £13 *per Cent.* when I sell them at 8*s.* *per* pair ready money?

	s. d.	
Sold at	8 0	s. d. d. £. £. s d. — *As* 7 4 : 8 :: 100 : 9 1 $9\frac{3}{4}$ *Gain per Cent. ready money.*
Prime cost	7 4	£. s. d. £. s. d. — *Then* £13—9 1 $9\frac{3}{4}$=3 18 $2\frac{1}{4}$ *Now,*
Gain'd	8 *per pair.*	£. Mo. £. s. d. Mo. d. h. — *As* 6 : 12 :: 3 18 $2\frac{1}{4}$: 7 24 13 *Answ.*

In casting up the amount of goods bought, imported or exported; to the prime cost of such goods we must add all the charges upon them, in order to fix the price they stand us in.

12. Suppose I import from France 12 Bales of Cloth, each Bale containing 10 Pieces, which, with the charges there, amounted to £108; I pay duty here 5/6 *per Piece,* for freight £3 10*s*, and porterage 7/6; what does it stand me in *per* piece, and how must I sell it *per* piece to gain £10 per Cent.?

12 *Bales.*
Each 10 *Pieces.*

In all 120 *Pieces at* 5/6 *per piece,*

	£		
First Cost	108	0	-
Duty	33	—	-
Freight	3	10	-
Porterage	—	7	6
Whole cost	£144	17	6

Pie. £. s. d. Pie.
As 120 : 144 17 6 :: 1
1

120 { 12 } 144 17 6
120 { 10 } 12 1 $5\frac{1}{2}$

£1 4 $1\frac{3}{4}$ *cost per Piece*

£. £. £. s. d.
As 10|0 : 1|0 :: 1 4 $1\frac{3}{4}$
Divis. 10)1 4 $1\frac{3}{4}$ *cost per Piece.*
2 5 *gain per piece, at* 10 *per Cent.*
So I must sell it at £1 6/$6\frac{3}{4}$ *per Piece*

Case

CASE 2.

To know how a Commodity must be sold, to gain or lose so much per Cent.

RULE. As £100 is to the price; so is £100 with the profit added, or loss subtracted, to the gaining or losing Price.

1. IF I buy a quantity of Serge, at 5*s.* *per* yard; How must I sell it *per* yard to gain £13 6*s.* 8*d.* *per Cent.*?

EXAMPLES.

```
    £.     s.      £.  s. d.
As 100  :  5  : : 113  6  8
    20              20
  ----            ----
  2000            2266
    12              12
  -----           -----
 24000           27200
                     5
                 ------
     24,000)136,000(5
            120
            ---
             16
             12
            ---
     24,000)192,000(8
            192
            ---
            000
```

Or thus,

```
      £.   s. d.
     113   6  8
              5
     -----------
  s 5|66  13  4
      12
     ---
     792
Add    8 = 2/3 of of 12 = 13/4
     ---
  d.8|00

              s. d.
         Ans. 5  8 per yard.
```

2. IF a barrel of Powder cost £4 how must it be sold to lose £10 *per Cent.*?

```
    £.   £.    £.
As 100 : 4 : : 90
                4
              ----
        100)360(3
            300
            ---
             60
             20
            ----
       100)1200(12 Ans. £3 12s.
           1200
           ----
```

Or thus

```
    90
     4
  -----
 £3,60
    20
  -----
 s. 12,00
```

3. BOUGHT Cloth, at 15*s.* *per* yard, which not proving so good as I expected, I am content to lose £17½ *per Cent.* by it; How must I sell it *per* yard?

£. s. £. s. d.
As 100 : 15 :: 82½ : 12 4½ *Ans.* 12*s.* 4½*d.*

Or thus:
82 10
3×5=15
247 10
5
s 12|37 10
12+6=½ *of* 12×10*s.*
d. 4|50
4
qrs. 2|0

4. If 120lb of Steel cost £7 how must I sell it *per* lb to gain £15½ *per Cent.*?

lb. £. lb. s. d. £. s. d. £. s. d.
As 120 : 7 :: 1 : 1 2 *As* 100 : 1 2 :: 115½ : 1 4 *per* lb *Ans.*

5. A Gentleman bought 10 Tons of Iron for £200, the freight and duties came to £25 and his own charges to £8 6*s.* 8*d*; How must he sell it *per* lb to gain £20 *per Cent.* by it?

£. £. £. s. d. £. s. d. £. s. d. £. s. d. £.
As 100 : 20 :: 233 6 8 : 46 13 4 *then* 233 6 8+46 13 4=280.
Tons £. lb. d.
As 10 : 280 :: 1 : 3 *per* lb *Answer.*

6. If a Bag of Cotton, weighing 8*Cwt*, 0*qr.* 20lb cost £13 13*s.* 4*d.* How must it be sold *per Cwt.* to lose £8 *per Cent.*?

Cwt. qr. lb. £. s. d. *Cwt.* £. s. d. £. £. s. d. £. £. s. d.
As 8 0 20 : 13 13 4 :: 1 : 1 13 5. *As* 100 : 1 13 5 :: 92 : 1 10 8¾
Ans. £1 10 8¾

7. Bought Fish in Newbury-port, at 10*s.* *per* Quintal, and sold it at Philadelphia, at 17*s.* 6*s.* *per* Quintal, now allowing the charges at an average, or one with another, to be 2*s.* 6*d.* *per* Quintal, and considering I must lose £20 *per Cent.* by remitting my money home; what do I gain *per Cent.*?

s. d.
Selling Price 17 6 *Philadelphia Currency per quintal.*
Charges = 2 6 *ditto.*

15 0 *ditto.*

£. s. £. s.
As 100 : 15 :: 80 : 12 New-England Currency.
Sold at 12*s.* *per Quintal.*
Bought at 10*s.* *per Quintal.*

s. s. £. £.
Gained 2*s.* *per Quintal*—*As* 10 : 2 :: 100 : 20 *per Cent. gained, Ans.*

8. Bought

8. BOUGHT 50 Gallons of Brandy, at 4*s.* *per Gallon*, but, by accident, 10 Gallons leaked out, at what rate must I sell the remainder *per* Gallon, to gain upon the whole prime cost, at the rate of £10 *per Cent.?*

		Gal.		£.		*Gal.*		*s.*	
50 Gall. at 4s. per Gall. = £10	*As*	40	:	10	::	1	:	5.	
10 Gallons leaked out.		£.		*s.*		£.		*s.*	*d.*
—	*As*	100	:	5	::	110	:	5	6 *Answer.*
40 Gallons remain.									

CASE 3.

When there is gained or lost per Cent. To know what the Commodity cost.

RULE. As £100 with the Gain *per Cent.* added, or loss *per Cent.* subtracted is to the price; So is £100 to the prime cost.

1. IF 1 yard of Cloth be sold, at 5*s.* 8*d.* and there is gained £13 6 8 *per Cent.* what did the yard cost?

EXAMPLES.

```
    £. s. d.   s. d.   £.
As 113 6  8  : 5  8 :: 100
       20       12      68
   ------      ---    -----
     2266       68     800
       12              600
   ------             -----
    27200     272,00)68|00(0
                        20
                      -----
           27200)1360(5s. Answer, 5s. prime cost.
                 1360
```

2. IF 12 yards of cloth are sold at 15*s.* *per* yard and there is £7 10*s.* Loss *per Cent* in the sale, what is the prime cost of the whole?

Yd. *s.* *Yds.* £. £. *s.* £. £. £. *s.* *d.*

As 1 : 15 :: 12 : 9 *As* 92 10 : 9 :: 100 : 9 13 6 *Ans.*

3. IF 40 lb. of Chocolate be sold at 1/6 *per* lb. and I gain £9 *per Cent.* what did the whole cost me?

lb. *s.* *d.* lb. £. £. £. £. £. *s.* *d.*

As 1 : 1 6 :: 40 : 3 *As* 109 : 3 :: 100 : 2 15 0½ *Ans.*

4. IF 19½ *Cwt.* Sugar be sold, at £4 5*s.* *per Cwt.* and I gain £15 *per Cent.* what did it cost *per Cwt.*?

£. £. *s.* £. £. *s.* *d.*

As 115 : 4 5 :: 100 : 3 13 10¼ — *Answer.*

CASE 4.

If by Wares sold at such a rate, there is so much gained or lost per Cent. To know what would be gained or lost per Cent. *if sold at another rate.*

RULE. As the first price is to £100 with the profit *per Cent.* added, or Loss *per Cent.* subtracted; so is the other price, to the Gain or loss *per Cent.* at the other rate.

N. B.

N. B. If your answer exceed £100 the Excess is your Gain *per Cent.* but if it be less than £100, that deficiency is the Loss *per Cent.*

EXAMPLES.

1. If Cloth, sold at 5/8 *per* yard, be £13 6/8 profit *per Cent.* what Gain or Loss *per Cent.* shall I have, if I sell the same at 5*s.* *per* yard?

```
    s.  d.   £.   s  d.     s.
As  5   8 : 113   6  8 ::   5
   12        20
   --       ----
   68       2266
              12
           -----
           27200
               5
           -----  2,0
68)136000(200,0     100—100=0. Ans. I neither gain nor lose.
   136      -----
   ---     £100
    ,000
```

2. If Cloth, sold at 4*s.* *per* yard, be £10 *per Cent.* profit; what shall I gain or lose *per Cent.* if sold at 3/6 *per* yard?

```
    s.  £.     s. d.
As  4 : 110 :: 3  6
   12          12               £.    £.    £.
   --          --         Then 100—96¼= 3¾
   48          42
              110
             ----
        48)4620(96¼   Ans. I lost £3¾ per Cent. by the last Sale.
           432
           ---
           300
           288
           ---
            12
```

3. If I sell a Gallon of Wine for 8*s.* and thereby lose £12 *per Cent.* what shall I gain or lose *per Cent.* if I sell 4 Gallons of the same Wine for £1 16*s.*?

```
    £. s.   £.     £. s.   £.
As  1  12 : 88 ::  1  16 : 99   And 100—99=£1 per Cent. loss.
```

4. I sold a Watch for £50, and by so doing, lost £17 *per Cent.* whereas I ought in trading to have cleared £20 *per Cent.* how much was it sold under its real value?

```
    £.   £.    £.   £.  s. d.        £.   £.  s. d.       £.  £.  s. d.
As  83 : 50 :: 100 : 60  4  9¾   As  100 : 60  4  9¾ :: 120 : 72  5  9¼
     £.  s. d.   £.   £.  s. d.
then 72  5  9¼—50=20  5  9¼  Answer.
```

EQUATION

EQUATION OF PAYMENTS

Is the finding of a time to pay, at once, several debts due at different times, so that no loss shall be sustained by either party.

RULE I. †

MULTIPLY each payment by the time at which it is due; then divide the sum of the Products by the sum of the payments, and the quotient will be the equated time, or that required.

EXAMPLES.

1. A owes B £380 to be paid as follows, *viz.* £100 in 6 months, £120 in 7 months, and £160 in 10 months: what is the equated time for the payment of the whole debt?

$$100 \times 6 = 600$$
$$120 \times 7 = 840$$
$$160 \times 10 = 1600$$

100 + 120 + 160 = 380) 3040 (8 *months, Answer.*
3040

2. A owes B £104 15*s.* to be paid in $4\frac{1}{2}$ months, £161 to be paid in $3\frac{1}{2}$ months, and £152 5*s.* to be paid in 5 months; what is the equated time for the payment of the whole?

Ans. 4 *months and* 8 *days.*

3. THERE is owing to a Merchant £698, to be paid £178 ready money, £200 at 3 months, and £320 in 8 months; I demand the indifferent time for the payment of the whole?

Ans. $4\frac{1}{2}$ *months.*

4. THE sum of £49 1*s.* 6*d.* is to be paid, $\frac{1}{2}$ at 6 months, $\frac{1}{3}$ at 8 months, and $\frac{1}{6}$ at 12 months; what is the mean time for the payment of the whole? *Ans.* $7\frac{2}{3}$ *months.*

RULE 2.

SEE by rule 1st. at what time the first man, mentioned, ought to pay in his whole money; then, as his money is to its time, so is the other's money, to his time, inversely, which, when found, must be added to, or subtracted from, the time at which the second ought to have paid in his money, as the case may require, and the sum, or remainder, will be the true time of the second's payment.

Nn EXAMPLES.

† THIS rule is founded upon a supposition, that the sum of the interests of the several debts, which are payable before the equated time, from their terms to that time, ought to be equal to the sum of the interests of the debts payable after the equated time, from that time to their terms. Some, who defend this principle, have endeavoured to prove it to be right by this argument; that what is gained by keeping some of the debts after they are due, is lost by paying others before they are due; but this cannot be the case; for though by keeping a debt after it is due, there is gained the interest of it for that time; yet by paying a debt before it is due, the payer does not lose the interest for that time, but the discount only, which is less than the interest, and therefore the rule is not accurately true; however, in most questions, which occur in business, the error is so trifling, that it will always be made use of as the most eligible method.

THE true Rule will be given in Equation of Payments by Decimals.

EXAMPLES.

A is indebted to B £150, to be paid, £50 at 4 months, and £100 at 8 months:—B owes A £250, to be paid at 10 months;—It is agreed between them, that A ſhall make preſent pay of his whole debt, and that B ſhall pay his ſo much the ſooner, as to balance that favor; I demand the time at which B muſt pay the £250, reckoning Simple Intereſt.

$$50 \times 4 = 200$$
$$100 \times 8 = 800$$

$50 + 100 = 15|0)\ 100|0\ (6\frac{2}{3}$ *months, A's equated time.*

$$\begin{array}{r} 90 \\ \hline 10 \\ \hline \end{array}$$

	£.	mo.		£.		mo.		mo.		mo.		mo.	
As	150 :	$6\frac{2}{3}$	: :	250	:	4	*then*	10	—	4	=	6	*time of B's payment.*

2. A Merchant has £120 due to him, to be paid at 7 months; but the debtor agrees to pay $\frac{1}{2}$ ready money, and $\frac{1}{3}$ at 4 months; I demand the time he muſt have to pay in the reſt, at ſimple intereſt, ſo that neither party may have the advantage of the other?

Debt £120

$\frac{1}{2}$ = 60 *muſt be paid down.*
$\frac{1}{3}$ = 40 *muſt be paid at 4 months.*
$\frac{1}{6}$ = 20 *unpaid.*

Now, as he pays £60, 7 months, and £40, 3 months before they are reſpectively due:—Say, As the intereſt of £20 for 1 month, is to 1 month, ſo is the ſum of the intereſt of £60 for 7 months, and of £40, for 4 months, to a 4th number, which, added to the 7 months, will give the time for which the £20 ought to be retained. *Anſ. 2 years and 10 months.*

3. A Merchant has £1200 due to him, to be paid $\frac{1}{6}$ at 2 months, $\frac{1}{3}$ at 3 months, and the reſt at 6 months; but the debtor agrees to pay $\frac{1}{2}$ down: How long may the debtor detain the other half, ſo that neither party may ſuſtain loſs?

	mo.		mo.
$\frac{1}{6}$ ×	2	=	$0\frac{1}{3}$
$\frac{1}{3}$ ×	3	=	1
$\frac{1}{2}$ ×	6	=	3
Equated time		=	$4\frac{1}{3}$

Now, as $\frac{1}{2}$ was paid $4\frac{1}{3}$ months before it was due, it is reaſonable that he ſhould detain the other $\frac{1}{2}$ $4\frac{1}{3}$ months after it became due, which, added, gives $8\frac{2}{3}$ months, the true time for the ſecond payment.

EQUATION

EQUATION OF PAYMENTS BY DECIMALS.

RULE.*

1. To the ſum of both payments add the continual product of the firſt payment, the ratio, and the time between the payments, and call this the firſt number.

2. MULTIPLY twice the firſt payment by the ratio, and call this the ſecond number.

3. DIVIDE the firſt number by the ſecond, and call the quotient the third number.

4. CALL the ſquare of the third number the fourth number.

5. DIVIDE the product of the ſecond payment and time between the payments by the product of the firſt payment and the ratio, and call the quotient the fifth number.

6. FROM the fourth number take the fifth, and call the ſquare root of the difference the ſixth number.

7. THEN the difference of the third and ſixth numbers is the equated time, after the firſt payment.

EXAMPLE.

THERE are £100 payable in 2 years, and £106 at 6 years hence; what is the equated time, allowing ſimple intereſt, at 6 *per Cent. per annum*?

1ſt. Payment = 100 — *1ſt Payment* 100
Ratio = ,06 — *Multiply by* 2

6,00 — 200
Time between the payments = 4 *years.* — *Mult. by the ratio* = ,06

24 — 12,00 = *2d. numb.*

Add both payments = { 100, 106 }

Div. by the 2d. numb. = 12)230 = *1ſt. number.*

19,166+ = *3d. number.*
19,166+

3d. number ſquared = 367,345556 = *4th. number.*

2d.

* SUPPOSE a ſum of money be due immediately and another ſum at the expiration of a certain given time forward, and it is propoſed to find a time, ſo that neither party ſhall ſuſtain loſs.

Now it is plain that the equated time muſt fall between the two payments; and that what is gotten by keeping the firſt debt after it is due, ſhould be equal to what is loſt by paying the ſecond debt before it is due; but the gain ariſing from the keeping of a ſum of money after it is due, is evidently equal to the *intereſt* of the debt for that time: And the loſs, which is ſuſtained by the paying of a ſum of money before it is due, is evidently equal to the *diſcount* of the debt for that time: therefore it is obvious that the debtor muſt retain the ſum immediately due, or the firſt payment, till its *intereſt* ſhall be equal to the *diſcount* of the ſecond ſum for the time it is paid before due; becauſe in that caſe the gain and loſs will be equal, and conſequently neither party can be a loſer.

2d. Payment = 106
Multiplied by the time = 4

1st Payment mult. by the ratio = 6)424 = { *Prod. of the 2d. Prod. and time between the payments.*

70,666+ = *5th. number.*

From the 4th. number = 367,345556
Take the 5th. number = 70,666666

296,678890(17,224 *square root* = *6th. number.*

From the 3d. number = 19,166
Take the 6th. number 17,224

1,942 = *Equated time from the first payment, therefore* 3,942 *years* = 3*y.* 11*m.* 14*d.* = *whole equated time.*

$$Or,\ \frac{100+106+\overline{100\times,06\times4}}{100\times2\times,06}-\left|\overline{\frac{100+106+100\times,06\times4}{100\times2\times,06}}\right|^{2}-\left.\frac{106\times4}{100\times,06}\right|^{\frac{1}{2}}=1,942.$$

COMMISSION OR FACTORAGE*

Is an allowance of so much *per Cent.* to a factor, or correspondent, abroad, for buying and selling goods, for his employer.

EXAMPLES.

1. What comes the Commission on £513 12/9 to, at $4\frac{1}{2}$ *per Cent.* ?

	£.	s.	d.
	539	12	9
			$4\frac{1}{2}$
	2158	11	0
Product of $\frac{1}{2}$ =	269	16	$4\frac{1}{2}$
	£.24|28	7	$4\frac{1}{2}$
	20		
	s.5|67		
	12		
	d. 8|08		

2. My Factor receives £1008 to lay out, after having deducted, his Commission of £5 *per Cent.* what does his commission amount to ?

HERE,

* The method of working questions in this Rule, Brokage, and the first case of Insurance, is the same as in Simple Interest.

Here, as his Commission is to be deducted from the given sum, it is evident that I ought not to pay him commission on his own money, (which, however, is often unjustly practised) therefore,

£. £. £. £.
As 105 : 5 :: 1008 : 48 *Answer.*

3. My Correspondent writes me that he has purchased goods to the value of £673 12*s*; what does his commission on that sum amount to, at £3½ *per Cent?* *Ans.* £23 11 6

4. What must I allow my correspondent, at £2¼ *per cent.* for disbursing on my account £395 15 5? *Ans.* £8 18 1

BROKERAGE

Is an allowance of so much *per cent.* to a person, called a Broker, for assisting Merchants or Factors in purchasing or selling goods.

EXAMPLES.

1. What is the brokerage upon £525 10*s.* at 5*s.* or £¼ *per Cent?*

5*s.*	¼	£. 525 *s.* 10
		1,31 7 6
		20
		6,27
		12
		3,30
		4
		1,20

5 shillings being an aliquot part of £1; divide by that part, and the quotient is the Answer.

Ans. £1 6 3½

2. What is the brokerage upon £673 16*s.* at £⅛ *per Cent?* *Ans.* £4 4 2¼

3. If a Broker sell goods to the amount of £709 19*s.* 11*d*; what is his demand, at £1½ *per cent?* *Ans.* £10 12 11½

BUYING AND SELLING STOCKS.

Stock is a general name for the Capitals of trading Companies, Banks, &c. and the buying and selling certain sums of money in those funds, is not unusual.

EXAMPLES.

1. What is the purchase of £275 15*s.* Bank-stock, at £74½ *per Cent?*

£74½

£74$\frac{1}{2}$ want £25$\frac{1}{2}$ of £100, therefore, take parts for the deficiency, and subtract the sum of those parts from the given sum.

		£.	s.	
£20	$\frac{1}{5}$	275	15	—
£5	$\frac{1}{4}$	55	3	
£$\frac{1}{2}$	$\frac{1}{10}$	13	15	9
		1	7	6$\frac{3}{4}$
	Subtract	70	6	3$\frac{3}{4}$
	Answer,	£204	8	8$\frac{1}{4}$

Or, If I had taken parts for £74$\frac{1}{2}$, the rate *per cent.* then the sum of the several quotients would have been the answer as above.

When the price is above 100, take parts for the surplus of the price above 100, and add them to the given sum for the answer.

2. What is the purchase of £1029 10*s.* 6*d.* bank-stock, at £110$\frac{1}{4}$ *per cent.*?

		£.	s.	d.			
10	$\frac{1}{10}$	1029	10	6	*at*	100	*per cent.*
£$\frac{1}{4}$	$\frac{1}{40}$	102	19	0$\frac{1}{2}$	*at*	10	*per cent.*
		2	11	5$\frac{1}{2}$	*at*	$\frac{1}{4}$	*per cent.*
Answer,		£1135	1		*at*	£110$\frac{1}{4}$	*per Cent.*

3. What is the purchase of £1058 12*s.* bank-stock, at £115$\frac{3}{4}$ *per Cent*? *Ans.* £1225 6 6$\frac{1}{2}$

4. What does £1600 capital stock, in the Massachusetts-Bank, come to, at £120$\frac{1}{8}$ *per Cent*? *Ans.* £1935 6 8

POLICIES OF INSURANCE.

Insurance is a security, or assurance, by mean of a Writ called a *Policy*, to indemnify the insured of such losses as shall be specified in the policy subscribed by the Insurer, or Insurers, by which the underwriters oblige themselves to make good and effectual the property insured, in consideration of a certain premium at a stipulated rate *per Cent.* (which varies according to the risque) to be immediately paid down, or otherwise secured according to the tenor of the agreement.

The average loss is 10 *per Cent.*; that is, if the insured suffer any damage or loss, not exceeding 10 *per cent.* he bears it himself, and the Insurers are free.

A Policy should be taken out for a sum sufficient to cover the principal and premium, and the business of this rule is, in general, to calculate what that sum should be.

Case

CASE I.

When the Premium, at a certain rate per cent. *for insuring a sum, is required, the operation is the same as in Interest, or Commission.*

1. WHAT is the premium upon £537 15 9 at $6\frac{1}{2}$ *per cent?*

	£.	s.	d.
	537	15	9
			$6\frac{1}{2}$
	3226	14	6
$\frac{1}{2}$ =	268	17	$10\frac{1}{2}$
	34|95	12	$4\frac{1}{2}$
	20		
	19|12		
	12		
	1|48		
	4		
	1|94		

Ans. £34 19/$1\frac{1}{4}$

CASE II.

To find the sum for which a Policy should be taken out to cover a given sum.

RULE. Take the premium from £100, and say, As the remainder is to 100; so is the sum adventured, to the policy. §

1. IT is required to cover £759, premium 8 *per Cent*; for what sum must the policy be taken?

```
100
  8
---
 92 : 100 :: 759
             100
      -----------
      92)75900(£825 Ans.
         736
         ---
          230
          184
          ---
           460
           460
           ---
```

2. A

§ Now it is plain that, if I want to recover £92, I must, in this case, insure upon £100; therefore to recover £759 I must insure upon £825; for when 8 *per Cent.* for premium is deducted, I shall have £759 remaining nett.

For £825 = *Sum insured upon at 8 per Cent.*
66 = *Premium to be deducted.*

£759 = *The first outset.*

IN this and the following cases, let x=100. p=premium, a=amount to be insured upon, and s=sum to be covered; then $x-p : x :: s : a$, or $\frac{xs}{x-p} = a$.

2. A Merchant sent a Vessel and Cargo to sea, valued at £1525; what sum must the Policy be taken out for, to cover his property, premium $19\frac{1}{2}$ *per cent?*

$$\begin{array}{r} 100 \\ \underline{19,5} \\ 80,5 \end{array}$$

80,5 : 100 :: 1525 : £1894 8s $2\frac{1}{4}$ *Answer.*

CASE III.

When a Policy is taken out for a certain sum in order to cover a given sum:—To find the Premium, say; As the Policy is to the covered sum; so is £100 to a fourth number, which, being taken from 100, will leave the premium. ‡

IF a Policy be taken out for £1250 to cover £500; what is the premium *per Cent.?*

$$\begin{array}{r} 1250 : 500 :: 100 \\ \underline{100} \end{array}$$

1250)50000(40 *and* £100—40= £60 *Answer.*

CASE IV.

When the Policy for covering any sum and the premium per cent. are given, to find the sum to be covered.

RULE.

DEDUCT the premium *per cent.* from 100, and say, As 100 is to the remainder; so is the policy to the sum required to be covered.

IF a Policy be taken out for £1250, at 60 *per cent*; what is the Adventure, or sum to be covered? †

$$\begin{array}{r} 100 \\ \underline{60} \end{array}$$

$$\begin{array}{r} 100 : 40 :: 1250 \\ \underline{40} \end{array}$$

100)50000(£500 *Answer.*

CASE V.

When a given sum is adventured several voyages round from one place to another, either at the same, or different Risques, from place to place, and it is required to take out a Policy for such a sum as will cover the adventure all round, supposing the risque out and home to be equal and tantamount to the several given Risques.

RULE.

‡ $a : s :: x : \overline{x-p}$. or $x - \frac{sx}{a} = p$.

† $x : x-p :: a : s$, or $\frac{a \times \overline{x-p}}{x} = s$.

RULE.

1. RAISE £100 to that power denoted by the number of Risques, and multiply the said power by the sum adventured, (or to be covered) for a dividend.

2. SUBTRACT the several premiums, each, from £100, and multiply the several remainders continually together for a divisor; and the quotient, arising from this division, will give the policy to cover the adventure the voyage round. *

1. A Merchant adventured £480 10*s.* from Newbury-port to South-Carolina, from thence to Jamaica, and from thence, home, and the premium was 5 *per cent.* from port to port; what sum must he take out a Policy for, to cover his adventure the voyage round, supposing the risque to be equal out and home, and tantamount to the several given risques?

$100 \times 100 \times 100 \times 480{,}5 = 480500000 =$ *Dividend.*

$$\begin{array}{ccccc} 100 & & 100 & & 100 \\ -5 & & -5 & & -5 \\ \hline 95 & \times & 95 & \times & 95 = 857375 = \textit{Divisor.} \end{array}$$

$857375)480500000(560\ 8/7\tfrac{1}{2}$ *Answer*

2. A Merchant adventured £500 from Boston to Philadelphia, at 3 *per cent.* from thence to Guadaloupe, at 4, from thence to Nantz, at 5, and from thence home, at 6 *per cent.*: For what sum must he take out a policy to cover his adventure the voyage round, supposing the risque to be equal out and home, and tantamount to the several given risques?

$$\frac{100 \times 100 \times 100 \times 100 \times 500}{\overline{100-3} \times \overline{100-4} \times \overline{100-5} \times \overline{100-6}} = \text{£}601{,}278, \textit{ Answer.}$$

* For the first Voyage.

$\overline{x-p} : x :: s : a.$

$\frac{xs}{\overline{x-p}} = a.$

$\frac{a \times \overline{x-p}}{x} = s.$

$x - \frac{sx}{a} = p.$

Second Voyage.

$\overline{x-p} : x :: \frac{xs}{x-p} : a$

$\frac{xxs}{\overline{x-p}^2} = a.$

$\frac{a \times \overline{x-p}^2}{x^2} = s.$

$x - \sqrt{\frac{xxs}{a}} = p.$

Third Voyage.

$\overline{x-p} : x :: \frac{xxs}{\overline{x-p}^2} : a.$

$\frac{xxxs}{\overline{x-p}^3} = a.$

$\frac{a \times \overline{x-p}^3}{x^3} = s.$

$x - \sqrt[3]{\frac{x^3 s}{a}} = p.$ and so on for as many Voyages as may be required.—Hence, making $m =$ exponent of any given power, $\frac{x^m s}{\overline{x-p} \times \overline{x-p} \times \overline{x-p}\ \&c.} =$ Sum to be insured upon, all round;—And $x - \sqrt[m]{\frac{x^m \times s}{a}} =$ the premium all round, tantamount to the several given premiums: s, in this Theorem, being equal to the first adventure, and $a =$ amount which will cover that adventure the Voyage round.

CASE VI.

When a given sum is adventured several voyages round, as in the last case, either at the same, or different risques, from Port to Port, and the premium for the voyage round is required, tantamount to the several given rates per Cent.

RULE.

1. FIND the sum for which the policy must be taken, by the last Case.
2. MULTIPLY the sum adventured by 100, and divide that product by the Policy.
3. TAKE the quotient from £100, and the remainder will be the premium *per cent.* on the policy, tantamount to the several premiums given in the question.

1. A Merchant adventured £480 10*s.* from Newbury-port to South-Carolina, from thence to Jamaica, and from thence, home, and the premium was 5 *per Cent.* from Port to Port, what will be the premium tantamount to the several given premiums, (allowing the risques out and home to be equal) on the policy which will cover the first adventure of £480 10*s*?

In Question 1, *Case* 5, *we found the Policy to be* £560 8*s*/7½=£560,43125

480,5 × 100 = 48050 *and* 560,43125)48050,00000(85,7375 *and* 100—85,7375=£14,2625 *the premium required, or thus*

$$100-\frac{480{,}5\times 100}{560{,}43125}=14{,}2625.$$

2. A Merchant adventured £500 from Boston to Philadelphia, at 3 *per cent.* from thence to Guadaloupe at 4; from thence to Nantz, at 5; and from thence, home, at 6 *per cent.*: What will be the premium, tantamount to those given in the question, on a Policy for covering the first adventure, the voyage, supposing the risques out and home equal?

In Question 2d. Case 5, *we found the Policy, which would cover the adventure the voyage round, to be* £601,278

Then, $100-\frac{500\times 100}{601{,}278}=$ £16,844 = *the premium* per cent. *on the policy the voyage round, and tantamount to the several given premiums.*

CASE VII.

If a policy be taken out for a given sum, to cover a certain adventure from one Port to another, on to several Ports, at equal premiums from one place to the other, to find what that equal premium is.

RULE.

1. INVOLVE 100 to that Power denoted by the number of Risques, and multiply this power by the sum adventured, (or covered.)
2. DIVIDE the last product by the policy.
3. EXTRACT that Root of the Quotient denoted by the number of Risques.

4. TAKE

4. TAKE this Root from £100, and the remainder will be the equal premium from one port to the other.

1. A Merchant adventured £480 10*s.* from Newbury-port to South-Carolina, from thence to Jamaica, and from thence home; To cover which all round he took out a policy for £560 8/7½, and the premium was equal from one place to the other; what was the premium *per cent.?*

$$100-\sqrt[3]{\frac{100\times100\times100\times480,5}{560,43125}}=5 \text{ per cent.}$$

CASE VIII.

When an adventure is insured out and home at one Risque, at a given rate per Cent. *and the voyage terminates short of what was at first intended: To find what the underwriter must receive* per cent.

RULE.

1. IF just half the voyage is performed, it must be considered as *two* equal Risques:—if one third, then, as *three* equal risques; if but one fourth, then, as *four* risques, and so on; and by case 2d. must be found the amount which will cover the adventure the voyage round.

2. INVOLVE 100 to that power denoted by the number of risques, and multiply this power by the sum adventured.

3. DIVIDE this product by the aforesaid amount.

4. EXTRACT that Root of the quotient denoted by the number of Risques

5. TAKE this root from £100, and the remainder will be the sum *per cent.* which the underwriter must receive.

1. A Merchant covers £200 at 6 *per Cent.* from Newbury-port to the West-Indies and home again; but the voyage terminating in the West-Indies, what must the Insurer receive *per Cent.?*

$$\begin{array}{l}100\\ \underline{\ \ 6}\\ 94 : 100 :: 200 : 212,765957 = \text{amount to cover £200 voyage round.}\end{array}$$

$$100\times100\times200=2000000 \text{ and } \frac{2000000}{212,765957}=9400.$$

and $100-\sqrt{9400}=3,0516$ *to be paid the Insurer* per Cent. *upon the above amount.*

2. A Merchant insures £350 to the West-Indies, from thence to France, and from thence home, at 10 *per cent.* the voyage round; but there is but one third part of the voyage performed; what must the Insurer receive *per cent.?*

There being but ⅓ of the voyage performed, we must suppose 3 *equal risques.*

$$\begin{array}{l}100\\ \underline{\ 10}\\ 90 : 100 :: 350 : 388,8889, \ \& \ 100\times100\times100\times340=350000000\end{array}$$

$$\frac{350000000}{388,8889}=899999,9999 \ \& \ 100-\sqrt[3]{899999,9999}=£3,4515 \text{ Ans.}$$

COMPOUND INTEREST

Is that which arises from the Interest being added to the Principal, and (continuing in the hands of the borrower) becomes a part of the principal, at the end of each stated time of payment.

METHOD I.

RULE. †

FIND the *amount* of the given principal, for the time of the first payment, by simple Interest; next, find the Interest of that sum, or principal, and add it as before, and thus proceed for any number of years, still accounting the last *amount* as the principal for the next payment.—The given principal being subtracted from the last *amount*, the remainder will be the compound Interest.

EXAMPLES.

1. WHAT will £480 amount to in 5 years, at £6 *per Cent. per Annum?*

	£.	s.	d.
Principal	480		
Rate of Interest	6		
	28|80		
	20		
	16|00		

	£.	s.	d.
Principal for the 1st year	480		
Interest of ditto	28	16	
Principal for the 2d. year	508	16	
		6	
	30|52	16	
		20	
		10|56	
		12	
		6|72	
		4	
		2|88	

	£.	s.	d.
Prin. for the 2d year	508	16	
Interest for ditto	30	10	$6\frac{1}{2}$
Prin. for the 3d year	539	6	$6\frac{1}{2}$
			6
	32|35	19	3
	20		
	7|19		
	12		
	2|31		
	4		
	1|24		

	£.	s.	d.
Principal for the 3d year	£539	6	$6\frac{1}{2}$
Interest for ditto	32	7	$2\frac{1}{4}$
Principal for the 4th year	571	13	$8\frac{3}{4}$

Principal

† As all the computations relating to simple Interest, are founded upon Arithmetical Progression, the simple Interest of one pound being a series of terms in arithmetical progression increasing; whose first term and common difference is the interest of one pound for one year, and the number of years shewing the number of all the terms; therefore, the last term will always be equal to the product of the time and rate, equal to the interest of one pound for any given time: So those relating to compound Interest are founded upon a series of Terms increasing in geometrical progression, wherein the number of years assigns the Index of the last and highest term; Therefore, as one pound is to the amount of one pound, for any given time; so is any proposed principal, or sum, to its amount for the same time.

Principal for the 4th year £571 13 8$\frac{3}{4}$
6

34|30 2 4$\frac{1}{2}$
20

6|02
12

0|28
4

1|14

	£	s.	d.
Principal for the 5th year	605	19	9
Interest for ditto	36	7	2
Amount for 5 years	642	6	11
Subtract the first Principal	480	-	—
Comp. Interest for 5 years	162	6	11

	£	s.	d.
Prin. for the 4th yr.	571	13	8$\frac{3}{4}$
Interest for ditto	34	6	-$\frac{1}{4}$
Prin. for the 5th year	605	19	9
			6

36|35 18 6
20

7|18
12

2|22

2. WHAT is the compound Interest of £740 for 6 years, at £4 *per Cent. per Annum?* *Ans.* £196 6 8$\frac{1}{2}$

3. WHAT will £400 amount to in 5 years, at £4 *per Cent. per Annum?* *Ans.* 486 13 2$\frac{1}{2}$

4. WHAT will £150 amount to in a year, at £2 *per Cent. per month?* *Ans.* £190 4 5

METHOD 2.

When the Rate is at 5 per Cent. per annum.

1. DIVIDE the Principal by 20, and this quotient, added to the principal, will be the amount for the first year, and the principal for the second.

2. IN like manner find the amount for every succeeding year.

When the Rate is at 6 per Cent. per annum.

1. DIVIDE the Principal by 20, and that quotient by 5: These quotients, added to the principal, will be the amount for the first year, and the principal for the second.

2. In like manner obtain the amount for every succeeding year.

EXAMPLES.

EXAMPLES.

WHAT is the amount of £480, at 6 *per cent. per annum*, for 5 years?

20)480			
5) 24			
4	16		
20)508	16		*Amount of the 1st year.*
5) 25	8	$9\frac{1}{2}$	
5	1	9	
20)539	6	$6\frac{1}{2}$	*ditto of the 2d.*
5) 26	19	$3\frac{3}{4}$	
5	7	$10\frac{1}{4}$	
20)571	13	$8\frac{1}{2}$	*ditto of the 3d.*
5) 28	11	8	
5	14	4	
20)605	19	$8\frac{1}{2}$	*ditto of the 4th.*
5) 30	5	$11\frac{3}{4}$	
6	1	$2\frac{1}{4}$	
£642	6	$10\frac{1}{2}$	*ditto of 5th, Ans.*

OF the same sum at 5 *per cent. per annum* for 5 years?

20)480			
24			
20)504			*Amount of 1st year.*
25	4		
20)529	4		= *ditto of 2d.*
26	9	$2\frac{1}{4}$	
20)555	13	$2\frac{1}{4}$	= *ditto of 3d.*
27	15	$7\frac{3}{4}$	
20)583	8	10	= *ditto of 4th.*
29	3	$5\frac{1}{4}$	
£612	12	$3\frac{1}{4}$	= *dit. of 5th, Ans.*

COMPOUND INTEREST BY DECIMALS.

A TABLE *of the amount of £1, at 6* per cent. per annum, for months.

Months.	£. Dec. parts.	Months.	£. Dec. parts.	Months.	£. Dec. parts.
1	1,00487	5	1,02457	9	1,04467
2	1,00976	6	1,02956	10	1,04975
3	1,01467	7	1,03457	11	1,05486
4	1,01961	8	1,03961	12	1,06

A TABLE *of the amount of £1, from* 1 *day to* 31 *days, at* 6 per cent. per annum.

Days.	£. Dec. parts.	Days.	£. Dec. parts.	Days.	£. Dec. parts.
1	1,00016	12	1,00192	22	1,00352
2	1,00032	13	1,00208	23	1,00368
3	1,00048	14	1,00224	24	1,00384
4	1,00064	15	1,0024	25	1,004
5	1,00 8	16	1,00256	26	1,00416
6	1,00096	17	1,00272	27	1,00432
7	1,00112	18	1,00288	28	1,00448
8	1,00128	19	1,00304	29	1,00464
9	1,00144	20	1,0032	30	1,0048
10	1,0016	21	1,00336	31	1,00496
11	1,00176				

A TABLE *of the Amount of* £1, *at* $\frac{1}{2}$ per Cent. per Month, *as practised at the Massachusetts-Bank.*

Months.	£. *Dec. parts.*	*Months.*	£. *Dec. parts.*	*Months.*	£. *Dec. parts.*
1	1,005	5	1,025	9	1,045
2	1,01	6	1,03	10	1,05
3	1,015	7	1,035	11	1,055
4	1,02	8	1,04	12	1,06

CASE I.†

When the principal, the rate of Interest, and time, are given, to find either the amount or Interest.

RULE

† LET r= amount of £1 for 1 year, and p= principal, or given sum; then, since r is the amount of £1 for 1 year, r^2 will be its amount for 2 years, r^3 for 3 years, and so on, therefore, it will be as $1 : r :: r : r^2$ =amount for the second year, or principal for the third;—Again, As $1 : r :: r^2 : r^3$ =amount for the third year, or principal for the fourth, &c. to any number of years.—And if the time, or number of years, be denoted by t, the amount of £1 for t years, will be r^t; from hence it will appear that the amount of any other principal sum p, for t years, is pr^t; for, as $1 : r^t :: p : pr^t$, the same as in the rule.

IF the rate of interest be determined to any other time than a year, as $\frac{1}{4}$, $\frac{1}{2}$, &c. the rule is the same, and then t will represent that stated time.

Let $\begin{cases} r = \text{Amount of £1 for 1 year, at the given rate } per\ Cent. \\ p = \text{Principal, or sum put out at interest.} \\ i = \text{Interest.} \\ t = \text{Time.} \\ m = \text{Amount for the time } t. \end{cases}$

THEN the following Theorems will exhibit the Solutions of all the cases in Compound Interest.

I. $pr^t = m$. II. $pr^t - p = i$. III. $\frac{m}{r^t} = p$. IV. $\overline{\frac{m}{p}}\Big|^{t} = r$.

THE most convenient way of giving the Theorems, especially that for the time, will be by Logarithms, as follows,

I. $t \times Log.\, r + Log.\, p = Log.\, m$. II. $Log.\, \overline{m - t \times L.\, r} = L.\, p$.

III. $\frac{L.\, m - L.\, p}{L.\, r} = t$. IV. $\frac{L.\, m - L.\, p}{t} = L.\, r$.

IF the compound interest, or amount of any sum be required for the parts of a year it may be determined as follows—

I. WHEN the Time is an aliquot part of a year.

RULE.

1. FIND the amount of £1 for 1 year, as before, and that root of it, which is denoted by the aliquot part, will be the amount of £1 for the time sought.
2. MULTIPLY the amount, thus found, by the principal, and it will be the amount of the given sum required.

II. WHEN the Time is not an aliquot part of a year.

RULE.

1. REDUCE the time into days, and the 365th root of the amount of £1 for 1 year is the amount for 1 day.
2. RAISE this amount to that power, whose Index is equal to the number of days, and it will be the amount of £1 for the given time.
3. MULTIPLY this amount by the principal, and it will be the amount of the given sum required.

To avoid extracting very high roots, the same may be done by Logarithms, thus; Divide the Logarithm of the rate, or amount of £1 for 1 year, by the denominator of the given aliquot part, and the quotient will be the logarithm of the root sought.

RULE.

1. FIND the amount of £1, for 1 year, at the given rate *per Cent.*

2. INVOLVE the amount, thus found, to such power, as is denoted by the number of years—or, in TABLE I, at the end of Annuities, under the rate, and against the given number of years, you will find the Power ‡

3. MULTIPLY this power by the Principal, or given sum, and the product will be the amount required, from which, if you subtract the principal, the remainder will be the Interest.

EXAMPLES.

1. WHAT is the compound Interest of £600 for 4 years, at 6 *per Cent. per Annum?*

$$\begin{array}{rl} & 1{,}06 = \left\{ \begin{array}{l} \textit{Amount of £1, for 1 year, at 6 per Cent.} \\ \textit{per Annum.} \end{array} \right. \\ \textit{Multiply by} & 1{,}06 \\ \hline & 1{,}1236 = \textit{2d. Power.} \\ \textit{Mult. by} & 1{,}1236 \\ \hline & 1{,}26247696 = \textit{4th Power.} \\ \textit{Multiply by} & 600 = \textit{Principal.} \\ \hline & 757{,}48617600 = \textit{Amount.} \\ \textit{Subtract} & 600 \\ \hline & 157{,}486176 = £157\ 9s.\ 8\tfrac{1}{2}d. = \textit{Interest required.} \end{array}$$

By TABLE I.

Tabular amount of £1 for 4 years, at 6 per Cent. per Annum = 1,2624769
Multiply by the Principal = 600

Amount = 757,4861400

2. WHAT is the amount of £570 10*s.* for 12 years, at $3\frac{1}{2}$ *per Cent. per Annum?* *Ans.* £1024 10*s.* $8\frac{1}{2}$*d.*

ANOTHER method of working compound Interest for *years, months and days,* which is much more concise than the preceding method.

RULE

‡ THE amounts of £1 in this Table, are so many powers of the amount of £1 for 1 year, whose Indices are denoted by the number of years.

Note, When the given time consists of years and months, or years, months and days; first seek the amount of £1 in the Table for years, then in the Table of months, &c. multiply these several amounts and the principal continually together, and the last product will be the amount required.

THUS, if the amount of £480 in $5\frac{1}{2}$ years, at 6 *per Cent. per Annum* were required; the amount of £1 for 5 years=£1,33822, ditto for 6 months = £1,02956
Now, 1,33822 x 1,02956,480 = £661,2341 *Answer.*

Rule.

To the logarithm of the Principal, found in any Table of logarithms, add the ſeveral logarithms, anſwering to the number of *years, months and days,* found in the following tables, and their ſum will be the logarithm of the amount for the given time, which being found in any Table of logarithms, the natural number correſponding thereto will be the anſwer. †

Logarithmic Tables, *at* 6 per Cent. per Annum, *for years, Months and Days.*

Years	*dec. pts.*	*Y.*	*dec. pts.*	*Y.*	*dec. pts*	*Y.*	*dec. pts.*	*Months*	*dec. pts.*
1	,025306	11	,278366	21	,531426	31	,784586	1	,002166
2	,050612	12	,303672	22	,556732	32	,809792	2	,004321
3	,075918	13	,328978	23	,582038	33	,835098	3	,006466
4	,101224	14	,354284	24	,607344	34	,860404	4	,0086
5	,12653	15	,37969	25	,63265	35	,88571	5	,010724
6	,151836	16	,404896	26	,657956	36	,911016	6	,012837
7	,177142	17	,430202	27	,683262	37	,936322	7	,01494
8	,202448	18	,455508	28	,708568	38	,961628	8	,017033
9	,227754	19	,480814	29	,733974	39	,986934	19	,019116
10	,25306	20	,50612	30	,75938	40	1,01224	10	,021189
								11	,023252
Days		*D*		*D*		*D*		*D*	
1	,000071	8	,000571	14	,000999	20	,001426	26	,001852
2	,000143	9	,000642	15	,00107	21	,001497	27	,001923
3	,000215	10	,000713	16	,001142	22	,001568	28	,001994
4	,000287	11	,000785	17	,001213	23	,001639	29	,002065
5	,000358	12	,000857	18	,001284	24	,00171	30	,002136
6	,000429	13	,000928	19	,001355	25	,001781	31	,002207
7	,0005								

6. What is the amount of £132 10*s.* at 6 *per Cent. per Annum* for 9 years, 8 months, and 15 days?

To the Log. of £132,5 = 2,122216

Add { *Log. for* 9 *years* = ,227754
ditto for 8 *months* = ,017033
ditto for 15 *days* = ,00107 }

2,368073

Becauſe 8 *months are paſt, deduct* 4 *per cent. upon the logarithm of* 15 *days* } = ,0000428

Remains · 2,3680302, *the neareſt to which, in the Table of Logarithms, is* 2,368101, *and the natural number anſwering thereto is* 233,4=£233 8*s. Anſwer.*

† Although there is a ſmall error in the logarithms for days, yet they are exact enough for common uſe,—And if after the firſt month you deduct ½ *per Cent.* for each month paſt (that is, ½ *per Cent.* after 1 month, 1½ *per Cent.* after 3 months, &c.) from the logarithm of the number of days, it will give the true anſwer.

Note, That, after 1 month, ½ *per Cent.* on the logarithm of 1 day, is ,000000355—on 2 days, is, 000000715;—After 2 months, 1 *per Cent.* on the logarithm of 1 day, is, ,00000071—on 2 days, ,00000143;—After 10 months, 5 *per Cent.* on the logarithm of 1 day, is, ,00000355, on 6 days, is, ,00002145, &c.

CASE 2.

When the Amount, Rate and Time are given, to find the Principal.

RULE.

Divide the amount by the amount of £1 for the given time, and the quotient will be the Principal.

Or, If you multiply the present value of £1 for the given number of years, at the given rate *per Cent.* by the amount, the product will be the principal, or present worth.†

EXAMPLES.

1. WHAT is the present worth of £757 9s 8½ due 4 years hence, discounting at the rate of £6 *per Cent. per Annum*?

By TABLE I.

Divide by the tabular amount of £1 for 1 year. } =1,2624769)757,4861400(£600, *Ans.*

By TABLE II.

Mult. by the present worth of £1 for 4 years, at 6 per cent. per annum } *Amount*= 757,48614
= ,79 20936

Answer 599,9999235827O4+ =£600.

2. WHAT Principal must be put to Interest 6 years, at 5½ *per Cent. per Annum*, to amount to £965 3*s*. 9½*d*,3616? *Ans.* £700.

CASE 3.

When the Principal, Rate and Amount, are given, to find the Time.

RULE.

DIVIDE the amount by the Principal; then divide this quotient by the amount of £1 for 1 year, this quotient by the same till nothing remain, and the number of the divisions will shew the time.

Or, Divide the amount by the principal, and the quotient will be the amount of £1 for the given time, which seek under the given rate in TABLE 1, and in a line with it, you will see the Time.

EXAMPLES.

1. IN what time will £600 amount to £757 9*s*. 8½ at 6 *per Cent. per Annum* Compound Interest?

Divide the amount by the Principal } = 600)757,486176(1,26247696 = *Quotient to be divided by* 1,06 *till it can be had no more; or you may find it in* TABLE 1. *under* 6 per Cent. *and against* 4 *years.*

Divide

† SEE TABLE 2. shewing the present value of £1, discounting at the rates of 4, 4½, &c. *per Cent.* the Construction of which is thus:

Amount.		*Pres. worth.*		*Amount.*		*Pres. worth.*
As 1,06	:	1	::	1	:	,9433962, and so on, for any other rate *per Cent.* and time.

Divide by 1,06)1,26247696

1,06)1,191016

1,06)1,1236

1,06)1,06

1

Four divisions shew the time to be 4 years.

CASE 4.

When the Principal, Amount and Time, are given, to find the Rate per Cent.

RULE.

DIVIDE the amount by the Principal, and the quotient will be the amount of £1 for the given time, then, extract such Root as the Time denotes, and that root will be the amount of £1 for 1 year, from which subtract unity, and the remainder will be the ratio.

Or, Having found the amount of £1 for the time, as above directed, look for it in TABLE 1, even with the given time, and directly over the amount you will find the ratio.

EXAMPLE.

AT what Rate *per Cent. per Annum* will £600 amount to £757 9/8½ in 4 years?

600)757,48614(1,2624769.—*Now, the time being* 4 *years, the* 4*th Root of this quotient minus* 1 *will be the ratio.*

1,26247690(1,123599+ *and* 1,123599(1,05999+ *and* 1,05999 —1 =,06 *Answer*.

DISCOUNT BY COMPOUND INTEREST.

CASE 1.*

The Sum, or Debt to be discounted, the Time, and Rate given, to find the Present worth.

RULE.

* LET m = sum or debt to be discounted, and the other letters as before: then the following Theorems will shew all the cases in Discount by Compound Interest.

I. $\frac{m}{r^t} = p$. $pr^t = m$. III. $\frac{m}{p} = r^t$ which being continually divided by r, till nothing remain, the number of these divisions will be equal to t.

IV. $\frac{m}{p} = r^t$ which being extracted, (the time, given in the question, shewing the power) will be equal to r.

Note, Case 2d may be wrought by TABLE 1, thus,

FIND that power of £1 for 1 year, denoted by the Time; multiply the present worth by it, and the Product will be the answer.

Or, by Table 2d, thus; find the present worth of £1 for the given time, by which divide the present worth, and the quotient will be the debt or principal.

CASE 3, thus; Divide the debt by its present worth, and seek the quotient in TABLE 1, under the given rate, and in a line with it you will see the Time.

CASE 4th is wrought in the same manner, only, seek the quotient in a line with the time, it will shew the rate a-top,

RULE.

DIVIDE the Debt by that Power of the amount of £1 for 1 year, denoted by the Time, and the quotient will be the present worth, which, subtracted from the debt, will leave the discount.

EXAMPLES.

1. WHAT is the present worth, and discount of £600, due 3 years hence, at £6 *per Cent. per Annum*, compound Interest?

Divide by $1{,}06|^3 = 1{,}19101)600{,}00000(503{,}7741 = £503\ 15/5\frac{3}{4}$ *present worth, and* £600—£503 15 $5\frac{3}{4}$ = £96 4*s*. $6\frac{1}{4}$*d*. = *Discount*.

$$Or,\ \frac{600}{1{,}19101} = £503{,}7741,\ \&\ 600 - \frac{600}{1{,}19101} = £96{,}2259$$

By TABLE II.

In this Table, corresponding to the Time and Rate,
we have ,839619 = *present worth of £1 for the Time and Rate.*
Multiply by 600 = *Debt, or Principal.*

503,771400 = *present worth of the debt.*

2. WHAT is the present worth of £312 10*s*. due 2 years hence, at $4\frac{1}{2}$ *per Cent. per Annum*, Compound Interest?

Ans. £286 3*s*. 3*d*. 2,97*qrs*.

3. WHAT ready money will discharge a debt of £1000, due 4 years hence, at £5 *per cent. per Annum* Compound Interest?

Ans. £822 14*s*. 0*d*. 2,304*qrs*.

ANNUITIES OR PENSIONS IN ARREARS AT COMPOUND INTEREST.

CASE I.

When the ANNUITY, *or* PENSION, *the* TIME *it continues, and the* RATE *per cent. are given, to find the* AMOUNT.

RULE. ‡ 1. Make 1 the first Term of a Geometrical Progression, and the amount of £1 for 1 year at the given Rate *per cent.* the Ratio.

2. CARRY

‡ It is plain that upon the first year's Annuity there will be due so many year's compound interest, as the given number of years less one, and gradually one year less, upon every succeeding year, to that preceding the last, which has but one year's interest, & the last bears none

LET r= rate, or amount of £1 for 1 year, then the series of amounts of £1 annuity for several years, from the first to the last, is 1, r, r^2 r^3, &c. to $r^t - 1$; And the sum of this, according to the rule in Geometrical Progression, will be $\frac{r^t - 1}{r - 1}$ = amount of £1 annuity for t years. And all annuities are proportional to their amounts; therefore, $1 : \frac{r^t - 1}{r - 1} :: n : \frac{r^t - 1}{r - 1} \times n$ = Amount of any given annuity n.

LET r= rate, or amount of £1 for 1 year, and the other letters as before, then, $\frac{r^t - 1}{r - 1} \times n = a$, and $\frac{ar - a}{r^t - 1} = n$.

And from these equations, all the cases relating to annuities or pensions in arrears, may be conveniently exhibited in Logarithmic Terms, thus,

I. L.

2. CARRY the series to so many Terms as the number of years, and find its sum.

3. MULTIPLY the sum, thus found, by the given annuity, and the Product will be the amount sought.

OR, Multiply the amount of £1 for 1 year into itself so many times as there are years, less by 1; then multiply this Product by the annuity; and subtract the annuity therefrom; *lastly*, divide the Remainder by the Ratio less 1, and the Quotient will be the amount.

EXAMPLES.

1. WHAT will an annuity of £60 *per Annum*, payable yearly, amount to in 4 years, at £6 *per Cent.*?

First Method.

$$1+1{,}06+\overline{1{,}06}|^2+\overline{1{,}06}|^3 = 4{,}374616 = \textit{Sum.}$$

Multiply by $60 = $ *Annuity.*

$$\begin{array}{r} 262{,}476960 \\ 20 \\ \hline 9{,}53920 \\ 12 \\ \hline 6{,}4704 \\ 4 \\ \hline 1{,}8816 \\ \hline \end{array}$$

Ans. £262 9*s.* 6¼*d.*

Or, $1+1{,}06+\overline{1{,}06}|^2+\overline{1{,}06}|^3 \times 60 =$ £262 9*s.* 6¼*d.*

Second

I. $L.\,n + L.\,r^t - 1 + L.\,\overline{r-1} = L.\,a.$

II. $L.\,a - L.\,r^t - 1 + L.\,\overline{r-1} = L.\,n.$

III. $\dfrac{L.\,\overline{ar-a+n} - L.\,n}{L.\,r} = t.$ IV. $r^t - \dfrac{ar}{n} - 1 = 0.$

Or thus, I. $\dfrac{nr^t - n}{r-1} = a.$ II. $\dfrac{ar-a}{r^t-1} = n.$ III. $\dfrac{ar+n-a}{n} = r^t$

which continually divided by r till nothing remain, the number of those divisions will be equal to t—Or, being extracted, (the given time shewing the power) will be equal to r.

Second Method.

$1{,}06 \times 1{,}06 \times 1{,}06 \times 1{,}06 = 1{,}26246$
Multiply by $60 =$ *Annuity.*

$75{,}74760$
Subtract 60

Divide by $1{,}06 - 1 = {,}06)\,15{,}7476(262{,}49 =$ £262 9*s.* 9½*d.* *Ans.*

12
37
36
14
12
27
24
36
36

Or, $\dfrac{1{,}06 \times 1{,}06 \times 1{,}06 \times 1{,}06 \times 60 - 60}{1{,}06 - 1} = 262{,}49.$

Or, By TABLE III. †

MULTIPLY the Tabular number under the Rate, and opposite to the time, by the annuity, and the Product will be the amount.

2. WHAT will an annuity of £60 *per Annum,* amount to in 20 years, allowing £6 *per Cent.* Compound Interest?

Under £6 per Cent. *and opposite* 20, *in Table* III. *you will find,*
Tabular number $= 36{,}78559$
Multiply by $60 =$ *Annuity.*

$2207{,}13540 =$ £2207 2*s.* 8¼ *the Ans.*

3. WHAT will a Pension of £75 *per Annum,* payable yearly, amount to in 9 years, at £5 *per Cent.* Compound Interest.

Ans. £826 19*s.* 10*d.*

4. IF a Salary of £100 *per Annum,* to be paid yearly, be forborne 5 years, at £6 *per Cent.* what is the amount? *Ans.* £563 14*s* 2

CASE

† TABLE III. is calculated thus; Take the first year's amount, which is £1, multiply it by $1{,}06 + 1 = 2{,}06 =$ second year's amount, which also multiply by $1{,}06 + 1 = 3{,}1836 =$ third year's amount, &c. and in this manner proceed in calculating Tables at any other rates.

CASE 2.

When the AMOUNT, RATE *per Cent. and* TIME *are given, to find the* ANNUITY, PENSION, *&c.*

RULE. Multiply the whole amount by the amount of £1 for 1 year, from which subtract the whole amount; divide the remainder by that power of the amount of £1 for 1 year, signified by the number of years, made less by unity, and the quotient will be the answer.

EXAMPLES.

1. WHAT annuity, being forborne 4 years, will amount to £262,47696, at £6 *per Cent.* Compound Interest?

262,47696 = *Amount.*
Multiply by 1,06 = *Amount of £1 for 1 year.*

157486176	1,06
262476960	1,06
278,2255776	636
Subtract 262,47696	1060
,26247696)15,7486176(£60 *the Answer.*	1,1236
15 7486176	1,06
0	67416
	112360
	1,191016
	1,06
	7146096
	11910160
	1,26247696
	Subtract 1
	Divisor = ,26247696

$$Or\ \frac{262,47696 \times 1,06 - 262,47696}{1,06 \times 1,06 \times 1,06 \times 1,06 - 1} = 60.$$

Or, By TABLE III.

DIVIDE the amount by the Tabular number under the Rate, and opposite to the Time, and the quotient will be the annuity.

2. WHAT annuity, being forborne 20 years, will amount to £2207,1354, at £6 *per Cent.* Compound Interest?

Tabular amount = 36,78559)2207,13540(£60 *Ans.*
2207,1354
0

CASE

CASE 3.

When the ANNUITY, AMOUNT *and* RATIO *are given, to find the* TIME.

RULE.

MULTIPLY the amount by the Ratio, to this Product add the Annuity, and from the sum subtract the amount; this remainder being divided by the Annuity, the *Quotient* will be that Power of the Ratio signified by the time, which being divided by the Amount of £1 for 1 year, and this *Quotient* by the same, till nothing remain, the number of those divisions will be equal to the time. Or look for this number under the given rate in TABLE I. and, in a line with it, you will see the time.

EXAMPLES.

1. IN what time will £60 *per Annum*, payable yearly, amount to £262,47696, allowing £6 *per Cent*, Compound Interest for the forbearance of payment?

	262,47696	= *Amount*.
Multiply by	1,06	= *Ratio*.
	157486176	
	262476960	
	278,2255776	
Add	60	= *Annuity*.
	338,2255776	
Subtract	262,47696	
Divide by	60)75,7486176	
Divide by	1,06)1,26247696	
Divide by	1,06)1,191016	
Divide by	1,06)1,1236	
Divide by	1,06)1,06	
	1	

The number of divisions by 1,06, *being* 4, *gives the number of years* = 4 *the answer*.

Or, IN TABLE 1. under the given Rate you will find 1,262476, and in a line, under years, you will find 4.

2. IN what time will an annuity of £60, payable yearly, amount to £2207,1354, allowing £6 *per Cent*. for the forbearance of payment?

Answer 20 years.

PRESENT

PRESENT WORTH of ANNUITIES, &c. at COMPOUND INTEREST.

Case I.

When the Annuity, &c. Rate *and* Time *are given, to find the* Present Worth.

Rule.*

1. Divide the Annuity by the Ratio, or the amount of £1 for 1 year, and the quotient will be the present worth of 1 year's annuity.

2. Divide the Annuity by the Square of the Ratio, and the Quotient will be the present worth for two years.

3. In like manner, find the present worth of each year by itself, and the sum of all these will be the present value of the Annuity, sought.

Or, Divide the Annuity, &c. by that Power of the Ratio signified by the number of years, and subtract the quotient from the Annuity: This remainder being divided by the Ratio less 1, the quotient will be the present worth.

 Examples.

* The reason of this rule is evident from the nature of the question, and what was said upon the same subject in the purchasing of annuities by Simple Interest.

Let p= present worth of the annuity, and the other letters as before, then,

$n \times \frac{r^t - 1}{r^{t+1} - r^t} = p$. And $p \times \frac{r^{t+1} - r^t}{r^t - 1} = n$.

And from these Theorems, all the cases, where the purchase of annuities is concerned, may be exhibited in logarithmic terms, as follows.

$$\text{I. } L.\,n + L.\,\overline{1 - \frac{1}{r^t}} - L.\,\overline{r-1} = L.\,p.$$

$$\text{II. } L.\,p + L.\,\overline{r-1} - L.\,\overline{1 - \frac{1}{r^t}} = n.$$

$$\text{III. } \frac{L.\,n - L.\,\overline{n+p-pr}}{L.\,r} = t.$$

Or, thus, I. $\frac{n - \frac{n}{r^t}}{r-1} = p$. II. $\frac{pr^t \times r - pr^t}{r^t - 1} = n$.

III. $\frac{n}{p+n-pr} = r^t$ which being continually divided by r till nothing remain, the number of those divisions will be equal to t.

Let t express the number of half years, or quarters, n the half year's, or quarter's payment, and r, the sum of £1, and $\frac{1}{2}$ or $\frac{1}{4}$ year's interest, then all the preceding rules will be applicable to half-yearly, and quarterly payments, the same as to whole years.

The *amount* of an annuity may also be found for years and parts of a year, thus:

1. Find the amount for the whole years, as before.
2. Find the interest of that amount for the given parts of a year.
3. Add this interest to the former account, and it will give the whole amount required.

The *present worth* of an annuity for years and parts of a year may be found thus,

1. Find the present worth for the whole years, as before.
2. Find the present worth of this present worth, discounting for the given parts of a year, and it will be the whole present worth required.

EXAMPLES.

1.§ WHAT ready money will purchase an Annuity of £60, to continue 4 years, at £6 *per Cent.* Compound Interest.

First Method.

Ratio = 1,06)60,00000(56,603 = *present worth for* 1 *year.*

$\overline{Ratio}|^2$ = 1,1236)60,00000(53,399 = *do. for* 2 *years.*

$\overline{Ratio}|^3$ = 1,191016)60,00000(50,377 = *do. for* 3 *years.*

$\overline{Ratio}|^4$ = 1,26247696)60,00000(47,525 = *do. for* 4 *years.*

207,904 = £207 18*s.* 0¾*d.* *Ans.*

Second Method.

4th Power of the Ratio } = 1,26247696)60,0000000(47,525

From 60
Subtract 47,525

Or $\frac{60}{,06|^4}$ = 47,525 60 − 47,525 = 12,475

And $\frac{12,475}{,06}$ = 207,916.

Divis. 1,06—1=,06)12,475

2 07,916 = £207 18*s.* 3¾*d.* *the Answer.*

By TABLE III.

Under £6 per Cent. *and opposite* 4, *we find*
4,37461 = *Amount of* £1 *Annuity for* 4 *years.*
Multiply by 60 = *Annuity.*

262,47660 = *Amount of* £60 *for* 4 *years.*

Then, opposite 4 *years, and under* £6 per Cent. *in* TABLE II.
We have ,792093
Multiply by 262,7466

4752558
4752558
3168372
5544651
1584186
4752558
1584186

208,1197426338 = £208 2*s.* 4½*d.*

Or, *Opposite* 4 *years, and under* £6 per Cent. *in* TABLE I. *we have* 1,26247 = *the amount of* £1 *for* 4 *years:*

Then, 262,7466÷1,26247=208,1209=£208 2*s.* 5*d.* *the Answer.*

By

§ QUESTIONS in this case may also be answered by first finding the amount of the given annuity by Case 1. of Annuities in Arrears, Page 300, and then the present worth, or principal, by Case 2. of Compound Interest, Page 298.

By TABLE IV. ¶

MULTIPLY the Tabular number, under the Rate, and opposite the time, into the annuity, and the product will be the present worth.

THUS, in EXAMPLE 1st. What ready money will purchase £60 Annuity, to continue 4 years, at £6 *per Cent.* Compound Interest?

Under £6 per Cent. *and even with* 4 *years,*

We have 3,4651 = *present worth* £1 *for* 4 *years.*
Multiply by 60 = *Annuity.*

Answer = 207,9060 = £207 18*s.* $1\frac{1}{4}$*d.*

2. WHAT is the present worth of an annuity of £60 *per Annum*, to continue 20 years, at £6 *per cent.* Compound Interest?

Ans. £688 3*s.* $10\frac{1}{4}$*d.*

CASE 2.

When the PRESENT WORTH, TIME *and* RATE *are given, to find the* ANNUITY, RENT, &c.

RULE. 1. From that Power of the Ratio, denoted by the number of years plus 1, subtract that power of it, denoted by the number of years.

2. DIVIDE the remainder by that power of the Ratio, signified by the Time made less by unity.

3. MULTIPLY the present worth into this quotient, and the product will be the annuity, pension, rent, &c.

OR, 1. Multiply that power of the Ratio, denoted by the number of years plus 1, by the present worth.

2. MULTIPLY that power of the Ratio, denoted by the time, by the present worth, and subtract this product from the former.

3. DIVIDE the remainder by that Power of the Ratio, denoted by the time made less by unity, and the quotient will be the annuity.

EXAMPLES.

1. WHAT annuity, to continue 4 years, will £207,904 purchase, Compound Interest, at £6 *per Cent.*?

First Method.

From 1,06 × 1,06 × 1,06 × 1,06 × 1,06 = 1,3382255776
Subt. 1,06 × 1,06 × 1,06 × 1,06 = 1,26247696

Div. by $\overline{1,06}|^4$ − 1 = ,26247696),0757486176(,2885898

,2885898
Multiply by 207,9 *present worth.*

25973082
20201286
57717960

Answer, 59,99781942 = £60.

¶ TABLE IV is thus made;—Divide £1 by 1,06 = ,94339 the present worth of the first year, which, divided by 1,06, is equal to ,88999, which, added to the first year's present worth, is = 1,83339, the second year's present worth, then ,88999 divided by 1,06, and the quotient added to 1,83339, gives 2,6701 for the third year's present worth, &c.

Second

Second Method.

From 1,06 x 1,06 x 1,06 x 1,06 x 1,06 x 207,9=278,217097573
Take 1,06 x 1,06 x 1,06 x 1,06 x 207,9 = 262,468959984

Divide by $\overline{1{,}061}^{4}$ —1=,26247696)15,748137589(59,998=£60.

By TABLE V.*

MULTIPLY the Tabular number, correſponding with the Rate and Time, by the purchaſe-money, and the product will be the Annuity.

Under £6 per Cent. *and oppoſite* 4 *years, you will find*
,28859=*Annuity which* £1 *will purchaſe in* 4 *years.*
Multiply by 207,9

259731
202013
577180

59,997861 =£60.

2. WHAT Salary, to continue 20 years, will £688 3*s*. 10½*d*. purchaſe at £6 *per Cent*. Compound Intereſt? *Anſwer* £60.

CASE 3.

When the ANNUITY, PRESENT WORTH, *and* RATIO *are given, to find the* TIME.

RULE. Divide the Annuity by the product of the preſent worth and ratio ſubtracted from the ſum of the preſent worth and annuity, and the quotient will be that power of the ratio, denoted by the number of years, which being divided by the Ratio, and this quotient by the ſame, till nothing remain, the number of diviſions will ſhew the time:—*Or*, the above quotient being ſought in TABLE I. under the given Rate, in a line with it, you will ſee the time.

EXAMPLES.

1. FOR how long may an annuity of £60 *per Annum* be purchaſed for £207,906336762, at £6 *per Cent*. Compound Intereſt?

Multiply

* TABLE V. is made in this manner;—Divide £1 by the preſent worth of £1 for 1 year, and the quotient will be the annuity, which £1 will purchaſe for 1 year; Divide £1 by the preſent worth of £1 for 2 years, and the quotient will be the annuity, which £1 will purchaſe for 2 years, &c.

Multiply 207,906336762 *To* 207,906336762 = *present worth.*
by 1,06 *Add* 60 = *Annuity.*

1247438020572 *From* 267,906336762
2079063367620 *Subt.* 220,380716967

220,380716967 72 47,525619795 = *Divisor.*

47,525619795)60,000000000(1,26247696
Divide by 1,06)1,26247696

1,06)1,191016

1,06)1,1236

1,06)1,06

1 { *The number of Divisions* = *Time* = 4 *years.*

Or, $\dfrac{60}{207{,}906336762+60-207{,}906336762\times 1{,}06}=1{,}26247696$, *which being sought in* TABLE I *under the given Rate, in a line with it, is* 4 = 4 *years.*

2. How long may a Lease of £75 yearly rent be had for £533 1*s.* 8½*d.* allowing £5 *per cent.* Compound Interest to the Purchaser? *Ans.* 9 *years.*

ANNUITIES, LEASES, &c. *taken in* REVERSION AT COMPOUND INTEREST.

CASE I.

When the ANNUITY, TIME *and* RATIO *are given, to find the* PRESENT WORTH *of the annuity in Reversion.*

RULE.‡ 1. Divide the annuity by that power of the Ratio denoted by the Time of its continuance.

2. SUBTRACT this Quotient from the annuity:—Divide the Remainder by the Ratio less 1, and the Quotient will be the present worth, to commence immediately. 3. DIVIDE

‡ LET v denote the time in reversion, and the other letters as before. Then, the two cases under this rule will be expressed by the following Theorems.

I. $n-\dfrac{n}{r^t}=p$. Then change p into m, and $\dfrac{m}{r^v}=p$.

II. $pr^v=m$, Change m into p, and $\dfrac{pr^t\times r-pr^t}{r^t-1}=n$.

Or, I. $\dfrac{r^t-1\times n}{r-1\times r^t\times r^v}=p$. II. $\dfrac{r-1\times r^t\times r^v\times p}{r-1}=n$.

3. DIVIDE this Quotient by that power of the Ratio denoted by the time of Reversion, (or, time to come, before the annuity commences) and the quotient will be the present worth of the annuity in Reversion.

OR 1. Multiply the annuity by that Power of the Ratio denoted by the time of its continuance, minus unity, for a Dividend.

2. MULTIPLY that Power of the Ratio denoted by the Time of its continuance, that Power of it, denoted by the Time of Reversion, and the Ratio less 1 continually together for a Divisor, and the Quotient arising from the Division of these two numbers will be the present worth of the annuity in Reversion.

EXAMPLES.

1. WHAT is the present worth of £60 payable yearly, for 4 years; but not to commence till 2 years hence at £6 *per Cent.*?

First Method.

Ratio = 1,06
1,06
636
1060
2d. Power = 1,1236
1,1236
67416
33708
22472
11236
11236

Or, In Table 4th, find the present value of £1 at the given rate, both for the time in being and the time in reversion added together, and subtract the present worth of the time in being from the other, multiply the remainder by the annuity, and the product will be the answer.

Pres. worth of the time in being & reversion = 4,91732
Pres. worth of the time in being = 1,8333
3,08402
60
£185,04120

Div. by 4th pow. = 1,26247696)60,0000000(47,525621378467
Subtract the quotient = 47,525621378467

Divide by 1,06—1 = ,06)12,4743786215333

Divide by 1,06 x 1,06 = 1,1236)207,906310358 8(185,035876 = £185 0s. 0¾. *the Present worth of the annuity in Reversion.*

Or, $\frac{60}{1.26247696} = 47,5256$ $\frac{60-47,5256}{1,06-1} = 207,906.$

And $\frac{207,906,}{1,1236} = 185,035876$

Second Method.

,26247696 = *4th. Power*—1
Multiply by 60 = *Annuity.*
15,74861760 *Dividend.*
1,26247696 *4th Power.*
1,1236 = *2d. Power.*
757486176
378743088
252495392
126247696
126247696
1,418519112256
,06 = *Ratio* — 1
,08511114673536 = *Divisor.*

,08511115)15,74861760(185,036 *Answ.*

Or, $\frac{\overline{1,06}|^4 - 1 \times 60}{\overline{1,06}|^4 \times \overline{1,06}|^2 \times \overline{1,06-1}} = 185,036$

2. WHAT

2. WHAT is the present worth of a Reversion of a Lease of £60 *per Annum*, to continue 20 years, but not to commence till the end of 8 years, allowing £6 *per Cent*. to the purchaser?

Answer £431 15*s*. 7*d*. 2,7819*qrs*.

An Annuity, several times in reversion, and rate being given, to find the several present values.

FIND the present value of £1 *per* TABLE IV. at the given rate, and for the several given times, which being severally multiplied by the annuity, the products will be the several present values of that annuity, for the several times given; subtract the several present values, the one from the other, and the several remainders will answer the question.

3. A has a Term of 6 years in an Estate of £60 *per Annum*. B has a Term of 14 years in the same estate, in reversion, after the 6 years are expired; and C hath a further Term of 16 years, after the expiration of 20 years. I demand the present values of the several Terms, at 6 *per Cent*.?

			£.	s.	d.	
Pres. value of £1 *for*	36	*years*=14,61722×60=	877	0	$7\frac{3}{4}$	
Ditto of ditto for	20	=11,46992×60=	688	3	$10\frac{3}{4}$	
Ditto of ditto for	6	= 4,91732×60=	295	0	$9\frac{1}{4}$	=*A's term*.

Therefore, 877 0 $7\frac{3}{4}$—688 3 $10\frac{3}{4}$=£188 16 9 *C's Term, and*
688 3 $10\frac{3}{4}$—295 0 $9\frac{1}{4}$=£393 3 $1\frac{1}{2}$=*B's Term*.

4. FOR a Lease of certain profits for 7 years, A offers to pay £300 gratuity, and £300 *per Annum*, B offers £800 gratuity and £250 *per Annum*, C bids £1300 gratuity and £200 *per Annum*, and D bids £2500 for the whole purchase, without any yearly rent; which is the best offer, computing at 6 *per Cent*.?

	£.
By Table 4, *the present worth of* £300 *per Annum, for* 7 *years, at* 6 *per Cent. is*	1674,714
To which add	300
Value of A's offer =	1974,714
Present worth of £250 per Annum, *for* 7 *years* =	1395,595
To which add	800
Value of B's offer =	2195,595
Present worth of £200 per Annum *for* 7 *years* =	1116,476
To which add	1300
Value of C's offer =	2416,476
D's offer =	2500

Hence it appears that D's offer is the best.

The

The above Questions may be answered by the 4th and 2d TABLES.
Take Question 1st for example.

1. MULTIPLY the Tabular number in Table IV. corresponding to the Rate and the Time of continuance, into the annuity, and the Product will be the present worth, to commence immediately.

2. MULTIPLY this present worth by the Tabular number in TABLE II. corresponding to the Rate and the time of Reversion, and the Product will be the present worth of the annuity in Reversion.

In Table 4th we have 3,4651
Multiply by 60 = *Annuity.*

207,9060
In Table 2d. we have ,889996

1247436
1871154
1871154
1871154
1663248
1663248

185,035508376 = *pres. worth of the revers.*

CASE 2.

When the PRESENT WORTH *of the Reversion,* RATE *and* TIME *are given, to find the* ANNUITY.

RULE. 1. Multiply that Power of the Ratio signified by the Time of Reversion, by the present worth, and the product will be the amount of the present worth for the Time before the annuity commences.

2. MULTIPLY that Power of the Ratio signified by the Time of continuance plus 1 by the last Product.

3. MULTIPLY that Power of the Ratio, signified by the Time by the aforesaid product, and this last Product, divided by that power of the Ratio denoted by the time, minus unity, will give the annuity.

OR, Divide the continual Product of the present worth, that Power of the Ratio denoted by the Time of continuance, that Power of it denoted by the Time of Reversion, and the Ratio minus 1, by that Power of the Ratio denoted by the Time of continuance minus 1, and the quotient will be the annuity.

EXAMPLES.

1. WHAT annuity, to be entered upon 2 years hence, and then to continue 4 years, may be purchased for £185,035876, at £6 *per Cent.*?

First

First Method.

1,06 x 1,06 = 1,1236 = *2d. Power of the Ratio.*
Multiply by 185,036 = *Present worth.*

67416
33708
561800
89888
11236

207,9064496 *Amount for the time of Reversion.*
Multiply by 1,33822 = *5th power of the Ratio.*

415812
415812
1663248
623718
623718
207906

From 278,22396732
Take 262,47508782

4th power of the Ratio = 1,26247
Multiply by 207,906

757482
11362230
883729
2524940

262,47508782

Divide by $\overline{1,06}|^4$—1=,26247)15,74887950(60 *the annuity required.*

Or, 185,036 x 1,1236 = 207,906

Then, $\dfrac{\overline{207,906 \times 1,33822} - \overline{207,906 \times 1,26247}}{1,26247-1} =$ £60 *Ans.*

Second Method.

185,036 = *present worth of the Reversion.*
1,26247 = *4th Power of the Ratio.*

1295252
740144
370072
1110216
370072
185036

233,6024
1,1236 = *2d. power of ditto.*

14016144
7008072
4672048
2336024
2336024

262,47565664
,06 = *Ratio* — 1

$\overline{1,06}|^4$—1=,26247)15,7485393984(60.

Or, $\dfrac{\overline{185,036 \times 1,26247 \times 1,1236 \times 1,06 - 1}}{1,26247-1} = 60.$

Or by Table 4. Divide the present worth of the Reversion by the difference between the present worth of £1 for the time both in being and reversion, and the time in being, and the quotient will be the annuity.

4,91732
1,8333

3,08402)185,0412(60 *Ans.*

2. The present worth of a Lease of an house is £431 15*s.* 7*d.* 2,7819 *qrs.* taken in Reversion for 20 years; but not to commence till the end of 8 years, allowing £6 *per Cent.* to the purchaser; what is the yearly Rent? *Ans.* £60.

Purchasing ANNUITIES *forever*, *or* FREEHOLD ESTATES, *at* COMPOUND INTEREST.

CASE 1.

When the ANNUITY, *or* YEARLY RENT, *and the* RATE *are given, to find the* PRESENT WORTH, *or* PRICE.

RULE.* As the Rate *per Cent.* is to £100; so is the yearly rent, to the value required.

OR, Divide the yearly rent by the Ratio less 1, and the Quotient will be the value required.

EXAMPLES.

1. WHAT is the worth of a freehold Estate of £60 *per Annum*, allowing £6 *per Cent.* to the purchaser?

```
£.   £.    £.
6 : 100 :: 60
      60
    ----
  6)6000
    ----
  £1000 the Answer.
```

Or, 1,06−1=,06)60,00 = 1000

2. AN Estate brings in yearly, £75; what would it sell for, allowing the purchaser £5 *per Cent.* Compound Interest?

Answer, £1500.

CASE 2.

When the PRICE, *or* PRESENT WORTH, *and* RATE *are given, to find the* ANNUITY, *or* YEARLY RENT.

RULE. As £100 is to the Rate, so is the present worth to its rent.

Or, Multiply the present worth by the Ratio less 1, and the Product will be the yearly Rent.

EXAMPLES.

1. IF a freehold Estate be bought for £1000, allowing £6 *per Cent.* to the Purchaser; what is the yearly Rent?

```
100 : 6 :: 1000
              6
           ----
     100)6000(£60 Ans.
         600
         ---
           0
```

Or, 1000×,06=£60.

2. IF

* THE reason of this rule is obvious; for since a year's interest of the price, which is given for it, is the annuity, there can neither more nor less be made of that price, than of the annuity, whether it be employed at simple or compound interest.

THE following Theorems shew all the varieties of this rule.

I. $\frac{n}{r-1}=p$. II. $\overline{r-1}\times p=n$. III. $\frac{n}{p}+1=r$, or $\frac{n}{p}=r-1$, or $\frac{p+n}{p}=r$.

2. If an Eſtate be ſold for £1500, and 5 *per Cent.* allowed to the buyer; what is the yearly rent? *Anſwer* £75.

Case 3.

When the Present Worth, *or* Price, *and yearly* Rent *are given, to find the* Rate.

Rule. As the Preſent Worth is to the Rent; ſo is £100 to the Rate.

Or, Divide the Rent by the preſent worth; add 1 to the quotient, and the ſum will be the ratio of the rate *per Cent.*

Or, Divide the ſum of the preſent worth and rent by the preſent worth, and the quotient will be the ratio.

Examples.

1. If an Eſtate of £60 *per Annum* be bought for £1000; what rate of Intereſt was allowed the Purchaſer for his money?

```
£.    £.    £.
1000 : 60 :: 100          Or, 1000)60,00(,06+1=1,06
       100                          6000
                                    ----
       ----
1000)6000(£6 Anſwer.
     6000                 Or, To 1000 = preſent worth.
     ----                    Add   60 = Rent.
                                 ----
                          1000)1060(1,06
                               1000
                               ----
                                 6000
                                 6000
                                 ----
```

2. An Eſtate of £75 *per Annum* was purchaſed for £1500; what Rate of Intereſt had the Buyer for his money? *Anſw.* £5.

To find at how many year's purchaſe an Eſtate may be bought.

Case 1.

When the Rate *of Intereſt is given, to find the* Number *of years.*

Rule. Divide £100 by the Rate, and the *Quotient* will be the years.

Examples.

1. How many year's purchaſe ſhould a gentleman offer for the purchaſe of an Eſtate, to have £6 *per Cent.* for his Money?

```
6)100
-----
16,666+=16 2/3 years.
```

2. How many year's purchaſe is an Eſtate worth, allowing £5 *per Cent.* to the purchaſer? *Anſwer* 20 *years.*

Case 2.

When the Number *of year's purchaſe, at which an Eſtate is bought, or ſold, is given, to find the* Rate *of Intereſt.*

Rule. Divide £100 by the number of years, and the Quotient will be the Rate.

Examples.

EXAMPLES.

1. A Gentleman gives $16\frac{2}{3}$ years purchase for a farm; what Interest is he allowed?

$16\frac{2}{3} = 16{,}666 +$)100,000(£6 *Answer.*

2. A Gentleman gives 20 year's purchase for an Estate, what Interest has he? *Ans.* £5.

PURCHASING FREEHOLD ESTATES IN REVERSION.

CASE I.

The Rate and Rent of a Freehold Estate being given, to find the present worth of Reversion.

RULE.§ Find the present worth of the Annuity or Rent, (by Case 1. of purchasing Freehold Estates, page 314th.) as though it were to be entered on immediately.

2. DIVIDE the last present worth by that Power of the Ratio denoted by the time of Reversion (by Case 1st. of Discount by Compound Interest) and the quotient will be the answer required.

Or, 1. Having found the present value of the Estate, supposing it to be immediate; Multiply the Annuity, or Rent, by the present worth of £1 corresponding with the time of Reversion and Rate in TABLE IV. and the product will be the present worth of the Annuity or Rent, for the time of Reversion; or the value of the present possession.

2. SUBTRACT the value of the Possession from the value of the Estate, and the remainder will be the value of Reversion.

EXAMPLES.

1. SUPPOSE a Freehold Estate of £60 *per Annum* to commence 2 years hence, be put up to sale; what is its value, allowing the Purchaser £6 *per Cent.?*

First Method.

1,06—1=,06)60,00 = *Rent per Annum.*

1000 = *Present worth, if entered on immediately.*

$\overline{1{,}06}|^2$ = 1,1236)1000,000(890,07653 = £890 1/6¼ = *present worth of* £1000 *for 2 years, or the whole present worth required.*

Second

§ THE following Theorems express all the Cases under this rule.

I. $\frac{n}{r-1} = p$; then change p into m, and $\frac{m}{r^v} = p$.

II. $pr^v = m$; then change m into p, and $\frac{prr-pr}{r} = n$.

Second Method.

1.06—1=,06)60,00

1000 = *present worth, for immediate possession.*

In Table 4th. we have, 1,83339 = *Value of* £1 *for* 2 *years.*

Multiply by 60 = *Rent.*

110,00340 = *Value of Possession.*

From 1000,0000

Subtract 110,0034

889,9966 = *Value required.*

2. Suppose an Estate of £75 *per Annum*, to commence 10 years hence, were to be sold, allowing the Purchaser £5 *per Cent.*; what is its worth? *Ans.* £920 17*s.* 5*d.*

Case 2.

The Value *of a Reversion, the* Time *prior to its Commencement, and* Rate *of Interest given, to find the* Annuity *or* Rent.

Rule. 1. Multiply the Price of the Reversion by that Power of the Amount of £1 for 1 year, denoted by the Time of Reversion, and the Product will be its amount (by Case 1, of Compound Interest.)

2. Find the Interest of the amount (by Case 1*st* Simple Interest) and it will be the Annuity, or yearly rent.

Examples.

1. A Freehold Estate is bought for £889,9966 which does not commence till the end of 2 years; the Buyer being allowed £6 *per Cent.* for his Money; I desire to know the yearly Income?

889,9966 = *Price of the Reversion.*

Multiply by $\overline{1,06}|^2$ = 1,1236 *Denoted by the time of Reversion.*

53399796
26699898
17799932
8899966
8899966

1000,00017976 = *Amount of the Reversion.*
,06

Answer, £60,00

2. If a freehold Estate, to commence 10 years hence, be sold for £920 17*s.* 5*d.* allowing the Purchaser £5 *per Cent.*; what is the yearly Income? *Ans.* £75.

Table

TABLE I. *Shewing the amount of £1 from 1 year to 50.*

ys.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
1	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000	1,0550000	1,0600000
2	1,0609000	1,0712250	1,1816000	1,0920250	1,1025000	1,1130250	1,1236000
3	1,0927270	1,1087178	1,1248640	1,1411661	1,1576250	1,1742413	1,1910160
4	1,1255088	1,1475230	1,1698585	1,1925186	1,2155062	1,2388245	1,2624769
5	1,1592740	1,1876863	1,2166529	1,2461819	1,2762815	1,3069[illegible]98	1,3382256
6	1,1940523	1,2292553	1,2653190	1,3022601	1,3400956	1,3788426	1,4185191
7	1,2298738	1,2722792	1,3159317	1,3608618	1,4071004	1,4546789	1,5036302
8	1,2667700	1,3168090	1,3685690	1,4221006	1,4774554	1,5346862	1,5938480
9	1,3047731	1,3628973	1,4233118	1,4860951	1,5513282	1,6190939	1,6894789
10	1,3439163	1,4105987	1,4802842	1,5529694	1,6288946	1,7081440	1,7908476
11	1,3842338	1,4599697	1,5394540	1,6228530	1,7103393	1,8020919	1,8982985
12	1,4257608	1,5110686	1,6010322	1,6958814	1,7958563	1,9012069	2,0121964
13	1,4685337	1,5639560	1,6650735	1,7721961	1,8856491	2,0057732	2,1329282
14	1,5125897	1,6186945	1,7316764	1,8519449	1,9799316	2,1160907	2,2609039
15	1,5579674	1,6753488	1,8009435	1,9352824	2,0789281	2,2324756	2,3965581
16	1,6047064	1,733986	1,8729812	2,0223701	2,1828745	2,3552617	2,5472716
17	1,6528476	1,7946755	1,9479005	2,1133768	2,2920183	2,4848011	2,6927727
18	1,7024330	1,8574892	2,0258161	2,2084787	2,4066192	2,6214652	2,8543391
19	1,7535060	1,9225013	2,1068491	2,3078603	2,5269502	2,7656458	3,0255995
20	1,8061112	1,9897888	2,1911231	2,4117140	2,6532977	2,9177563	3,2071355
21	1,8602945	2,0594314	2,2787680	2,5202411	2,7859625	3,0782329	3,3995636
22	1,9161034	2,1315115	2,3699187	2,6336520	2,9252607	3,2475357	3,6035374
23	1,9735865	2,2061144	2,4647155	2,7521663	3,0715237	3,4261502	3,8197496
24	2,0327941	2,2833284	2,5633041	2,8760138	3,2250999	3,6145885	4,0489346
25	2,0937779	2,3632449	2,6658363	3,0054344	3,3863549	3,8133910	4,2918707
26	2,1565912	2,4459585	2,7724697	3,1406790	3,5556726	4,0231279	4,5493829
27	2,2212890	2,5315671	2,8833685	3,2820095	3,7334563	4,2443999	4,8223459
28	2,2879276	2,6201719	2,9987033	3,4296999	3,9201291	4,4778419	5,1116867
29	2,3565655	2,7118779	3,1186514	3,5840364	4,1161356	4,7241232	5,4183879
30	2,4272624	2,8067937	3,2433975	3,7453181	4,3219423	4,9839499	5,7434912
31	2,5000803	2,9050314	3,3731334	3,9138574	4,5380394	5,2580671	6,0881007
32	2,5750827	3,0067075	3,5080587	4,0899810	4,7649414	5,5472608	6,4533867
33	2,6523352	3,1119423	3,6483811	4,2740301	5,0031885	5,8523600	6,8405899
34	2,7319053	3,2208603	3,7943163	4,4663615	5,2533479	6,1742398	7,2510253
35	2,8138624	3,3335904	3,9460889	4,6673478	5,5160152	6,5138230	7,6860868
36	2,8982783	3,4502661	4,1039325	4,8773784	5,7918101	6,8720832	8,147252
37	2,9852266	3,5710254	4,2680898	5,0968604	6,0814069	7,2500478	8,6360871
38	3,0747834	3,6960113	4,4388134	5,3262192	6,3854772	7,6488004	9,1542523
39	3,1670269	3,8253717	4,6163659	5,5658990	6,7047511	8,0694844	9,7035074
40	3,2620377	3,9592597	4,8010206	5,8164645	7,0399887	8,5133060	10,2857178
41	3,3598988	4,0978337	4,9930614	6,0782054	7,3919881	8,9815378	10,9028608
42	3,4606958	4,2412579	5,1927838	6,3517246	7,7615879	9,4755224	11,5570325
43	3,5645167	4,3897020	5,4004952	6,6375522	8,1496669	9,9966761	12,2504547
44	3,6714522	4,5433415	5,616515	6,9362421	8,5571502	10,5464933	12,9854817
45	3,7815957	4,7023585	5,8411756	7,248373	8,9850077	11,1265504	13,7626109
46	3,8950436	4,8669411	6,0748236	7,5745497	9,4342581	11,7385217	14,5883673
47	4,0118949	5,0372840	6,3168166	7,9154045	9,9059710	12,3841404	15,4636693
48	4,1322518	5,2135889	6,5694892	8,2715977	10,4012696	13,0652681	16,3914894
49	4,2562193	5,3960645	6,8322688	8,6438196	10,9213331	13,7838579	17,3749788
50	4,3839059	5,5849268	7,1055596	9,0327915	11,4673697	14,5419700	18,2174775

TABLE II. *Shewing the present value of £1, due at the end of any number of years, from 1 to 40.*

yrs	4 per Cent.	4½ per cent.	5 pr cent.	5½ pr cent.	6 per cent.	yrs
1	,961538	,956938	,952381	,947867	,943396	1
2	,924556	,91573	,90703	,898513	,889996	2
3	,888996	,876297	,863838	,851728	,839619	3
4	,854804	,838561	,822702	,807397	,792093	4
5	,821927	,802451	,783526	,765392	,747258	5
6	,790314	,767896	,746215	,725587	,70496	6
7	,759918	,734828	,710681	,687869	,665057	7
8	,730690	,703185	,676839	,652125	,627412	8
9	,702587	,672904	,644609	,618253	,591898	9
10	,675564	,643928	,613913	,586153	,558394	10
11	,649581	,616199	,584679	,573733	,562787	11
12	,624597	,589664	,556837	,526903	,496969	12
13	,600574	,564271	,530321	,49958	,468839	13
14	,577475	,539973	,505068	,473684	,442301	14
15	,555264	,516720	,481017	,449141	,417265	15
16	,533908	,494469	,458311	,425979	,393647	16
17	,513373	,473176	,436297	,40383	,371364	17
18	,493628	,4528	,415521	,382932	,350343	18
19	,474642	,433302	,395734	,363123	,330513	19
20	,456387	,414643	,376889	,344346	,311804	20
21	,438833	,396787	,358942	,326568	,294155	21
22	,421955	,379701	,34185	,309677	,277505	22
23	,405726	,36335	325571	,293684	,261797	23
24	,390121	,347703	,310068	,278523	,246978	24
25	,375117	,332731	,305303	,26915	,232998	25
26	,360689	,318402	,281241	,250525	,21981	26
27	,340816	,304691	,267848	,237608	,207368	27
28	,333477	,291571	,255094	,225362	,19563	28
29	,320651	,279015	,242946	,213715	,184556	29
30	,308309	,267	,231377	,202743	,17411	30
31	,290460	,255502	,220359	,192307	,164255	31
32	,285058	,2445	,209866	,182411	,154957	32
33	,274094	,233971	,199872	,173029	,146186	33
34	,263552	,223896	,190355	,164133	,137912	34
35	,254415	,214251	,18129	,155692	,130105	35
36	,243669	,205028	,172057	,147399	,122741	36
37	,234297	,196299	,164436	,140114	,115793	37
38	,225285	,18775	,156605	,132893	,109182	38
39	,216671	,179665	,149148	,126075	,103002	39
40	,208289	,171929	,142046	,119608	,09717	40

TABLE III. *Shewing the amount of £1 annuity for any number of years, from 1 to 40.*

yrs	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.	ys
1	1,	1,	1,	1,	1,	1
2	2,04	2,045	2,05	2,055	2,06	2
3	3,1216	3,137025	3,1525	3,16802	3,1836	3
4	4,246464	4,278191	4,310125	4,34226	4,374616	4
5	5,416322	5,47071	5,525631	5,58109	5,637093	5
6	6,632975	6,716892	6,801913	6,888051	6,975318	6
7	7,898294	8,019152	8,142008	8,26689	8,393837	7
8	9,214266	9,380014	9,549109	9,721573	9,897467	8
9	10,582795	10,802114	11,026564	11,256259	11,491315	9
10	12,006107	12,2882	12,577892	12,875354	13,180794	10
11	13,486351	13,841179	14,206787	14,583498	14,971642	11
12	15,025805	15,464032	15,917126	16,38559	16,86994	12
13	16,626838	17,159913	17,712983	18,286798	18,882132	13
14	18,291911	18,932109	19,598632	20,292572	21,015064	14
15	20,023588	20,784054	21,578563	22,408663	23,275968	15
16	21,824531	22,719337	23,657492	24,64114	25,672527	16
17	23,697512	24,741707	25,840366	26,996402	28,212879	17
18	25,645413	26,855084	28,132385	29,481205	30,905652	11
19	27,671229	29,063562	30,529004	32,102671	33,759991	19
20	29,778078	31,371423	33,065954	34,868318	36,78559	20
21	31,969202	33,783137	35,719252	37,786075	39,992725	21
22	34,24797	36,303378	38,505214	40,864309	43,392289	22
23	36,617888	38,93703	41,430475	44,111846	46,995826	23
24	39,082604	41,689196	44,501999	47,537998	50,815576	24
25	41,645908	44,56521	47,727099	51,152588	54,86451	25
26	44,311745	47,570645	51,113454	54,96598	59,156381	26
27	47,084214	50,711324	54,669126	58,989109	63,705763	27
28	49,967583	53,993333	58,402583	63,23351	68,528109	28
29	52,966286	57,423033	62,322712	67,711353	73,639796	29
30	56,084938	61,007067	66,438847	72,435478	79,058183	30
31	59,328335	64,752388	70,76079	77,419429	84,801674	31
32	62,701469	68,666245	75,298829	82,677498	90,889775	32
33	66,209527	72,756226	80,063771	88,22476	97,343161	33
34	69,857908	77,030256	85,066959	94,077122	104,183751	34
35	73,652225	81,496618	90,320307	100,251363	111,434776	35
36	77,598314	86,163966	95,836323	106,765188	119,120863	36
37	81,702246	91,041344	101,628139	113,637274	127,268114	37
38	85,970336	96,138205	107,709546	120,887324	135,904201	38
39	90,40915	101,464424	114,095025	128,536127	145,058453	39
40	95,025516	107,030323	120,799774	136,605614	154,761961	40

Table IV. *Shewing the present worth of £1, annuity, for any number of years, from 1 to 40.*

yrs.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent	6 per cent.
1	0,96154	0,95694	0,95238	0,94786	0,94339
2	1,88609	1,87267	1,85941	1,8463	1,83339
3	2,77509	2,74896	2,72325	2,6979	2,67301
4	3,62989	3,58752	3,54595	3,49862	3,4651
5	4,45182	4,38997	4,32948	4,25759	4,21236
6	5,24214	5,15787	5,07569	4,97699	4,91732
7	6,00205	5,8927	5,78637	5,65888	5,58238
8	6,73274	6,59589	6,46321	6,30522	6,20979
9	7,43533	7,26879	7,10782	6,91786	6,80169
10	8,11089	7,91272	7,72173	7,49856	7,36008
11	8,76048	8,52892	8,30641	8,04898	7,88687
12	9,38507	9,11858	8,86325	8,5707	8,38384
13	9,98565	9,68285	9,39357	9,06522	8,85268
14	10,56312	10,22282	9,89864	9,53395	9,29498
15	11,11839	10,73954	10,37966	9,97824	9,71225
16	11,65229	11,23401	10,83777	10,39936	10,10589
17	12,16567	11,70719	11,27407	10,79852	10,47726
18	12,65929	12,15099	11,68958	11,17687	10,8276
19	13,13394	12,59329	12,08532	11,53549	11,15811
20	13,59032	13,00793	12,46221	11,87541	11,46992
21	14,02916	13,40472	12,82115	12,1976	11,76407
22	14,45111	13,78442	13,163	12,50299	12,04158
23	14,85684	14,14777	13,48807	12,79245	12,30338
24	15,24696	14,49548	13,79864	13,06682	12,55035
25	15,62208	14,82821	14,09394	13,3688	12,78335
26	15,98277	15,14661	14,37518	13,57338	13,00316
27	16,32959	15,4513	14,64303	13,80702	13,21053
28	16,66306	15,74287	14,89813	14,02848	13,40616
29	16,98371	16,02189	15,14107	14,23838	13,59072
30	17,20202	16,28889	15,37245	14,43733	13,76483
31	17,58849	16,54439	15,59281	14,6259	13,92908
32	17,87355	16,78889	15,80268	14,80463	14,08398
33	18,14764	17,02286	16,00255	14,97404	14,22917
34	18,4112	17,24676	16,1929	15,13461	14,36613
35	18,66461	17,46101	16,37419	15,2868	14,49533
36	18,90828	17,66604	16,54685	15,43105	14,61722
37	19,14258	17,86224	16,71129	15,56779	14,73211
38	19,36787	18,04999	16,86789	15,6974	14,84048
39	19,58448	18,22965	17,01704	15,82024	14,9427
40	19,79277	18,40158	17,15909	15,93667	15,03913

TABLE V. *The annuity which £1 will purchase for any number of years to come, from 1 to 40.*

yrs.	4 pr cent.	$4\frac{1}{2}$ per cent.	5 per cent.	$5\frac{1}{2}$ per cent.	6 per cent.
1	1,04	1,045	1,05	1,055	1,06
2	,5302	,534	,5378	,54162	,54544
3	,36035	,36377	,36721	,37065	,37411
4	,27549	,27874	,28201	,28582	,28859
5	,22463	,22779	,23097	,23487	,23739
6	,19076	,19388	,19702	,20092	,20336
7	,16661	,1697	,17282	,17671	,17913
8	,14853	,15161	,15473	,15859	,16103
9	,13449	,13757	,14069	,14455	,14702
10	,12329	,12638	,1295	,13334	,13587
11	,11415	,11725	,12039	,12424	,12679
12	,10655	,10967	,11282	,11667	,11927
13	,10014	,10327	,10645	,11031	,11296
14	,09467	,09782	,10102	,10489	,10758
15	,08994	,09311	,09624	,10022	,10296
16	,08582	,08901	,09227	,0962	,09895
17	,0822	,08542	,0887	,0926	,09544
18	,07899	,08224	,08555	,08947	,09235
19	,07614	,07941	,08274	,08699	,08962
20	,07359	,07688	,08024	,08427	,08718
21	,07128	,0746	,078	,08198	,085
22	,0692	,07254	,07597	,07998	,08303
23	,06731	,07068	,07414	,07825	,08128
24	,06559	,06899	,07247	,07653	,07968
25	,06401	,06744	,07095	,07503	,07823
26	,06257	,06602	,06956	,07367	,0769
27	,06124	,06472	,06829	,07242	,0757
28	,06001	,06352	,06712	,07128	,07459
29	,05888	,06241	,06604	,07023	,07358
30	,05783	,06139	,06505	,06926	,07272
31	,05685	,06044	,06413	,06837	,07179
32	,05595	,05956	,06328	,06754	,071
33	,0551	,05874	,06249	,06678	,07027
34	,05431	,05798	,06175	,06607	,06959
35	,05358	,05727	,06107	,06541	,06899
36	,05289	,0566	,06043	,0648	,06839
37	,05224	,05224	,05984	,06423	,06785
38	,05163	,0554	,05928	,0637	,06735
39	,05106	,05485	,05876	,06321	,06689
40	,05052	,05434	,05828	,06274	,06646

CIRCULATING DECIMALS

ARE produced from Vulgar Fractions, whose denominators do not measure their numerators, and are distinguished by the continual repetition of the same figures.

1. THE circulating figures are called *repetends*; and, if one figure only repeats, it is called a *single repetend*; As ,1111 &c. ,6666 &c.

2. A *compound repetend* has the same figures circulating alternately, As ,010101 &c. ,379379379 &c.

3. IF other figures arise before those which circulate, the decimal is called a *mixed repetend*; thus, ,375555 &c. is a *mixed single repetend*, and ,378123123 &c. a *mixed compound repetend*.

4. A single repetend is expressed by writing only the circulating figure with a point over it; thus, ,1111 &c. is denoted by $,\dot{1}$, and ,6666 &c. by $,\dot{6}$.

5. COMPOUND repetends are distinguished by putting a point over the first and last repeating figures; thus, ,010101 &c. is written $,\dot{0}\dot{1}$ and ,379379379 &c. thus $,\dot{3}7\dot{9}$.

6. *Similar circulating decimals* are such as consist of the same number of figures, and begin at the same place, either before or after the decimal point; thus; $,\dot{3}$ and $,\dot{5}$ are similar circulates; as are also $\dot{3},5\dot{4}$ and $7,\dot{3}\dot{6}$, &c.

7. *Dissimilar repetends* consist of an unequal number of figures, and begin at different places.

8. *Similar and conterminous circulates* are such as begin and end at the same place; as $47,3\dot{4}57\dot{6}$, $9,7\dot{3}52\dot{8}$ and $,0\dot{5}46\dot{3}$, &c.

REDUCTION OF CIRCULATING DECIMALS.

CASE I.

To reduce a simple repetend to its equivalent Vulgar Fraction.

RULE.† 1. Make the given decimal the numerator, and let the denominator be a number, consisting of so many nines as there are recurring places in the repetend.

2. IF

† IF unity, with cyphers annexed, be divided by 9 *ad infinitum*, the quotient will be 1 continually; that is, if $\frac{1}{9}$ be reduced to a decimal, it will produce the circulate $,\dot{1}$, and since $,\dot{1}$ is the decimal equivalent to $\frac{1}{9}$, $,\dot{2}$ will $=\frac{2}{9}$, $\dot{3}=\frac{3}{9}$, and so on till $\dot{9}=\frac{9}{9}=1$.—Therefore every single repetend is equal to a vulgar fraction, whose numerator is the repeating figure, and denominator 9.

AGAIN, $\frac{1}{99}$ or $\frac{1}{999}$, being reduced to decimals, make ,010101 &c. & ,001001001 &c. *ad infinitum* $=,\dot{0}\dot{1}$ and $,\dot{0}0\dot{1}$; that is, $\frac{1}{99}=,\dot{0}\dot{1}$, and $\frac{1}{999}=,\dot{0}0\dot{1}$, consequently $\frac{2}{99}=,\dot{0}\dot{2}$, $\frac{3}{99}=,\dot{0}\dot{3}$, &c. and $\frac{2}{999}=,\dot{0}0\dot{2}$, $\frac{3}{999}=,\dot{0}0\dot{3}$, &c. and the same will hold universally.

2. If there be integral figures in the circulate, ſo many cyphers muſt be annexed to the numerator as the higheſt place of the repetend is diſtant from the decimal point.

Examples.

1. Required the leaſt vulgar fractions equal to $,\dot{3}$ and $,\dot{3}2\dot{4}$.

$,\dot{3}=\frac{3}{9}=\frac{1}{3}$; *and* $,\dot{3}2\dot{4}=\frac{324}{999}=\frac{12}{37}$ *Anſw.* $\frac{1}{3}$ *and* $\frac{12}{37}$.

2. Reduce $,\dot{7}$ to its equivalent vulgar fraction. *Anſw.* $\frac{7}{9}$.

3. Reduce $2,\dot{3}\dot{7}$ to its equivalent vulgar fraction. *Anſ.* $\frac{2370}{999}$.

4. Required the leaſt vulgar fraction equal to $,\dot{3}8461\dot{5}$.

Anſw. $\frac{5}{13}$.

Case 2.

To reduce a mixed repetend to its equivalent vulgar fraction.

Rule.‡ 1. To ſo many nines as there are figures in the repetend, annex ſo many cyphers as there are finite places, (that is, as there are decimal places before the repetend) for a denominator.

2. Multiply the nines in the ſaid denominator by the finite part, and add the repeating decimals to the product for the numerator.

3. If the repetend begins in ſome integral place, the finite value of the circulating part muſt be added to the finite part.

Examples.

1. What is the vulgar fraction equivalent to $,15\dot{3}$?

There being 1 *figure in the repetend, and* 2 *finite places, I annex* 2 *Cyphers to* 9 *for a denominator, viz.* 900 ; *then I multiply the* 9 *in the denominator by the two figures in the finite part, and add the repeating figure for a numerator* ; *thus,* $9\times15+3=138$ *Numerator.*

Therefore, $,15\dot{3}=\frac{138}{900}=\frac{23}{150}$ *the Anſwer.*

2. What is the leaſt vulgar fraction equal to $,41\dot{2}\dot{3}$? *Anſ.* $\frac{4079}{9990}$

Case 3.

To make any number of diſſimilar repetends ſimilar and conterminous ; *that is, of an equal number of places.*

Rule.

‡ In like manner for a mixed circulate; conſider it as diviſible into its finite and circulating parts, and the ſame principle will be ſeen to run through them alſo; thus the mixed circulate $1\dot{3}$ is diviſible into the finite decimal ,1, and the repetend $,0\dot{3}$; but $,1=\frac{1}{10}$, and $,0\dot{3}$ would be equal to $\frac{3}{9}$ provided the circulation began immediately after the place of units; but as it begins after the place of tenths, it is $\frac{3}{9}$ of $\frac{1}{10}=\frac{3}{90}$, and ſo the vulgar fraction $=,1\dot{3}$ is $\frac{1}{10}+\frac{3}{90}=\frac{9}{90}+\frac{3}{90}=\frac{12}{90}$, & is the ſame as by the rule

Rule.† Change them into other repetends, which ſhall each conſiſt of ſo many figures, as the leaſt common multiple of the ſums of the ſeveral numbers of places, found in all the repetends, contains Units.

E x a m p l e s.

1. Make $6,\dot{3}1\dot{7}$; $3,4\dot{5}$; $52,3$; $191,\dot{0}\dot{3}$; $,\dot{0}5\dot{7}$; $5,\dot{3}$ and $1,35\dot{9}$ ſimilar and conterminous.

Here, in the firſt repetend, there are 3 places, in the ſecond, 1, in the third, none, in the fourth, 2, in the fifth, 3, in the ſixth, 1, and in the ſeventh, 1.

Now find the leaſt common multiple of theſe ſeveral ſums, thus,

$3\,\big)\,\underline{3,\ 1,\ 2,\ 3,\ 1,\ 1}$
$\quad 1,\ 1,\ 2,\ 1,\ 1,\ 1$

and $2 \times 3 = 6$ units; therefore, the ſimilar and conterminous repetends muſt contain 6 places. ‡

Diſſimilar		*Made ſimilar and conterminous.*
$6,\dot{3}1\dot{7}$	=	$6,31\dot{7}3173\dot{1}$
$3,4\dot{5}$	=	$3,45\dot{5}5555\dot{5}$
$52,3$	=	$52,30\dot{0}0000\dot{0}$
$191,\dot{0}\dot{3}$	=	$191,03\dot{0}3030\dot{3}$
$,\dot{0}5\dot{7}$	=	$,05\dot{7}0570\dot{5}$
$5,\dot{3}$	=	$5,33\dot{3}3333\dot{3}$
$1,35\dot{9}$	=	$1,35\dot{9}9999\dot{9}$

2. *Make* $,5\dot{3}\dot{1}$, $,73\dot{4}\dot{8}$, $,\dot{0}\dot{7}$ *and* $,05\dot{0}\dot{3}$ *ſimilar and conterminous.*

C a s e 4.

To find whether the decimal fraction, equal to a given vulgar one, be finite or infinite, and how many places the repetend will conſiſt of.

Rule.

† Any given repetend whatever, whether ſingle, compound, pure, or mixed, may be transformed into another repetend, which ſhall conſiſt of an equal or greater number of figures at pleaſure: thus, $\dot{3}$ may be transformed into $,\dot{3}3$, or $,\dot{3}\dot{3}$, or $,\dot{3}3\dot{3}$, &c. alſo $,\dot{7}\dot{9} = \dot{7}9\dot{7}9 = \dot{7}9\dot{7}$, and ſo on.

‡ The learner may obſerve that the ſimilar and conterminous repetends begin juſt ſo far from unity, as is the fartheſt among the diſſimilar repetends; and is ſo in all caſes.

RULE. † 1. REDUCE the given fraction to its least Terms, and divide the denominator by by 2, 5 or 10 as often as possible.

2. DIVIDE 9999, &c. by the former result, till nothing remain, and the number of 9's used will shew the number of places in the repetend; which will begin after so many places of figures as there were 10's, 2's, or 5's, divided by.

IF the whole denominator vanish in dividing by 2, 5 or 10, the decimal will be finite, and will consist of so many places as you perform divisions.

EXAMPLES.

1. REQUIRED to find whether the decimal equal to $\frac{420}{2240}$ be finite or infinite, and if infinite, how many places that repetend will consist of.

First $10\overline{)\frac{420}{2240}} = \frac{42}{224}$ *and* $2\overline{)\frac{42}{224}} = \frac{21}{112}$ $\quad 2\overline{)112} = \overset{(2)}{56} = \overset{(2)}{28} = \overset{(2)}{14} = 7.$

Then $\frac{7)999999}{142857}$; *therefore, because the Denominator* 112 *did not vanish in dividing by* 2, *the decimal is infinite, and, as six* 9*s were used, the circulate consists of* 6 *places, beginning at the decimal point.*

2. *Let* $\frac{2}{11}$ *be the fraction proposed.*

3. *Let* $\frac{3}{7}$ *be the fraction proposed.*

ADDITION OF CIRCULATING DECIMALS.

RULE. 1. Make the repetends *similar and conterminous,* and find their sum as in common addition.

2. DIVIDE this sum (of the repetends only) by so many nines as there are places in the repetend, and the remainder is the repetend of their sum; which must be set under the figures added, with cyphers on the left-hand, when it has not so many places as the repetends.

3. CARRY the quotient of this division to the next Column, and proceed with the rest, as infinite decimals. EXAMPLES.

† IN dividing 1,000, &c. by any prime number whatever, except 2 or 5, the figures in the quotient will begin to repeat over again as soon as the remainder is 1: and since 999, &c. is less than 1000, &c. by 1, therefore 999, &c. divided by any number whatever, will, when the repeating figures are at their period, leave 0 for a remainder.

NOW, whatever number of repeating figures we have, when the dividend is 1, there will be exactly the same number, when the dividend is any other number whatever.

THUS, let ,390539053905 &c. be a circulate, whose repeating part is 3905. Now every repetend (3905,) being equally multiplied, must give the same product: for although these products will consist of more places, yet the over-plus in each, being alike, will be carried to the next, by which mean, each product will be equally increased, and consequently every four places will continue alike. And the same will hold for any other number.

NOW from hence it appears that the dividend may be altered at pleasure, and the number of places in the repetend will still be the same; thus $\frac{1}{11} = \dot{9}\dot{0}$ and $\frac{4}{11}$, or $\frac{1}{11} \times 4 = \dot{3}\dot{6}$, whence the number of places in each are alike.

EXAMPLES.

1. Let $5,\dot{3}+59,4\dot{3}5\dot{6}+397,\dot{6}+519+,\dot{3}\dot{9}+217,\dot{5}$ be added together.

$5,\dot{3}$	=	$5,3\dot{3}3333\dot{3}$
$59,4\dot{3}5\dot{6}$	=	$59,4\dot{3}5635\dot{6}$
$397,\dot{6}$	=	$397,6\dot{6}6666\dot{6}$
519	=	$519,0\dot{0}0000\dot{0}$
$,\dot{3}\dot{9}$	=	$,3\dot{9}3939\dot{3}$
$217,\dot{5}$	=	$217,5\dot{5}5555\dot{5}$
		$1199,3851305$ *the Sum.*

In this question, the sum of the repetends is 2851303, *which, divided by* 999999, *gives* 2 *to carry to the next column* 5,3,0 &c. *and the remainder is* 851305.

2. Let $3275,319+36,4\dot{5}+123,\dot{1}9+5,317\dot{3}+112,35\dot{1}3+11,13\dot{1}$ $+,\dot{1}25+29,1005\dot{3}$ be added together. *Ans.* $3593,0004\dot{2}$.

SUBTRACTION of CIRCULATING DECIMALS.

Rule. Make the repetends *similar and conterminous*, and subtract as usual, observing, that if the repetend of the number to be subtracted be greater than the repetend of the number it is to be taken from, then the right-hand of the remainder must be less by unity than it would be if the expressions were finite.

EXAMPLES.

1. From $57,\dot{0}\dot{3}$ take $29,73\dot{5}8\dot{7}$

$57,\dot{0}\dot{3}$	=	$57,0\dot{3}03\dot{0}$
$29,73\dot{5}8\dot{7}$	=	$29,73\dot{5}8\dot{7}$
		$27,2\dot{9}44\dot{2}$ *the Difference.*

2. From $325,\dot{1}7$ take $137,581\dot{9}$— *Ans.* $187,595\dot{7}$.

MULTI-

MULTIPLICATION of CIRCULATING DECIMALS.

Rule. 1. Turn both the Terms into their equivalent vulgar Fractions, and find the Product of those fractions as usual.

2. Turn the vulgar fraction, expressing the Product, into an equivalent decimal one, and it will be the Product required.

Examples.

1. Multiply $\dot{5}\dot{4}$ by ,1$\dot{5}$. $\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11}$ and ,1$\dot{5} = \frac{14}{90} = \frac{7}{45}$

$\frac{6}{11} \times \frac{7}{45} = \frac{42}{495} =$,0$\dot{8}\dot{4}$ *the Product.*

2. Multiply 378,$\dot{5}$ by 23,$\dot{6}$— *Ans.* 8959,1$\dot{4}\dot{8}$.

DIVISION of CIRCULATING DECIMALS.

Rule. 1. Change both the Divisor and Dividend into their equivalent vulgar fractions, and find their Quotient as usual.

2. Turn the vulgar fraction, expressing the quotient, into its equivalent decimal, and it will be the quotient required.

Examples.

1. Divide ,$\dot{5}\dot{4}$ by ,1$\dot{5}$

,$\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11}$ *and* ,1$\dot{5} = \frac{14}{90} = \frac{7}{45}$

$\frac{6}{11} \div \frac{7}{45} = \frac{6}{11} \times \frac{45}{7} = \frac{270}{77} = 3\frac{39}{77} =$ 3,$\dot{5}0649\dot{3}$ *the Quotient.*

2. Divide 34$\dot{5}$,$\dot{8}$ by 6. *Ans.* 518,8$\dot{3}$

ALLIGATION

Is the method of mixing two or more Simples of different qualities, so that the Composition may be of a mean or middle quality: It consists of two kinds, *viz.* Alligation Medial and Alligation Alternate.

ALLIGATION MEDIAL

Is, when the Quantities and Prices of several things are given, to find the mean price of the mixture compounded of those things.

Rule.

As the sum of the Quantities, or the whole Composition, is to their total value; so is any part of the Composition to its mean Price or Value.

Examples.

EXAMPLES.

1. A Tobacconist would mix 60 ℔. of Tobacco, at 6*d.* *per* ℔. with 50 ℔. at 1*s.* 40 ℔. *at* 1/6 and 30 ℔. at 2*s.* *per* ℔.; What is 1 ℔. of this mixture worth?

℔.		*s.*	*d.*		£.	*s.*
60	*at*	0	6	*is*	1	10
50	—	1	0	—	2	10
40	—	1	6	—	3	0
30	—	2	0	—	3	0
Sum of the simples 180		*Total value*			10	0

℔. £. ℔.
As 180 : 10 :: 1

```
        10
         1
        ——
        10
        20
        ——
180)200(1s.
    180
    ———
     20
     12
    ———
180)240(1⅓d.      s. d.
    180     Ans. 1 1⅓ per ℔.
    ———
     60
```

2. A Farmer would mix 20 Bushels of Wheat at 6*s.* *per* Bushel, 16 Bushels of Rye at 4*s.* *per* Bushel, 12 Bushels of Barley at 3*s.* *per* bushel, and 8 Bushels of Oats at 2*s.* *per* Bushel; What is the value of 1 Bushel of this Mixture? *Ans.* 4*s.* $2\frac{1}{2}$*d.*

3. A Wine-Merchant mixes 12 Gallons of Wine, at 4*s.* 10*d.* *per* Gallon, with 24 Gallons, at 5*s.* 6*d.* and 16 Gallons, at 6*s.* $3\frac{1}{4}$*d.* What is a Gallon of this Composition worth? *Ans.* 5*s.* 7*d.*

4. A Goldsmith melted together 8*oz.* of Gold of 22 Carats fine, 1 ℔. 8*oz.* of 21 Carats fine, and 10*oz.* of 18 Carats fine: Pray what is the quality, or fineness of the Composition?

$$\frac{8\times22+20\times21+10\times18}{8+20+10}=20\tfrac{8}{19}\ \textit{Carats fine, Answer.}$$

5. A Refiner melts 5 ℔ of Gold of 20 carats fine with 8 ℔. of 18 carats fine; How much alloy must he put to it, to make it 22 carats fine?

$$22-\overline{5\times20+8\times18}\div\overline{5+8}=3\tfrac{3}{13}$$

Answer, It is not fine enough by $3\frac{3}{13}$ *carats; so that no alloy must be added, but more Gold.*

ALLIGATION ALTERNATE*

Is the method of finding what quantity of each of the ingredients, whose rates are given, will compose a mixture of a given rate: so that it is the reverse of Alligation medial, and may be proved by it.

CASE

* *Demon.* By connecting the less rate with the greater, and placing the difference between them and the mean rate alternately, or one after the other in turn, the quantities resulting are such that there is precisely as much gained by one quantity as is lost by the other, and therefore the gain and loss, upon the whole, are equal, and are exactly the proposed rate.

IN

CASE I.

THE whole work of this case consists in linking the extremes truly together and taking the differences between them and the mean price, which differences are the quantities sought.

RULE.

1. PLACE the several prices of the simples, being reduced to one denomination, in a column under each other, the least uppermost, and so gradually downward, as they increase, with a line of connection at the left hand, and the mean price at the left hand of all.

2. CONNECT, with a continued line, the price of each simple, or ingredient, which is less than that of the compound, with one or any number of those, which are greater than the compound, and each greater rate or price with one, or any number, of the less.

3. PLACE the difference, between the mean price (or mixture-rate) and that of each of the simples, opposite to the rates with which they are connected.

4. THEN, if only one difference stand against any rate, it will be the quantity belonging to that rate, but if there be several, their sum will be the quantity.

EXAMPLES.

1. A Merchant has Spices, some at 1*s.* 6*d.* *per* ℔. some at 2*s.* some at 4*s.* and some at 5*s.* *per* ℔. how much of each sort must he mix, that he may sell the mixture at 3*s.* 4*d.* *per* ℔.?

	d.	℔.	s. d.	
Mean rate 40*d.*	18	20	*at* 1 6	*per* ℔.
	24	8	-- 2	
	48	16	-- 4	
	60	22	-- 5	

	d.	℔.	s. d.	
40*d.*	18	8	*at* 1 6	*per* ℔
	24	20	-- 2	
	48	22	-- 4	
	60	16	-- 5	

	d.		℔.		
40*d.*	18	20+8	28	*at* 1/6	*per lb.*
	24	8	8	-- 2	
	48	16+22	38	-- 4	
	60	22	22	-- 5	

	d.		℔.		
40*d.*	18	20	20	*at* 1/6	*per lb.*
	24	8+20	28	-- 2	
	48	16	16	-- 4	
	60	22+16	38	-- 5	40*d.*

IN like manner, let the number of simples be what it may, and with how many soever each one is linked, since it is always a less with a greater than the mean price, there will be an equal balance of loss and gain between every two, and consequently an equal balance on the whole.

IT is obvious from the rule, that questions of this sort admit of a great variety of Answers; for having found 1 answer, we may find as many more as we please by only multiplying or dividing each of the quantities found by 2, 3, 4, &c. the reason of which is evident; for if two quantities of two simples make a balance of loss and gain with respect to the mean price, so must also the double or triple, the half or third part, or any other ratio of these quantities, and so on *ad infinitum*.

IF any one of the simples be of little or no value with respect to the rest, its rate is supposed to be nothing, as water mixed with wine, and alloy with gold and silver.

40*d.*	18	20 + 8	28 *at* 1 6	*per lb.*
	24	8 + 20	28 -- 2	
	48	16 + 22	38 -- 4	
	60	22 + 16	38 -- 5	

Note, *These five answers arise from as many various ways of linking the rates of the ingredients together.*

2.† A Merchant has Canary Wine, at 3*s.* *per* gallon; Sherry at 2*s.* 1*d.* and Claret at 1*s.* 5*d.* *per* gallon; How much of each sort must he take, to sell it at 2*s.* 4*d.* *per* gallon?

Mean rate 28*d.*	36	3 + 11	14 *at* 3 -	*per gallon.*
	25	8	8 — 2 1	
	17	8	8 — 1 5	

3. How much Barley at 2*s.* 4*d.* Rye at 3*s.* 9*d.* and Wheat at 5*s.* *per* bushel, must be mixed together, that the compound may be worth 4*s.* 4*d.* *per* bushel?

Ans. 8 *bushels of Barley,* 8 *of Rye, and* 31 *of Wheat.*

4. A Goldsmith would mix gold of 19 carats fine, with some of 16, 18, 23 and 24 carats fine, so that the compound may be 21 carats fine; What quantity of each must he take?

21	16
	18
	19
	23
	24

5. It is required to mix several sorts of wine at 3*s.* 5*s.* and 7*s.* *per* gallon, with water, that the mixture may be worth 4*s.* *per* gallon; How much of each sort must the mixture consist of?

4	0
	3
	5
	7

Ans. 3 *gal. Water,* 1 *gal. Wine, at* 3*s,* 1 *do. at* 5*s and* 4 *ditto, at* 7*s.*

Case 2.

When the *rates* of *all* the *ingredients*, the *quantity* of but *one* of them, and the *mean rate* of the whole mixtures are given, to find the several quantities of the rest, in proportion to the quantity given.

Rule.

Take the differences between each price, and the mean rate, and place them alternately, as in case 1. Then, As the difference standing against that simple, whose quantity is given, is to that quantity; so is each of the other differences, severally, to the several quantities required.

Examples.

1. A Merchant has 40 lb. of Tea, at 6*s.* *per* lb. which he would mix with some at 5*s.* 8*d.* some at 5*s.* 2*d.* and some at 4*s.* 6*d.* How much

† Note, the 2d. and 3d. questions admit but of one way of linking, and so but of one Answer; yet all numbers in the same proportion between themselves, as the numbers, which compose the Answer, will likewise satisfy the condition of the question.

much of each ſort muſt he take, to mix with the 40lb. that he may ſell the mixture at 5*s.* 5*d. per* lb.

$$65\left\{\begin{array}{l}54\\62\\68\\72\end{array}\right.\quad\begin{array}{l}7+3\\3+7\\3+11\\11+3\end{array}\left|\begin{array}{l}\text{lb.}\\10\\10\\14\\14\ \textit{ſtands against the given quantity.}\end{array}\right.$$

$$\textit{As}\ 14 : 40 :: \left\{\begin{array}{l}\text{lb.}\quad\ \text{lb.}\\10 : 28\tfrac{8}{14}\ \textit{at}\ 4s.\ 6d.\\10 : 28\tfrac{8}{14}\ —\ 5s.\ 2d.\\14 : 40\ ——\ 5s.\ 8d.\end{array}\right\}\ \textit{per}\ \text{lb.}$$

2. A Farmer being determined to mix 20 buſhels of Oats, at 2*s.* *per* buſhel, with barley, at 2*s.* 6*d.* rye, at 4*s.* and wheat, at 5*s.* 6*d.* *per* buſhel; I demand the quantity of each, which muſt be mixed with the 20 buſhels of oats, that the whole quantity may be worth 4*s.* 6*d. per* buſhel? *Anſ.* 20 *of barley,* 20 *of rye, and* 100 *of wheat.*

3. How much Gold of 16, 20 and 24 carats fine, and how much alloy, muſt be mixed with 10 *oz.* of 18 carats fine, that the compoſition may be 22 carats fine?

Anſ. 10 *oz. of* 16 *carats fine,* 10 *of* 20, 170 *of* 24, *and* 10 *of alloy.*

ALTERNATION TOTAL.*

CASE 3.

WHEN the *rates* of the ſeveral ingredients, the *quantity* to be compounded, and the *mean rate* of the whole mixture are given, to find how much of each ſort will make up the quantity.

RULE.

PLACE the differences between the mean rate, and the ſeveral prices alternately, as in caſe 1;—then, As the ſum of the quantities, or differences thus determined, is to the given quantity, or whole

* To this Caſe belongs that curious queſtion concerning King Hiero's Crown.

HIERO, king of Syracuſe, gave orders for a crown to be made, entirely of pure gold; but ſuſpecting the workmen had debaſed it by mixing with it ſilver or copper, he recommended the diſcovery of the fraud to the famous Archimides, and deſired to know the exact quantity of alloy in the Crown.

ARCHIMIDES, in order to detect the impoſition, procured two other maſſes, the one of pure gold, and the other of ſilver, or copper, and each of the ſame weight with the former; and, by putting each ſeparately into a veſſel *full* of water, the quantity of Water expelled by them, determined their ſpecific bulks; from which and their given weights it is eaſier to determine the quantities of gold and alloy in the crown by this caſe of Alligation, than by an Algebraic proceſs.

SUPPOSE the weight of each maſs to have been 5*lb.* the weight of the water expelled by the alloy, 23*oz.* by the gold 13 *oz.* and by the crown 16 *oz.* that is, that their ſpecific bulks were as 23, 13, and 16, then, what were the quantities of gold and alloy reſpectively in the crown?

HERE, the rates of the ſimples are 23 and 13, and of the compound 16; whence,

$$16\left\{\begin{array}{l}13\\23\end{array}\right.\quad\begin{array}{l}7\ \text{of gold}\\3\ \text{of alloy}\end{array}\Big\}\ \text{And the sum of these is}\ 7+3=10,\ \text{which should have been but}\ 5,\ \text{whence by the rule,}$$

$$10 : 5 :: \left\{\begin{array}{l}7 : 3\tfrac{1}{2}\ \textit{lb.}\ \text{of gold}\\3 : 1\tfrac{1}{2}\ \textit{lb.}\ \text{of alloy}\end{array}\right\}\ \text{the Anſwer.}$$

whole compoſition; ſo is the difference of each rate, to the required quantity of each rate.

E x a m p l e s.

1. Suppose I have 4 ſorts of currants at 8*d.* 12*d.* 18*d.* and 22*d.* *per* ℔; the worſt would not ſell, and the beſt were too dear; I therefore conclude to mix 120 ℔. and ſo much of each ſort, as to ſell them at 16*d.* *per* ℔: How much of each ſort muſt I take?

$$16d.\left\{\begin{array}{c} d. \\ 8 \\ 12 \\ 18 \\ 22 \end{array}\right. \begin{array}{c} ℔. \\ 6 \\ 2 \\ 4 \\ 8 \\ \hline \end{array} \quad As\ \begin{array}{c} ℔. \\ 20 \end{array} : \begin{array}{c} ℔. \\ 120 \end{array} :: \left\{\begin{array}{l} ℔.\ \ ℔. \\ 6 : 36\ at\ 8d. \\ 2 : 12 - 12d. \\ 4 : 24 - 18d. \\ 8 : 48 - 22d. \end{array}\right\} per\ ℔.$$

$$Sum = 20 \qquad\qquad 120$$

2. A Goldſmith has ſeveral ſorts of Gold; *viz.* of 15, 17, 20 and 22 carats fine, and would melt together, of all theſe ſorts, ſo much, as may make a maſs of 40 *oz.* 18 carats fine; how much of each ſort is required?

Anſ. 16 *oz.* 15 *carats fine*, 4 *oz.* 17, 8 *oz* 20, *and* 12 *oz.* *of* 22 *carats fine.*

3. A Merchant would mix 4 ſorts of Wine, of ſeveral prices, viz. at 4*s.* 6*s.* 8*s.* and 9*s.* *per* Gallon; of theſe he would have a mixture of 60 Gallons, worth 7*s.* *per* Gallon; what quantity of each ſort muſt he have?

Anſ. $17\frac{1}{7}$ *gal. at* 4*s.* $8\frac{4}{7}$ *at* 6*s.* $8\frac{4}{7}$ *at* 8*s.* *and* $25\frac{5}{7}$ *at* 9*s.*

4. How many Gallons of water, of no value, muſt be mixed with wine at 4*s.* *per* Gallon, ſo as to fill a veſſel of 80 Gallons, that may be afforded at 2*s.* 9*d.* *per* Gallon?

$$33\left\{\begin{array}{c} 0 \\ 48 \end{array}\right. \begin{array}{c} Gal. \\ 15 \\ 33 \\ \hline 48 \end{array} \quad As\ \begin{array}{c} Gal. \\ 48 \end{array} : \begin{array}{c} Gal. \\ 80 \end{array} :: \left\{\begin{array}{l} Gal. \\ 15 : 25\ Gallons\ of\ Water \\ 33 : 55\ Gallons\ of\ Wine \end{array}\right\} Anſ.$$

Sum 48

C a s e 4.*

When more than one of the ſimples are limited.

R u l e.

Find, by Alligation medial, what will be the *rate* of a mixture made of the *given quantities* of the *limited ſimples* only; then, conſider this as the *rate* of a *limited ſimple*, whoſe *quantity* is the ſum of the firſt given *limited ſimples*, from *which* and the *rates* of the *unlimited ſimples*, by Caſe 2d. calculate the quantity.

E x a m p l e s.

1. How much Wine, at 4*s.* 6*d.* and at 5*s.* *per* Gallon, muſt be mixed with 6 gallons at 4*s.* and 6 Gallons at 3*s.* *per* Gallon, that the mixture may be worth 4*s.* 4*d.* *per* Gallon? *Limited*

* The three laſt caſes need no demonſtration, as the 2d. and 3d, evidently reſult from the firſt, and the laſt, from Alligation medial and the ſecond caſe in alternate.

$$\text{Limited simples}\left\{\begin{array}{l}6\text{ Gallons at }4s. = 24\\ 6\text{ Gallons at }3s. = 18\end{array}\right\}\ \text{As } \overset{Gal.}{12} : \overset{s.}{42} :: \overset{Gal.}{1} : 3/6 \text{ per Gal.}$$

$$\underline{\quad}\qquad\underline{\quad}$$
$$12\qquad 42$$

Now, having found the rate of the limited simples; the question may stand thus: How much Wine at 4*s.* 6*d.* and 5*s.* *per* Gallon, must be mixed with 12 Gallons, at 3*s.* 6*d.* *per* Gallon, that the mixture may be worth 4*s.* 4*d.* *per* Gallon?

$$52\left\{\begin{array}{l}42\\54\\60\end{array}\right.\ \begin{array}{l|l}2+8 & 10\\ 10 & 10\\ 10 & 10\end{array}\quad \text{As } 10 : \left\{\begin{array}{l}10\\10\end{array}\right\} :: 12 : \left\{\begin{array}{l}12\text{ gal. at }4/6\\12\text{ gal. at }5s\end{array}\right\}\ \begin{array}{l}\text{per gal.}\\ \text{Ans.}\end{array}$$

2. How much gold, of 14 and 16 carats fine, must be mixed with 6 *oz.* of 19, and 12 of 22 carats fine, that the composition may be 20 carats fine?

Ans. $1\frac{8}{10}$ *oz. of each sort.*

POSITION.

Position is a rule, which, by false, or supposed numbers, taken at pleasure, discovers the true ones required. It is divided into two parts; Single and Double.

SINGLE POSITION.

Single Position teaches to resolve those questions, whose results are proportional to their suppositions: such are those which require the multiplication or division of the number sought by any proposed number; or when it is to be increased or diminished by itself a certain proposed number of times

Rule.* 1. Take any number, and perform the same operations with it, as are described to be performed in the question.

2. Then say; as the sum of the Errors: is to the given sum; so is the supposed number: to the true one required.

Proof. Add the several parts of the sum together, and if it agrees with the sum, it is right.

Examples.

1. A School-master being asked how many scholars he had, said, if I had as many more as I now have, three quarters as many, half as

* The reason of this rule is obvious, it being evident that the Results are proportional to the suppositions;

$$\text{Thus}\left\{\begin{array}{l} nx : x :: na : a\\ \frac{x}{n} : x :: \frac{a}{n} : a\\ \frac{x}{n} \pm \frac{x}{m}\&c. : x :: \frac{a}{n} \pm \frac{a}{m}\ \&c. : a,\ \text{and so on.}\end{array}\right.$$

as many, one fourth and one eighth as many, I should then have 435; Of what number did his School consist?

Suppose he had 80	*As* 290 : 435 :: 80	
As many = 80	80	
$\frac{3}{4}$ *as many* = 60	———	120
$\frac{1}{2}$ *as many* = 40	29|0)3480|0(120 *Answer.*	120
$\frac{1}{4}$ *as many* = 20	29	90
$\frac{1}{8}$ *as many* = 10	———	60
———	58	30
290	58	15
	—	———
	0	435 *Proof.*

2. A Person lent his friend a sum of money unknown, to receive Interest for the same at £6 *per Cent. per Annum*, Simple Interest, and at the end of 12 years, received for principal and Interest £860; what was the sum lent? *Ans* £500.

3. A, B and C joined their Stocks, and gained £350, of which A took up a certain sum, B took up four times so much as A, and C, eight times so much as B; what share of the gain had each?

	£.	s.	d.	qrs.	
Ans.	9	9	2	$1\frac{3}{37}$	*A's share.*
	37	16	9	$0\frac{12}{37}$	*B's ditto.*
	302	14	0	$2\frac{22}{37}$	*C's ditto.*

4. A, B, C, and D spent 35*s.* at a reckoning, and, being a little dipped, they agreed that A should pay $\frac{2}{3}$, B $\frac{1}{2}$, C $\frac{1}{3}$, and D $\frac{1}{4}$; what did each pay in the above proportion?

		s.	d.
Ans.	*A.*	13	4
	B.	10	
	C.	6	8
	D.	5	

5. A certain sum of money is to be divided between 5 men, in such a manner as that A shall have $\frac{1}{4}$, B. $\frac{1}{5}$ C $\frac{1}{10}$, D $\frac{1}{20}$, and E the remainder, which is £40; what is the sum?

Suppose £200, *then* $\frac{1}{4} + \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = 120$.

$200 - 120 = 80$. *As* 80 : 40 :: 200 : 100. *Ans.*

6. A Person, after spending $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had £26 $\frac{2}{3}$ left; what had he at first? *Ans.* £160.

7. A and B talking of their ages, B said his age was once and an half the age of A; C said his was twice and one tenth the age of both, and that the sum of their ages was 93; what was the age of each? *Ans. A's* 12, *B's* 18 *and C's* 63 *years.*

8. A vessel has 3 Cocks, A, B and C; A can fill it in $\frac{1}{2}$ an hour, B in $\frac{1}{4}$ of an hour, and C in $\frac{1}{3}$ of an hour; In what time will they all fill it, together? *Ans.* $\frac{1}{9}$ *hour.*

9. A person having about him a certain number of dollars, said

said that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of them would make 57; Pray, how many had he? *Ans.* 60.

10. A Gentleman bought a Chaise, horse and harness for £100, the horse cost $\frac{1}{4}$ more than the harness, and the Chaise $\frac{1}{3}$ more than the horse; what was the price of each?

Ans. Harness £$25\frac{25}{47}$. *Horse* £$31\frac{43}{47}$. *Chaise* £$42\frac{26}{47}$.

11. A and B, having found a purse of money, disputed who should have it: A said that $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{20}$ of it amounted to £35, and if B could tell him how much was in it, he should have the whole, otherwise he should have nothing; How much did the purse contain? *Ans.* £100.

12. A Gentleman divided his fortune among his sons, to A he gave £9 as often as to B £5; and to C, but £3 as often as to B £7, yet C's portion came to £$1050\frac{4}{5}$; what was the whole Estate? *Ans.* $7916\frac{2}{35}$.

13. Seven-Eighths of a certain number exceeds four-fifths by 6; What is that number? *Ans.* 80.

14. What number is that, which, being increased by $\frac{2}{5}$, $\frac{3}{8}$ and $\frac{5}{6}$ of itself, the sum will be $234\frac{3}{4}$? *Ans.* 90.

DOUBLE POSITION.

Double Position teacheth to resolve questions by making two suppositions of false numbers.

Those questions, in which the results are not proportional to their positions, belong to this rule: such are those, in which the number sought is increased or diminished by some given number, which is no known part of the number required.

Rule.* 1. Take any two convenient numbers, and proceed with each according to the conditions of the question.

2. Place the result or errors against their positions or supposed numbers, thus,

$$\begin{array}{ccc} \textit{Pos.} & & \textit{Err.} \\ 30 & \times & 12 \\ 20 & & 6 \end{array}$$

and if the error be too great, mark it with $+$; *and if too small with* $-$.

3. Multiply them cross-wise; that is, the first position by the last error, and the last position by the first error. 4. If

* The rule is founded on this supposition, that the first error is to the second, as the difference between the true and first supposed number is to the difference between the true and second supposed number: when that is not the case, the exact answer to the question cannot be found by this rule.

That the Rule is true, according to the supposition, may be thus demonstrated.

Let A and B be any two numbers produced from a and b by similar operations, it is required to find the number from which N is produced by a like operation.

Put x = number required, and let $N - A = r$, and $N - B = s$. Then, according to the supposition on which the rule is founded, $r : s :: x - a : x - b$, whence, by multiplying means and extremes, $rx - rb = sx - sa$; and by transposition, $rx - sx = rb - sa$; and by division, $x = \frac{rb - sa}{r - s}$ = number sought; and if r and s be both negative, the Theorem is the same, and if r or s be negative, x will be equal to $\frac{rb + sa}{r + s}$ which is the rule.

4. If the errors be alike, that is, both too ſmall, or both too great, divide the difference of the Products by the difference of the errors, and the quotient will be the anſwer.

5. If the errors be unlike; that is, one too ſmall, and the other too great, divide the ſum of the products by the ſum of the errors, and the quotient will be the anſwer.

Note, When the errors are the ſame in quantity, and unlike in quality, half the ſum of the ſuppoſitions is the number ſought.

Examples.

1. A Lady bought damaſk for a gown, at 8*s*. *per* yard, and lining for it, at 3*s*. *per* yard; the gown and lining contained 15 yards, and the price of the whole was £3 10*s*.; How many yards were there of each?

Suppoſe 6 *yards damaſk, value* 48*s*.
Then ſhe muſt have 9 *yards of lining, value* 27*s*.

Sum of their values = 75*s*.
So that the firſt error is 5 *too much, or* + 5
Again, ſuppoſe ſhe had 4 *yards of damaſk, value* 32*s*.
Then ſhe muſt have 11 *yards of lining, value* 33*s*.

Sum of their values = 65*s*.
So that the ſecond error is 5 *too little, or* —5*s*.

Sup.		*Errors.*					
6	X	5+					
Then 4		5—		5 *yds. at* 8*s*. =	£2	0	0
				10 *yds. at* 3*s*. =	1	10	0
20		30			£3	10	0 *Proof.*
		20					

Sum of errors. = 5 + 5 = 10) 50

Anſ. 5 *yards damaſk, and* 15—5 = 10 *yds. lining*.
Or 6 + 4 ÷ 2 = 5 *as before*.

2. A and B have the ſame Income; A ſaves $\frac{1}{8}$ of his; but B, by ſpending £30 *per Annum* more than A, at the end of 8 years finds himſelf £40 in debt; what is their Income, and what does each ſpend *per Annum*?

Suppoſe { 80 X 120+ / 160 X 40+ }

Anſ. Their Income is £200 *per Annum*.
Alſo, A ſpends £175, *and B* £205 *per Annum*.

3. A and B laid out equal ſums of money in trade: A gained a ſum equal to $\frac{1}{4}$ of his Stock, and B loſt £225, then A's money was double that of B; what did each lay out?

Suppose $\left\{\begin{array}{ccc} 300 & & 225+ \\ & \times & \\ 900 & & 225- \end{array}\right.$ *Ans.* £600.

4. A Labourer was hired for 60 days upon this condition, that, for every day he wrought, he should receive 3*s.* 4*d.*; and for every day he was idle, should forfeit 1*s.* 8*d.*; at the expiration of the time he received £3 15; how many days did he work, and how many was he idle?

Suppose he worked $\left\{\begin{array}{ccc} 20 & & 900- \\ & \times & \\ 40 & & 300+ \end{array}\right.$

Ans. He was employed 35 *days, and was idle* 25.

5. A Gentleman has two Horses of considerable value, and a Carriage worth £100; now, if the first horse be harnessed in it, he and the carriage together will be triple the value of the second; but if the second be put in, they will be 7 times the value of the first; what is the value of each horse?

Ans. One £20, *and the other* £40.

6. There is a fish, whose head is 10 feet long; his tail is as long as his head and half the length of his body, and his body as long as the head and tail; what is the whole length of the fish?

First, suppose the body 20 ✕ 10 —
2d. suppose it 30 ✕ 5 —

Head = 10
Tail = 30
Body = 40

Ans. 80 *feet.*

7. What number is that, which, being increased by its $\frac{1}{2}$, its $\frac{1}{4}$, and 5 more, will be doubled? *Ans.* 20.

8. A Farmer having driven his Cattle to market, received for them all, £80, being paid at the rate of £6 *per* Ox, £4 *per* Cow, and £1 10*s.* *per* Calf; there were as many oxen as Cows, and 4 times as many Calves as Cows; how many were there of each sort?

Ans. 5 *Oxen,* 5 *Cows, and* 20 *Calves.*

10. A, B and C built a ship, which cost them £1000, of which A paid a certain sum, B paid £100 more than A, and C £100 more than both; having finished her, they fixed her for sea with a cargo worth twice the value of the ship: the outfits and charges of the voyage amounted to $\frac{1}{8}$ of the ship; upon the return of which, they found their clear gain to be $\frac{2}{3}$ of $\frac{3}{5}$ of the vessel, Cargo and expences; Please to inform me what the ship cost them, severally; what share each had in her, and what, upon the final adjustment of their accompts, they had severally gained?

A owned $\frac{7}{40}$ *of the Ship, which cost him* £175, *and his share of the gain was* £227 10*s.*—*B owned,* $\frac{11}{40}$, *which cost* £275, *and his gain was* £357 10*s.*—*C owned* $\frac{11}{20}$, *which cost* £550, *and his gain was* £715.

PERMU-

PERMUTATIONS AND COMBINATIONS.

THE PERMUTATION OF QUANTITIES is, the ſhewing how many different ways any given number of things may be changed.

THIS is alſo called *variation*, *alternation*, or *changes*; and the only thing to be regarded here is the order they ſtand in; for no two parcels are to have all their quantities placed in the ſame ſituation.

THE COMBINATION OF QUANTITIES is the ſhewing how often a leſs number of things can be taken out of a greater, and combined together, without conſidering their places, or the order they ſtand in.

THIS is, ſometimes called *election*, or *choice*; and here every parcel muſt be different from all the reſt, and no two are to have preciſely the ſame quantities, or things.

THE COMPOSITION OF QUANTITIES is the taking of a given number of quantities out of as many equal rows of different quantities, one out of every row, and combining them together.

HERE no regard is had to their places; and it differs from combination only as that admits but of one row of things.

COMBINATIONS OF THE SAME FORM are thoſe in which there are the ſame number of quantities, and the ſame repetitions; thus, *abcc*, *bbad*, *deef*, &c. are of the ſame form; but *abbc*, *abbb*, *aacc* are of different forms.

PROBLEM I.

To find the number of permutations, or changes, that can be made of any given number of things, all different from each other.

RULE.† Multiply all the terms of the natural ſeries of numbers, from 1 up to the given number, continually together, and the laſt product will be the anſwer required.

EXAMPLES.

1. CHRIST-CHURCH, in Boſton, has 8 bells; how many changes may be rung on them?

$1\times2\times3\times4\times5\times6\times7\times8=40320$ *the Anſwer.*

2. NINE Gentlemen met at an Inn, and were ſo pleaſed with their hoſt, and with each other, that, in a frolic they agreed to tarry ſo long as they, together with their hoſt, could ſit every day in a different poſition at dinner; pray how long, had they kept their agreement, would their frolic have laſted? *Anſ.* $9941\frac{335}{365}$ *years.*

3. How many changes, or variations will the Alphabet admit of? *Anſ.* 620448401733239439360000.

PROBLEM

† THE reaſon of this rule may be ſhewn thus, any one thing *a* is capable of one poſition only, as *a*.

ANY two things *a* and *b* are capable of two variations only; as *ab*, *ba*; whoſe number is expreſſed by 1×2.

IF there be three things *a*, *b* and *c*; then any two of them, leaving out the third, will have 1×2 variations; and conſequently when the third is taken in, there will be $1\times2\times3$ variations; and ſo on, as far as you pleaſe.

PROBLEM 2.

Any number of different things being given; to find how many changes can be made out of them, by taking any given number of quantities at a time.

RULE.* Take a ſeries of numbers, beginning at the number of things given, and decreaſing by 1, to the number of quantities to be taken at a time: the product of all the terms will be the anſwer required.

EXAMPLES.

1. How many changes may be rung with 4 bells out of 8?

$$\begin{array}{r} 8 \\ 7 \\ \hline 56 \\ 6 \\ \hline 336 \\ 5 \\ \hline 1680 \end{array}$$

Or $8 \times 7 \times 6 \times 5$ ($=4$ *terms*) $=1680$ *the Anſwer.*

2. How many words can be made with 6 letters of the alphabet, admitting a number of conſonants may make a word?

Anſ. 106909120.

PROBLEM 3.

Any number of things being given; whereof there are ſeveral things of one ſort, ſeveral of another, &c. To find how many changes may be made out of them all.

RULE.§ 1. Take the ſeries $1 \times 2 \times 3 \times 4$ &c. up to the number of things given, and find the product of all the terms.

2. TAKE

* THIS Rule, expreſſed in terms, is as follows; $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$ &c. to n terms; whence $m =$ number of things given, and $n =$ quantities to be taken at a time.

§ THIS Rule is expreſſed in Terms thus; $\dfrac{1 \times 2 \times 3 \times 4 \times 5, \text{ \&c. to } m.}{1 \times 2 \times 3 \text{ \&c. to } p. \times 1 \times 2 \times 3 \text{ \&c. to } q \text{ \&c.}}$ whence $m =$ number of things given, $p =$ number of things of the firſt ſort, $q =$ number of things of the ſecond ſort, &c.

ANY 2 quantities, a, b, both different, admit of 2 changes; but if the quantities are the ſame, or, ab become aa, there will be only one alteration, which may be expreſſed by $\dfrac{1 \times 2}{1 \times 2} = 1$.

ANY 3 quantities, a, b, c, all different from each other, admit of 6 variations; but if the quantities are all alike, or, $a\,b\,c$ become aaa, then the 6 variations will be reduced to 1, which may be expreſſed by $\dfrac{1 \times 2 \times 3}{1 \times 2 \times 3} = 1$. Again, if two quantities out of three are alike, or abc become aac; then the 6 variations will be reduced to theſe 3, aac, caa, aca, which may be expreſſed by $\dfrac{1 \times 2 \times 3}{1 \times 2} = 3$, and ſo of any greater number.

2. TAKE the series $1\times2\times3\times4$ &c. up to the number of the given things of the first sort, and the series, $1\times2\times3\times4$ &c. up to the number of the given things of the second sort, &c.

3. DIVIDE the product of all the terms of the first series by the *joint* product of all the terms of the remaining ones, and the quotient will be the answer required.

EXAMPLES.

1. How many variations may be made of the Letters in the word *Zaphnathpaaneah?*

$1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11\times12\times13\times14\times15$ (= *number of Letters in the word*) $=1307674368000$.

$1\times2\times3\times4\times5$ (= *number of a's*) $= 120$
1×2 (= *number of p's*) $= 2$
1 (= *number of t's*) $= 1$
$1\times2\times3$ (= *number of h's*) $= 6$
1×2 (= *number of n's*) $= 2$

$2\times6\times1\times2\times120 = 2880$)1307674368000(454053600 *the answer*.

2. How many different numbers can be made of the following figures 1223334444? *Answ.* 12600.

PROBLEM 4.

To find the number of combinations of any given number of things, all different from one another, taken any given number at a time.

RULE.† 1. Take the series 1, 2, 3, 4 &c. up to the number to be taken at a time, and find the product of all the terms.

2. TAKE a series of as many terms, decreasing by 1, from the given number, out of which the election is to be made, and find the product of all the terms.

3. DIVIDE the last product by the former, and the quotient will be the number sought.

EXAMPLES.

1. How many combinations may be made of 7 letters out of 12?

$1\times2\times3\times4\times5\times6\times7$ (= *the number to be taken at a time*) $= 5040$.

$12\times11\times10\times9\times8\times7\times6$ (= *same number from* 12) $= 3991680$.

5040)3991680(792 *the Answer*.

2. How many combinations can be made of 6 letters out of the 24 letters of the Alphabet? *Ans.* 134596.

3. A General was asked by his king, what reward he should confer on him for his services; The General only required a penny for

† THIS Rule, expressed algebraically, is $\frac{m}{1} \times \frac{m-1}{1} \times \frac{m-2}{3} \times \frac{m-3}{4}$, &c. to n terms; where m is the number of given quantities, and n, those to be taken at a time.

Note, In any given nnmber of quantities, the number of Combinations increases gradually till you come about the mean numbers, and then gradually decrease. If the number of quantities, be *even*, half the number of places will shew the greatest number of Combinations, that can be made of those quantities; but if odd, then those 2 numbers, which are the middle, and whose sum is equal to the given number of quantities, will shew the greatest number of combinations.

for every file, of 10 men in a file, which he could make out of a company of 90 men; What did it amount to?

Anſwer £238360228413 2/5$\frac{268}{367}$.

4. A Farmer bargained with a Gentleman for a dozen Sheep, (at 2 dollars *per* head) which were to be picked out of 2 dozen; but, being long in chuſing them, the Gentleman told him that if he would give him a penny for every different dozen which might be choſen out of the two dozen, he ſhould have the whole, to which the farmer readily agreed; Pray what did they coſt him?

Anſwer £11268 3ſ.

5. How many Locks, whoſe wards differ, may be unlocked with a key of 6 ſeveral wards? *Anſw.* 63: 6 *of which may have one ſingle ward,* 15 *double wards,* 20 *triple wards,* 15 *four wards,* 6 *five wards, and* 1 *lock,* 6 *wards.*

$$\text{Wards. } \left\{\begin{matrix}1\\2\\3\\4\\5\\6\end{matrix}\right\} \textit{ in } 6 = \left\{\begin{matrix}6\\15\\20\\15\\6\\1\end{matrix}\right\} \text{ Locks.} \qquad \text{Wards. } \left\{\begin{matrix}1\\2\\3\\4\\5\end{matrix}\right\} \textit{ in } 5 = \left\{\begin{matrix}5\\10\\10\\5\\1\end{matrix}\right\} \text{ Locks.}$$

PROBLEM 5.

To find the number of Combinations of any given number of things, by taking any given number at a time; in which there are ſeveral things of one ſort, ſeveral of another, &c.

RULE. Find the number of different *forms,* which the things, to be taken at a time, will admit of, in the following manner:

1. PLACE the things ſo that the greateſt indices may be firſt, and the reſt in order.

2. BEGIN with the firſt letter, and join it to the ſecond, third, fourth, &c. to the laſt.

3. JOIN the ſecond letter to the third, fourth, &c. to the laſt; and ſo on 'till they are all done, always rejecting ſuch combinations as have occured before; and this will give the combinations of all the twos.

4. JOIN the firſt letter to every one of the twos; then join the ſecond, third, &c. as before; and it will give the combinations of all the threes.

5. PROCEED in the ſame manner to get the combinations of all the fours, fives, &c. and you will at laſt get all the ſeveral *forms* of combination, and the number in each form.

6. HAVING found the number of combinations in each *form,* add them all together, and the ſum will be the number required.

EXAMPLE.

1. LET the things propoſed be *aaabbc*; It is required to find the number of combinations of every 2, of every 3, and of every 4 of theſe quantities.

Combinations

Combinations at large.	*Forms.*	*Comb. in each form.*
aa,aa,ab,ab,ac	$a^2,b^2,$	2
aa,ab,ab,ac	*ab,ac,bc*	3
ab,ab,ac		—
bb,bc		5 = *Sum of the twos.*
bc		
	a^3	1
aaa,aab,aab,aac	a^2b,a^2c,b^2a,b^2c	4
aab,aab,aac	*abc*	1
abb,abc		—
bbc		6 = *Sum of the threes.*
		—
aaab,aaab,aaac	a^3b,a^3c	2
aabb,aabc	a^2b^2	1
abbc	$a^2bc,b^2ac,$	2
		—
		5 = *Sum of the fours.*

Anſ. 5 *Combinations of every* 2; 6, *of every* 3, *and* 5 *of every* 4 *quantities.*

PROBLEM 6.

To find the changes of any given number of things, taken a given number at a time; in which there are ſeveral given things of one ſort, ſeveral of another, &c.

RULE. 1. Find all the different forms of combination of all the given things, taken, as many at a time, as in the queſtion, by Problem 5.

2. FIND the number of changes in any form, (by Problem 3) and multiply it by the number of combinations in that form.

3. Do the ſame for every diſtinct form, and the ſum of all the Products will give the whole number of changes required.

EXAMPLE.

1. How many changes can be made of every 4 letters out of theſe 6; *aaabbc?*

$$\begin{array}{lcl}
\textit{No. of forms.} & \textit{Comb.} & \textit{Changes.} \\
 & & \left\{\begin{array}{l} 1\times2\times3\times4=24 \\ 1\times2\times3 \quad =6 \end{array}\right\} = 4 \\
\left.\begin{array}{l} a^3b,\ a^3c \\ a^2b^2 \\ a^2bc,\ b^2ac, \end{array}\right. & \left.\begin{array}{r} 2 \\ 1 \\ 2 \end{array}\right\} & \left\{\begin{array}{l} 1\times2\times3\times4=24 \\ 1\times2\times1\times2=4 \end{array}\right\} = 6 \\
 & & \left\{\begin{array}{l} 1\times2\times3\times4=24 \\ 1\times2 \quad =2 \end{array}\right\} = 12
\end{array}$$

$$\textit{Therefore} \left\{\begin{array}{l} 2\times4=8 \\ 1\times6=6 \\ 2\times12=24 \end{array}\right.$$

$38 =$ *number of changes required.*

PROBLEM

PROBLEM 7.

To find the Compositions of any number, in an equal number of sets, the things being all different.*

RULE. Multiply the number of things in every set continually together, and the Product will be the Answer required.

EXAMPLES.

1. SUPPOSE there are 5 Companies, each consisting of 12 Men; It is required to find how many ways 9 men may be chosen, one out of each Company?

Multiply 9 into itself continually as many times as there are companies.

$9 \times 9 \times 9 \times 9 = 61049$ *different ways.*

2. How many chances are there in throwing 4 Dice?

As a Die has 6 sides, multiply 6 into itself four times continually.

$6 \times 6 \times 6 \times 6 = 1296$ *Chances, Answer.*

3. SUPPOSE a Man undertakes to throw an Ace at one throw with 4 Dice; what is the Probability of his effecting it?

First, $6 \times 6 \times 6 \times 6 = 1296$ *different ways with and without the Ace; Then, if we exclude the Ace-side of the Die, there will be* 5 *sides left, and* $5 \times 5 \times 5 \times 5 = 625$, *ways without the Ace; Therefore there are* 1296 —625 = 671 *ways wherein one or more of them may turn up an Ace: And the probability that he will do it, as* 671 *to* 625, *Answer.*

MISCELLANEOUS MATTERS.

A ſhort method of reducing a Vulgar Fraction into its equivalent Decimal, by Multiplication.

RULE.

DIVIDE Unity or 1 by the denominator, till the remainder is a ſingle figure, 10, 100, &c. if convenient, then multiply the whole quotient, including the remainder after Diviſion, by the remainder (which is now the numerator, and the diviſor, the denominator) and annex the product to the quotient, then multiply the quotient, thus increaſed, by the laſt numerator, and annex the product to the increaſed quotient; and thus it may be reduced to what exactneſs you pleaſe. But if the numerator of the given fraction exceed 1, you muſt finally multiply the laſt Product by the ſaid Numerator.

EXAMPLES.

REDUCE $\frac{1}{26}$ to its equivalent Decimal.

26)1,00(,03846$\frac{4}{26}$
78
———
220
208
———
120
104
———
160
156
———
4

This multiplied by 4 (*the numerator*) *is* ,15384$\frac{16}{26}$ = $\frac{8}{13}$
Which annexed to the Quotient ,03846 *is* ,0384615384$\frac{8}{13}$
And ,0384615384$\frac{8}{13}$ x 8 *and annexed to the laſt product*
= ,038461538430769230 76$\frac{12}{13}$ *&c.*

REDUCE $\frac{5}{246}$.

246)1,000000(,004065$\frac{10}{246}$ *and* ,004065$\frac{10}{246}$ x 10 = ,0040650$\frac{100}{246}$ *and* *this annexed to the quotient is* ,00406540650$\frac{100}{246}$, *and this multiplied by the given numerator* 5, *is* ,02032703252$\frac{8}{246}$.

FOR any number of pounds, Avoirdupois, under 28, multiply the decimal ,00892857 by the given number of pounds, which generally gives the decimal true to the ſixth place.

A ſhort method of finding the duplicate, triplicate, &c. Ratio of any two numbers, whoſe difference is ſmall, compared with the two numbers.

For the Duplicate Ratio.

RULE. Aſſume two numbers, whoſe difference is ſmall; ſubtract *half* their difference from the leaſt, and add it to the greateſt, and the two numbers, thus found, will be in the ſame proportion nearly as the ſquares of the aſſumed numbers.

EXAMPLE.—Let the aſſumed numbers be 10 and 11;—Then 11—10=1. 10—,5=9,5 and 11+,5=11,5.

Proof, As $10^2 : 11^2 :: 9,5 : 11,5$ *nearly.*

For a Triplicate Ratio.

RULE. Subtract the *difference* of the aſſumed numbers from the leaſt,

and

and add it to the greatest, and the numbers, thus obtained, will be in the same proportion nearly as the Cubes of the assumed numbers.

LET the numbers be 164 and 165; Then 165—164=1. 164—1 =163 & 165+1=166. *Proof, As* $164^3 : 165^3 :: 163 : 166$ *nearly.*

FOR a Quadruplicate Proportion subtract, and add once and a half the difference, and so on, for each higher Power, increasing the number to be subtracted and added by ,5.

To reduce a Ratio, consisting of large Numbers, to its least Terms, and very nearly of the same value.

RULE. 1. Divide the greater of the Terms by the less, and the last Divisor by the Remainder, and so on continually till nothing remain, in the same manner as we get the greatest common measure for reducing a Vulgar Fraction: This will give a number of Ratio's, from which we can choose one that will suit our purpose.

2. PLACE the first Quotient under unit for the first Ratio; multiply that by the next Quotient, adding nothing to the Numerator, and 1 to the Product of the Denominator for a new denominator, and it will give a second Ratio, nearer than the first; Then, multiply the last Ratio by the next Quotient, adding the preceding Ratio, and so on continually till you have gone through.

EXAMPLES.

1. SIR ISAAC NEWTON has demonstrated, in his Principia, that the velocity of a Comet, moving in a Parabola, is to that of a Planet, moving in a circular Orb, at the same distance from the Sun, as $\sqrt{2}$ to 1. Let this be taken for an Example.

$\sqrt{2} = 1{,}4142$; *Those Motions, then, are as* 1,4142 *to* 1; *or as* 14142 to 10000?

```
10000)14142(1
      10000
      -----
       4142)10000(2
             8284
             ----
             1716)4142(2
                  3432
                  ----
                   710)1716(2
                       1420
                       ----
                        296)710(2
                            592
                            ---
                            118)296(2
                                236
                                ---
                                 60)118(1
                                     60
                                     --
                                     58)60(1
                                        58
                                        --
                                         2)58(29
                                           58
```

Then $\frac{1}{1}$ = *First Ratio.*

$\frac{1\times2+0\,=\,2}{1\times2+1\,=\,3}$ = *Second.*

$\frac{2\times2+1\,=\,5}{3\times2+1\,=\,7}$ = *Third.**

$\frac{5\times2+2\,=\,12}{7\times2+3\,=\,17}$ = *Fourth.*

$\frac{12\times2+5\,=\,29}{17\times2+7\,=\,41}$ = *Fifth, &c.*

* THE late Professor Winthrop chose 7 to 5 for a Proportion.

2. Geometers have found the proportion of the circumference of a circle to its diameter, to be as 3,1416 to 1:—Let this ratio be reduced.

10000)31416(3
30000
—
1416)10000(7
9912
—
88)1416(16
88
—
536
528
—
8)88(11
88
—

Then $\frac{1}{3}$ = *First Ratio.*

$1 \times 7 + 0 = 7$
$3 \times 7 + 1 = 22$ $\frac{7}{22}$ = *Second.*

$7 \times 16 + 1 = 113$
$22 \times 16 + 3 = 355$ $\frac{113}{355}$ = *Third:—This is the Ratio generally made use of, and is sufficiently exact for very nice calculations.*

3. The Area of a circle is to its circumscribing Square, as, 7854 to 1, very nearly;—Let this be reduced.

7854)10000(1
7854
—
2146)7854(3
6438
—
1416)2146(1
1416
—
730)1416(1
730
—
686)730(1
686
—
44)686(15
44
—
246
220
—
26 &c.

Then $\frac{1}{1}$ = *First Ratio.*

$1 \times 3 + 0 = 3$
$1 \times 3 + 1 = 4$ $\frac{3}{4}$ = *Second.*

$3 \times 1 + 1 = 4$
$4 \times 1 + 1 = 5$ $\frac{4}{5}$ = *Third.*

$4 \times 1 + 3 = 7$
$5 \times 1 + 4 = 9$ $\frac{7}{9}$ = *Fourth.*

$7 \times 1 + 4 = 11$
$9 \times 1 + 5 = 14$ $\frac{11}{14}$ = *Fifth.—This is very exact, and the proportion generally used.*

Therefore; As 14 : 11 :: *the square of the diameter of a circle to its area.*

To estimate the distance of Objects on level ground, or at sea, having only the height given.

Rule. 1. To the Earth's diameter (viz. 42056462 feet,) add the height of the Eye, and multiply the sum by that height, then the square root of the Product is the distance, at which an object on the surface of the earth or water, can be seen by an Eye so elevated.

2. As objects are seen in a strait line, and that line is a Tangent to the Earth's Surface, therefore; *To find the distance of two elevated Objects, when the right line joining them touches the Earth's Surface between those objects, (for instance, the line from the Eye of the Observer to the distance found by the first part of the rule, and from thence to the Object.)* Work for each object separately, and the sum of the square roots of the Products is the distance of the two objects from each other.

How

How far may a mountain be ſeen on level ground, or at Sea, which is a mile high, ſuppoſing the Eye of the Obſerver elevated 5 feet above the ſurface?

$\sqrt{42056462 + 5 \times 5} = 2{,}746$ *miles.*

$\sqrt{42056462 + 5280 \times 5280} = 80{,}817$ *miles.*

Anſwer, 83,563 *miles.*

To eſtimate the height *of Objects on level ground, or at ſea, having only the diſtance given.*

Rule. 1. From the given diſtance take the diſtance, which the elevation of your eye above the ſurface will give, found by the laſt Problem.

2. Divide the ſquare of the remainder in feet by 42056462 feet, and the quotient will be the height required.

Being on my return from a foreign voyage, and finding by my reckoning I was about $5\frac{1}{2}$ leagues from Boſton Light-houſe, it being in the duſk of the evening, with my Teleſcope I deſcried the Lamp of the Light-houſe in the horizon, at which time my eye was elevated 6 feet above the ſurface of the water: Now, ſuppoſing my reckoning to be true; what is the height of the Light-houſe above the water?

$5\frac{1}{2}$ *Leagues* $= 17{,}5$ *miles, then* $17{,}5 - \sqrt{42056462 + 6 \times 6} = 14{,}492$ *miles,* & $14{,}492$ *miles* $= 76518$ *feet nearly;* & $76518 \times 76518 \div 42056462 = 140$ *feet nearly, Anſwer.*

MISCELLANEOUS QUESTIONS, *with the Method of Solution.*

1. What Part of 9*d.* is $\frac{2}{5}$ of 7*d.*?

$\frac{2}{5}$ *of* $\frac{7}{1} = \frac{14}{5}$, *and* $\frac{9}{1} \div \frac{14}{5} = \frac{1 \times 14}{9 \times 5} = \frac{14}{45}$ *Anſ.*

2. What number is that, from which $\frac{3}{7}$ being taken, the Remainder will be $\frac{1}{5}$?

$\frac{1}{5} + \frac{3}{7} = \frac{1 \times 7 + 3 \times 5}{5 \times 7} = \frac{22}{35}$ *Anſwer.*

3. What number is that, to which if $\frac{3}{7}$ of $\frac{12}{5}$ of $\frac{129}{313}$ be added, the total will be 1?

$\frac{3}{7}$ of $\frac{12}{5}$ of $\frac{129}{313} = \frac{4644}{10955}$, and $\frac{1}{1} - \frac{4644}{10955} = \frac{1 \times 10955 - 1 \times 4644}{1 \times 10955} = \frac{6311}{10955}$ *Anſ.*

4. What number is that, of which $19\frac{3}{13}$ is $\frac{5}{7}$ of it?

$19\frac{3}{13} = \frac{250}{13}$, *Then, As* $\frac{5}{7} : \frac{250}{13} :: \frac{1 \times 250 \times 7}{5 \times 13 \times 1} = 26\frac{60}{65}$.

5. In

5. In an Orchard of Fruit-trees, $\frac{1}{2}$ of them bear Apples, $\frac{1}{4}$ Pears, $\frac{1}{6}$ Plumbs, 60 of them Peaches, and 40, Cherries; How many Trees does the Orchard contain?

$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{11}{12}$, and $\frac{12}{12}-\frac{11}{12}=\frac{1}{12}$, *therefore, As* $\frac{1}{12} : \frac{60+40}{1} :: \frac{12}{12} : 1200$ *Ans.*

6. A Person, who was possessed of $\frac{2}{5}$ of a Vessel, sold $\frac{5}{8}$ of his Interest for £375; what was the Ship worth at that rate?

$\frac{5}{8}$ of $\frac{2}{5}=\frac{1}{4}$. *As* $\frac{1}{4} : \frac{375}{1} :: \frac{1}{1}$: £1500 *Ans.*

7. If $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{4}{5}$ of a Ship be worth $\frac{2}{9}$ of $\frac{7}{8}$ of $\frac{12}{13}$ of the Cargo, valued at £1000: what did both Ship and Cargo cost?

$\frac{5}{7}$ *of* $\frac{3}{8}$ *of* $\frac{4}{5}=\frac{6}{28}$, *and* $\frac{2}{9}$ of $\frac{7}{8}$ *of* $\frac{12}{13}$ *of* $\frac{1000}{1}=\frac{7000}{39}$, *then,*

As $\frac{6}{28} : \frac{7000}{39} :: \frac{28}{28} : \frac{28\times7000\times28}{6\times39\times28}=$ £837 12*s* 1 $\frac{25}{39}$ *the Cost of the Ship, and* £1000 + £837 12*s* 1 $\frac{25}{39}$ = £1837 12*s* 1 $\frac{25}{39}$ *Value of the Ship and Cargo, Ans.*

8. Two Ships A and B sailed from a certain Port at the same time; A sailed North 8 miles an hour, and B East 6 miles an hour: Required, by an easy method, their Distance asunder at every hour's end?

$\sqrt{8\times8+6\times6}=10$ *miles distant in* 1 *hour, and* $10\times2=20$ *miles in* 2 *hours, &c. Answer.*

9. *If a Body be weighed in each Scale of a Balance, whose Beam is unequally divided, and those different weights of the body be multiplied together, the square root of the Product will be the true weight of that Body.*

Suppose the weight of a Bar of Silver, in one Scale, to be 10*oz.* and in the other Scale 12 *oz.*; Required the true weight of the Bar?

$\sqrt{12\times10}=10{,}954$ *oz.* = 10 *oz.* 19 *pwt.* 1,92 *gr.* *Ans.*

10. A younger Brother received £1560, which was just $\frac{7}{12}$ of his elder Brother's fortune; and $5\frac{3}{8}$ times the elder's money was $\frac{2}{3}$ as much again as the Father was worth; Pray, what was his Estate valued at?

As $7 : 1560 :: 12 : 2674\frac{2}{7}$ *the elder Brother's fortune, then,*

$2674\frac{2}{7}\times5\frac{3}{8}\div\frac{2}{3}=$ £19165 14*s* 3$\frac{3}{7}$ *Ans.*

11. A Gentleman divided his Fortune among his Sons, giving A £9 as often as B £5, and to C but £3 as often as to B £7; and yet C's dividend was £1537$\frac{5}{8}$; what did the whole Estate amount to?

As $7 : 5 :: 3 : 2\frac{1}{7}$, *then, As* $2\frac{1}{7} : 1537\frac{5}{8} :: 9+5+2\frac{1}{7}$: £11583 8*s* 10 *Answer.*

12. A Gentleman left his son a fortune; $\frac{5}{16}$ of which he spent in 3 months; $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder lasted him 9 months longer, when he had only £537 left; Pray, what did his Father bequeath him?

$\frac{16}{16}=$ *whole legacy,* $\frac{16}{16}-\frac{5}{16}=\frac{11}{16}$ *left at* 3 *months, then,* $\frac{3}{4}$ *of* $\frac{5}{6}$ *of* $\frac{11}{16}=\frac{165}{384}$, *and* $\frac{11}{16}-\frac{165}{384}=\frac{1584}{6144}=$ £537, *therefore, As* $\frac{1584}{6144} : \frac{537}{8} :: \frac{1}{1}$: £2082 18*s* 2$\frac{2}{11}$ *Ans.*

13. A

13. A gay young fellow ſoon got the better of $\frac{2}{7}$ of his fortune; he then gave £1500 for a Commiſſion, and his Profuſion continued till he had but £450 left, which he found to be juſt $\frac{6}{16}$ of his money, after he had purchaſed his Commiſſion; what was his fortune at firſt?

As 6 : 450 :: 16 : 1200, *and* 1200 + 1500 = £2700 = $\frac{5}{7}$ *of his fortune, and, As* 5 : 2700 :: 7 : £3780 *Anſ.*

14. A Merchant begins the world with £1500, and finds that by his diſtillery he clears £1500 in 7 years; by his Navigation £1500 in 9 years, and that he ſpends in gaming £1500 in $3\frac{1}{2}$ years; How long will his Eſtate laſt?

$$As\left\{\begin{array}{l} 7 : 1500 :: 1 : 214\frac{2}{7} \\ 9 : 1500 :: 1 : 166\frac{2}{3} \\ 3\frac{1}{2} : 1500 :: 1 : 428\frac{4}{7} \end{array}\right\} As\ 428\tfrac{4}{7} - \overline{214\tfrac{2}{7} + 166\tfrac{2}{3}} : \overset{Y.}{1} :: \overset{£.}{1500} : \overset{Y.}{31\tfrac{1}{2}}$$

15. A has £100 of B's money in his hands, for the remittance of which B allows him 9 *per Cent.*: What Sum muſt he remit, to diſcharge himſelf of the £100?

As 100+9 : 100 :: 100 : £$91\frac{81}{109}$ *or,* $\frac{100 \times 100}{100+9}$ = £$91\frac{81}{109}$ *to be remitted, and* $\frac{100 \times 9}{100+9}$ = £$8\frac{28}{109}$ *his Commiſſion.*

16. Said Harry to Edmund, I can place four 1's ſo that, when added, they ſhall make preciſely 12; Can you do ſo too?

17. A and B are on oppoſite ſides of a circular field 268 Poles about; They begin to go round it, both the ſame way, at the ſame inſtant of Time; A goes 22 rods in 2 minutes, and B 34 rods in 3 minutes; How many times will they go round the field, before the ſwifter overtakes the ſlower?

Pole. min. Pol. min. Pole. min. Pol. min.
As 22 : 2 :: 1 : $\frac{1}{11}$ *and, As* 34 : 3 :: 1 : $\frac{3}{34}$, *then,* $\frac{1}{11} - \frac{3}{34} = \frac{1}{374}$ *minutes.*

Pole. min. Pole. min. Time. Round. Time.
As 1 : $\frac{1}{374}$ :: 22 : $\frac{11}{187}$, *As* $\frac{11}{187}$: $\frac{1}{1}$ :: $\frac{1}{1}$: 17 *times round, Anſ.*

18. If 15 men can perform a piece of work in 11 days: How many men will accompliſh another piece of work four times ſo large, in a fifth part of the time?

Work. Men. Works. Men. Time. Men. Time. Men.
As 1 : 15 :: 4 : 60 *As* $\frac{1}{1}$: $\frac{60}{1}$ :: $\frac{1}{5}$: 300 *Anſ.*

19. If A can do a piece of work alone in 7 days, and B in 12; ſet them both about it together; in what time will they finiſh it?

Days. Work. Day. Work. — Work. Work. Work. Work. Day. Work. Day.
$$As\left\{\begin{array}{l} 7 : 1 :: 1 : \frac{1}{7} \\ 12 : 1 :: 1 : \frac{1}{12} \end{array}\right\} Then,\ \tfrac{1}{7}+\tfrac{1}{12}=\tfrac{19}{84}.\quad As\ \tfrac{19}{84} : \tfrac{1}{1} :: \tfrac{1}{1} : 4\tfrac{8}{19}\ Anſ.$$

20. A and B together can build a Boat in 20 days; with the aſſiſtance of C they can do it in 12; In what time would C do it by himſelf?

As

D. W. D. W.   *W. W. W.*   *W. D. W. D.*

As $\left\{\begin{array}{l} 20 : 1 :: 1 : \frac{1}{20} \\ 12 : 1 :: 1 : \frac{1}{12} \end{array}\right\}$ *Then,* $\frac{1}{12} - \frac{1}{20} = \frac{8}{240}$, & *As* 8 : 1 :: 240 : 30 *Ans.*

21. A can do a piece of work alone in 13 days, and A and B together in 8 days; In what time can B do it alone?

D. W. D. W.   *W. W. W.*   *W. D. W. D.*

As $\left\{\begin{array}{l} 13 : 1 :: 1 : \frac{1}{13} \\ 8 : 1 :: 1 : \frac{1}{8} \end{array}\right\}$ *Then,* $\frac{1}{8} - \frac{1}{13} = \frac{5}{104}$, *and, As* 5 : 1 :: 104 : $20\frac{4}{5}$

22. A, B and C can complete a piece of work in 15 days; A can do it alone in 23 days, and B in 37 days; In what time can C do it by himself?

D. W. D. W.

As $\left\{\begin{array}{l} 15 : 1 :: 1 : \frac{1}{15} \\ \hline 23 : 1 :: 1 : \frac{1}{23} \\ 37 : 1 :: 1 : \frac{1}{37} \end{array}\right\}$

W. W. W. W.

Then, $\frac{1}{15} - \frac{1}{23} + \frac{1}{37} = \frac{49}{1765}$, *and,*

Works. Day. Works. Days.

As 49 : 1 :: 1765 : $36\frac{1}{49}$ *Ans.*

23. A Ciſtern, for water, has two Cocks to ſupply it, by the firſt it may be filled in 45 minutes, and by the ſecond, in 55 minutes; it has likewiſe a diſcharging Cock, by which it may, when full, be emptied in 30 minutes; Now, if theſe three Cocks be all left open when the water comes in; In what time will the Ciſtern be filled?

Min. Ciſt. Min. Ciſt.

45 : 1 :: 60 : 1,3333
55 : 1 :: 60 : 1,0909

2,4242

30 : 1 :: 60 : 2

Gains in an hour ,4242 *of a Ciſtern.*

Ciſt. Hour. Ciſt. h. m. s.

As ,4242 : 1 :: 1 : 2 21 $26\frac{1}{2}$ *Anſ.*

24. A Water-Tub holds 73 Gallons; the Pipe, which conveys the water to it, uſually admits 7 Gallons in 5 minutes; and the Tap diſcharges 20 Gallons in 17 minutes; Now, ſuppoſing theſe both to be careleſſly left open, and the water to be turned on at 4 o'clock in the morning; a ſervant, at 6, finding the water running, puts in the Tap; In what time, after this accident, will the Tub be full?

Min. Gal. Min. Gal.   *h.*

As $\left\{\begin{array}{l} 5 : 7 :: 60 : 84 \\ 17 : 20 :: 60 : 70\frac{10}{17} \end{array}\right\}$ $\overline{84 - 70\frac{10}{17}} \times 2 = 29\frac{14}{17}$ *gal.* & $73 - 26\frac{14}{17} =$ $46\frac{3}{17}$ *gal. which now remain to be filled.*

Gal. Min. Gal. M. s.

Therefore, As 7 : 5 :: $46\frac{3}{17}$: 32 $58\frac{118}{119}$, *and therefore the Tub will*

m. s.

be full at 32 $58\frac{118}{119}$ *after* 6.

25. A has a Cheſt of Tea, weighing $3\frac{1}{2}$*Cwt.* the prime coſt of which is £60; Now, allowing intereſt at 6 *per Cent. per Annum,*

How

How muſt he rate it *per* ℔. to B, ſo that by taking his note of hand, payable at 6 months, he may clear 50 dollars by the bargain ?

Int. £2 5*ſ*. *Then, As* $3\frac{1}{2}$*Cwt.* : £60 + £15 + £2 5*ſ*. : : 1 ℔ : 3*ſ*$11\frac{17}{98}$ *Anſwer.*

26. Suppose the American Continental Debt to be 18 millions; what annuity, at 6 *per Cent. per Annum*, will diſcharge it in 25 years ?

By Table V. of annuities, Page 322*d.* ,07823 *is the annuity, which* £1 *will purchaſe in* 25 *years, then,* ,07823 × 18000000 = £1408140 *Anſ.*

The annual Intereſt of the Debt = 1080000

Therefore, there muſt be a ſinking fund of £328140 *pr ann.*

27. The hour and minute hand of a watch are exactly together at 12 o'clock; when are they next together ?

The velocities of the two hands of a watch, or Clock, are to each other, As 12 *to* 1; *therefore, the difference of velocities is* 12—1 = 11.

$$As\ 11 : 1 : : \left\{\begin{array}{lrrl} & h. & m. & s. \\ 12\times1 : & 1 & 5 & 27\frac{3}{11} \\ 12\times2 : & 2 & 10 & 54\frac{6}{11} \\ 12\times3 : & 3 & 16 & 21\frac{9}{11} \end{array}\right\} \begin{array}{l} Anſwer. \\ \&c. \end{array}$$

28. A Hare ſtarts 12 Rods before a Hound; but is not perceived by him till ſhe has been up 45 ſeconds; ſhe ſcuds away at the rate of 10 miles an hour, and the dog, on view, makes after, at the rate of 16 miles an hour; How long will the courſe hold, and what ſpace will be run over, from the ſpot where the dog ſtarted ?

$\frac{16-10}{16} = \frac{6}{16} = \frac{3}{8}$, *or, as* 8 *to* 3 *againſt the Hare,* 1 *hour* = 3600 *ſeconds.*

Sec. *Feet.* *Sec.* *Feet.* 10 *miles* = 52800 *feet.*

As 3600 : 52800 : : 45 : 660 *diſtance the Hare had run before the dog* (*diſcovered her.*

Add 12 *Rods* = 198

858 = *the diſtance of the hare when the Dog ſtarted.*
8

3)6864

Feet 2288 = *the ground run over by the Dog.*

Miles. *Feet.* *Sec.* *Feet.* *Sec.*

Now, As 16 = 84480 : 3600 : : 2288 : $97\frac{1}{2}$

29. In a Series of proportional numbers, the firſt is 4, the third 12, and the Product of the ſecond and third is 112,8; what is the difference of the ſecond and fourth ?

112,8 ÷ 12 = 9,4 *the ſecond. As* 4 : 9,4 : : 12 : 28,2, *and* 28,2—9,4 = 18,8 *Anſ.*

30. A Fellow ſaid that when he counted his Nuts, two by two, three by three, four by four, five by five, and ſix by ſix, there was ſtill

ſtill an odd one ; but when he told them ſeven by ſeven, they came out even ; How many had he?

$2\times3\times4\times5\times6=720$, *and* $\overline{720+1}\div7=103$ *even*, *Anſ.* 721.

$\frac{721}{2.3.4.5 \text{ and } 6}$ *reſpectively, will leave an odd one.*

31. There is an Iſland, 50 miles in circumference, and 3 men ſtart together to travel the ſame way about it: A goes 7 miles *per* day, B 8 and C 9; when will they all come together again, and how far will each travel?

$\overline{50\times7}+\overline{50\times8}+\overline{50\times9}\div\overline{7+8+9}=50$ *days.—A* 350 *miles, B* 400 *and C* 450, *Anſ.*

32. Suppose A leaves Newbury-port at 6 o'clock on Monday morning, and travels towards Providence, at the rate of 4 miles *per* hour, without intermiſſion; and that, at 3 in the afternoon, B ſets out from Providence for Newbury-port, and travels conſtantly at the rate of 7 miles an hour; Now, ſuppoſe the diſtance between the two Towns to be 90 miles; whereabout on the road will they meet?

$6+3=9$ *hours, and* $9\times4=36$ *miles, the time and diſtance A had travelled before B ſtarted. Then* $90-36=54$ *miles remain to be travelled by both, now, as both together leſſen the diſtance* $7+4=11$ *miles an hour, therefore* $\frac{4}{11}$ *of* $54+36=55\frac{7}{11}$ *miles from Newbury-port; which is near Ames' at Dedham.*

33. If, during Ebb-tide, a wherry ſhould ſet out from Haverhill to come down the river, and, at the ſame time, another ſhould ſet out from Newbury-port, to go up the river, allowing the diſtance to be 18 miles; ſuppoſe the current forwards one and retards the other $1\frac{1}{2}$ mile *per* hour; the boats are equally laden, the Rowers equally good, and, in the common way of working in ſtill water, would proceed at the rate of 4 miles *per* hour; where, in the River, will the two Boats meet?

M. M. M. M. M. M. M. M. M.
$4+1\frac{1}{2}=5\frac{1}{2}$, *and* $4-1\frac{1}{2}=2\frac{1}{2}$, *then,* $5\frac{1}{2}+2\frac{1}{2}=8$ *in one hour by*
M. H. M. H. M. H M.
both. As $8 : 1 :: 18 : 2\frac{1}{4}$, *then* $5\frac{1}{2}\times2\frac{1}{4}=12\frac{3}{8}$ *from Haverhill,*
M. H. M.
and $2\frac{1}{2}\times2\frac{1}{4}=5\frac{5}{8}$ *from Newbury-port.*

34. A Gentleman making his addreſſes in a Lady's family, who had 5 daughters; ſhe told him that their father had made a will, which imported that the firſt four of the Girls' Fortunes were, together, to make £50000; the laſt four £66000, the three laſt with the firſt £60000, the three firſt with the laſt £56000, and the two firſt with the two laſt £64000, which, if he would unravel, and make it appear what each was to have, as he appeared to have a partiality for Harriet, her third daughter, he ſhould be welcome to her; Pray, what was Miſs Harriet's fortune?

$$\left.\begin{array}{r} A+B+C+D = 50000 \\ B+C+D+E = 66000 \\ A +C+D+E = 60000 \\ A+B+C +E = 56000 \\ A+B +D+E = 64000 \\ \hline 296000 \end{array}\right\}$$

Then, 296000÷4 *the number of combinations* = 74000 *the sum of their fortunes.*
Then, $A+B+C+D+E = 74000$
and $A+B \quad +D+E = 64000$

Harriet's fortune = £10000 *Ans.*

35. Three Persons purchase a vessel in company, towards the payment whereof A advanced $\frac{2}{5}$, B $\frac{3}{7}$ and C, £256; what did A and B pay, each, and what part of the vessel had C?

$\frac{2}{5}+\frac{3}{7}=\frac{14+15}{35}=\frac{29}{35}$, *and* $\frac{35}{35}-\frac{29}{35}=\frac{6}{35}$ *C's part of the vessel.*

$$As\ \frac{6}{35} : \frac{256}{1} :: \begin{cases} \frac{14}{35} : £597\ 6/8 \ \textit{A advanced.} \\ \frac{15}{35} : £640 \ \textit{B advanced.} \end{cases}$$

36. A and B cleared, by an adventure at sea, 45 Guineas, which was £35 *per Cent.* upon the money advanced, and with which they agreed to purchase a genteel Horse and Carriage, whereof they were to have the use in proportion to the sums adventured, which was found to be 11 to A, as often as 8 to B; what money did each adventure?

As £35 : 100 :: 45 *Guin.* : £180 = *the whole adventure.*

$$As\ 11+8 : 180 :: \begin{cases} 11 : £104\ 4/2\ \frac{10}{19}\ \textit{A's.} \\ 8 : £75\ 15/9\ \frac{9}{19}\ \textit{B's.} \end{cases}$$

37. A, B and C are to share £100 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively; but C dying, it is required to divide the whole sum properly between the other two?

$$As\ \tfrac{1}{3}+\tfrac{1}{4}+\tfrac{1}{5} : 100 :: \left\{\begin{array}{l} \tfrac{1}{3} : 42\tfrac{26}{47}\ \textit{A's} \\ \tfrac{1}{4} : 31\tfrac{43}{47}\ \textit{B's} \\ \tfrac{1}{5} : 25\tfrac{25}{47}\ \textit{C's} \end{array}\right\}$$

Again, As $\frac{1}{3}+\frac{1}{4} : 25\frac{25}{47} :: \frac{1}{3} : 14\frac{582}{947}$.
Then, $42\frac{26}{47}+14\frac{582}{987}$ = £57 2/10¼ *A's share.*
And £100—57 2/10¼ = £42 17/1¼ *B's share.*

38. A, B and C have among them 135 Guineas; A's + B's are to B's + C's, as 5 to 7, and C's—B's to C's + B's as 1 to 7; How many had each?

A+B. B+C.

Suppose A's+B's=50; *then, as* 5 *to* 7 :: 50 : 70, *As* 7 : 1 :: 70 : 10 = *C's—B's*; *then,* 70—10=60, *and* 60÷2=30=*B's*, 30—10=20= *A's, and* 30+10=40=*C's by the supposition*; *Now* 20+30+40=90, *which should have been* 135, *therefore,*

$$As\ 90 : 135 :: \begin{cases} 20 : 30 = \textit{A's.} \\ 30 : 45 = \textit{B's.} \\ 40 : 60 = \textit{C's.} \end{cases}$$

Sum = 135 *Proof.*

39. THERE

39. THERE are 3 horſes, belonging to different men, employed as a Team to draw a load of ſalt from Newbury-port to Boſton for £2 10s. A and B are ſuppoſed to do $\frac{3}{11}$ of the work; A and C $\frac{5}{13}$ and B and C $\frac{4}{14}$ of it; they are to be paid proportionally; Can you divide it as it ſhould be?

$$\left.\begin{array}{l} A+B=\frac{3}{11}=,2727 \\ A+C=\frac{5}{13}=,3846 \\ B+C=\frac{4}{14}=,2857 \\ \hline \textit{Sum}=,943 \end{array}\right\}$$

And ,943÷2, *the number combined* $= ,4715 = A+B+C$
$- ,2727 = A+B$

Then, As ,4715 : 50 :: ,1988 : £1 1s/0¾ = ,1988 = C

And in the ſame manner proceed for the reſt.

40. I would put 20 hogſheads of London Beer into 10 wine-pipes, and deſire to know what the Caſk muſt contain, which will receive the difference, 231 ſolid Inches being the wine-gallon, and 282 that of Beer?

Beer-Hogſhead = 54 *Gal. and* $54\times282\times20=304560$ *ſolid inches.*
Wine-Pipe = 126 *Gal. and* $126\times231\times10=291060$ *ſolid inches and*
$\frac{304560-291060}{282}=47\frac{41}{47}$ *Beer-Gallons, Anſ.*

41. BEING about to plant 5292 Trees equally diſtant in rows, the length of the Grove is to be 3 times the breadth; how many of the ſhorter rows will there be?

$\sqrt{\frac{5292}{3}}\times3=126$ *rows. Anſ. viz.* $\frac{1}{3}$ *of the Trees are to form an exact ſquare, the ſide whereof being* 42, *ſhews how many come into a ſhort row.*

42. A General, diſpoſing his Army into a ſquare Battalion, found he had 231 over and above; but increaſing each ſide with one Soldier, he wanted 44 to fill up the ſquare; How many men did his army conſiſt of?

$\overline{231+44}=275$, *and* $\overline{275-1}\div2=137$, *then* $\overline{137\times137}+231=19000$ *Anſwer.*

43. I want the length of a ſhoar, the bottom of which, being ſet 9 feet from the perpendicular ſide of a houſe, will ſupport a weak place in the wall $22\frac{1}{2}$ feet from the ground?

$\sqrt{\overline{22,5\times22,5}+\overline{9\times9}}=24$ *feet,* $2\frac{3}{4}$ *inches, Anſwer.*

44. A line 35 yards long will exactly reach from the Top of a fort, ſtanding on the brink of a River, to the oppoſite bank, known to be 27 yards broad; what is the height of the wall?

$\sqrt{\overline{35\times35}-\overline{27\times27}}=22$ *feet,* $3\frac{1}{4}$ *inches, nearly.*

45. SUPPOSE a Light-houſe built on the Top of a Rock; the diſtance between the place of obſervation and that part of the rock level

level with the eye 620 yards; the diſtance from the top of the rock to the place of obſervation 846 yards, and from the top of the Light-houſe 900 yards: The height of the Light-houſe is required?

$\sqrt{\overline{900\times900-620\times620}}-\sqrt{\overline{846\times846-620\times620}}=55,42$ *yards, Anſ.*

46. *The Sum and difference of the Squares of two numbers given, to find thoſe numbers.*

RULE. *From the Sum take the difference, and half the remainder is the Square of the leſs, which, taken from the Sum of the Squares, will give the Square of the greater.*

A and B have between them a number of Guineas, which are to be ſo divided, that the Sum of their Squares may be 208, and the difference of their ſquares 80; ſuppoſing A's the greater number, how many has he more than B?

$\overline{208-80}\div2=64$ *the Square of B's, and* $208-64=144$ *the Square of A's; therefore* $\sqrt{144}-\sqrt{64}=4$ *Anſwer.*

47. *Having the Sum of two numbers, aud the Sum of their Squares given, to find thoſe numbers.*

RULE. *From the Square of their Sum take the Sum of their Squares: then, from the Sum of their Squares take this remainder, and the Square Root of the difference will be the difference of the two numbers. To half their ſum add half their difference, and the ſum will be the greater. From half the ſum take half their difference, and the remainder will be the leſs.*

A and B have 50 Guineas between them, which are to be ſo divided, as that the Sum of the Squares of the two numbers ſhall be 1300; How many had each, ſuppoſing A to have the greater number?

$\overline{50\times50}-1300=1200$; *Then* $\sqrt{\overline{1300-1200}}=10$ *difference,*

Now $\overline{50\div2}+\overline{10\div2}=30=$ *A's. And* $\overline{50\div2}-\overline{10\div2}=20=$ *B's, Anſ.*

48. *Having the Difference of two numbers, and the Sum of their Squares given, to find thoſe numbers.*

RULE. *From the Sum of their Squares take the Square of their Difference: To the Sum of the Squares add the remainder, and the Square Root of this ſum will be the ſum of the required numbers; then, with the half ſum and half difference proceed as in the laſt queſtion.*

A number of guineas are to be divided between A and B in ſuch a manner, that A may have 50 more than B, and that the Sum of the ſquares of the reſpective Shares may be 12500; What number had each?

$12500-\overline{50\times50}=10000$, *and* $\sqrt{\overline{12500+10000}}=150=$ *Sum of their Shares. Then,* $\overline{150\div2}+\overline{50\div2}=100$ *A's; and* $\overline{150\div2}-\overline{50\div2}=50$ *B's, Anſwer.*

49. *Having the Sum of the Squares of two numbers, and the Square of their Half-ſum given, to find thoſe numbers.*

RULE.

RULE. *From the Sum of the Squares take* twice *the Square of the Half-sum, and the Square Root of half the remainder will be their Half-difference, with which and the half-sum proceed as before directed.*

LET the Sum of the Squares of two numbers be 3161, and the Square of their Half-sum 1560,25; Required those numbers?

$\overline{3161-1560,25}\times2=\overline{40,5}$ $40,5\div2=20,25$, *and* $\sqrt{20,25}=4,5=\frac{1}{2}$ *difference, and* $\sqrt{1560,25}=39,5=\frac{1}{2}$*Sum*; *then*, $39,5+4,5=44$ *the greater, and* $39,5-4,5=35$ *the less, Answer.*

50. 1. *If the quantity of matter, (or weights) of any two Bodies, put in motion, be equal, the force by which they are moved will be in proportion to their Velocities, or swiftness of motion.*

2. *If the Velocities of these bodies be equal, their forces will be directly as the quantities of matter contained in them, that is, as their weights.*

3. *If both the quantities of matter and the Velocities be unequal, the forces, with which the Bodies are moved, will be in a proportion compounded of their quantities of matter and Velocities.*

SUPPOSE the Battering-Ram of Vespasian weighed 60000 ℔; that it was moved at the rate of 24 feet in one second, and that this was sufficient to demolish the walls of *Jerusalem*; with what velocity must a Cannon-ball, which weighs 42℔. be moved, to do the same Execution?

THE velocity of the Ram being 24, and the weight of the Ball 42, compounded will make a fraction$=\frac{24}{42}=\frac{4}{7}$, and $\frac{4}{7}\times60000=$ $34285\frac{5}{7}$ feet in a second, *Answer.*

51. A Body weighing 30℔. is impelled by such a force as to send it 20 rods in a second; with what velocity would a Body weighing 12℔. move, if it were impelled by the same force?

$\frac{30\times20}{12}=50$ *Rods in a Second, Answ.*

OF GRAVITY.

52. *The Gravity of Bodies above the Surface of the Earth decreases in a duplicate ratio (or as the squares of their distances) in Semidiameters of the Earth, from the Earth's Centre.*

SUPPOSING a Body to weigh 400℔. at 2000 miles above the Earth's Surface; what would it weigh at the Surface, estimating the earth's Semidiameter at 4000 miles?

From the Centre to the given height being $1\frac{1}{2}$ *Semidiameters; multiply the Square of* $1\frac{1}{2}$ *by the weight, and the product will be the answer.*

$1,5\times1,5\times400=900$℔. *Answ.*

53. IF a Body weigh 900℔. at the Surface of the Earth; what will it weigh at 2000 miles above the Surface?

This being the reverse of the last, therefore, $1+,5=1,5$ *and* $900\div\overline{1,5\times1,5}=400$℔. *Answer.*

54. A Certain Body on the Surface of the Earth, weighs 180℔: how high must it be carried, to weigh but 20℔?

$\sqrt{180 \div 20} = 3$, *Answer, 3 Semidiameters from the Earth's Centre, that is,* 8000 *above it's Surface.*

55. How high must a Ball be raised, to lose *half* it's weight?

As 1 : 4000×4000 : : 2 : 32000000, *and* $\sqrt{32000000} = 5656{,}85$, *and* 5656,85—4000=1656,85 *miles, Answer.*

56. At what distance from the Earth would a Balloon be suspended between the Earth and Moon?

Rule. *As the sum of the Masses of the Earth and Moon to the mean-distance of their Centres; so is the mass of the Earth, to the distance required.*

Suppose the Earth's Diameter 8000 *miles, the Moon's* 2182,—*the Moon's mean distance from the Earth* 240400 *miles,—the Earth's Density* 400, *and the moon's* 494,—*then* 240400+4000+1091=245491 *miles, the distance of their Centres, therefore,*

As 8000×8000×8000×400 : 245491 : : 2182×2182×2182×494 : 638,77 *miles from the Earth, Answ.*

57. If the attraction of the moon raise a tide on the Earth 5 feet high: what will be the height of a Tide, raised by the Earth on the surface of the moon under similar circumstances?

The attraction of one of those Bodies to the other's surface is directly as its quantity of matter, and inversely as its diameter; therefore,

As 2182×2182×2182×494 : 5 : : 8000×8000×8000×400 : 199,5 *feet, Answer.*

58. 1. *If the Diameters of two Globes be equal, and their Densities, (compactness, or closeness) different; the weight of a body on their Surfaces will be as their Densities.*

2. *If their Densities be equal, and their diameters different; the weight of a body will be as* $\frac{1}{3}$ *of their Circumferences.*

3. *If their Diameters and Densities be both different; The weight will be as* $\frac{2}{3}$ *of their Semidiameters multiplied by their Densities.*

If a Stone weigh 100℔. at the Surface of the Earth: What will it weigh at the Surfaces of the Sun, and the several Planets, whose densities are known respectively?

	Sun.	*Jupiter.*	*Saturn.*	*Earth.*	*Moon.*
Their Densities	100.	94,5.	67.	400.	494
Diam. in geog. miles	776970.	135079.	98566.	6875.	1869,35
Semi-Diam.	388485.	67539,5.	49283.	3437,5.	934,67
$\frac{2}{3}$ *Sem. Diam.*	258990.	44693.	32855,33.	2295.	623,11

Then, As 2295×400 : 100 : :
- 258990 ×100 : 2821,24℔ *at the Sun.*
- 44693 × 94,5 : 460℔. *at Jupiter.*
- 32855,33× 67 : 239,8℔. *at Saturn.*
- 623,11×494 : 33,5 *at the Moon.*

Of

OF THE FALL OF BODIES.

59. *Heavy bodies, near the surface of the Earth, fall one foot the first Quarter of a second; three feet in the second Quarter; five feet in the third, and seven feet in the fourth Quarter; that is* 16 *feet in the first second.* *

The velocities, acquired by Bodies in falling, are in proportion to the squares of the Times in which they fall; For instance, *Let go three bullets together; Stop the first at one second, and it will have fallen* 16 *feet. Stop the next at the end of the second Second, and it will have fallen* ($2\times2=4$) *four times* 16, *or* 64 *feet, and Stop the last at the end of the third Second, and the distance fallen will be* ($3\times3=9$) *nine times* 16, *or* 144 *feet, and so on.*

Or, which is the same, the space fallen through (in feet) is always equal to the square of the Time in 4ths of a second.

Or, By multiplying 16 *feet by so many of the odd numbers, beginning at unity, as there are seconds in any given time; viz. by* 1 *for the first second, by* 3 *for the second, by* 5 *for the third, and so on, these several Products will give the spaces fallen through, in each of the several Seconds, and their sum will be the whole distance fallen.*

The Velocity given, To find the space fallen through.

RULE. 1. *The square root of the feet, in the space fallen through, will ever be equal to one eighth of the velocity acquired at the end of the fall; Therefore,*

2. *Divide the velocity by* 8, *and the square of the Quotient will be the distance fallen through, to acquire that velocity.*

SUPPOSE the velocity of a Cannon-ball to be about $\frac{1}{8}$ of a mile, or 660 feet *per* second; From what height must a Body fall, to acquire the same Velocity *per* Second?

$660\div8=82{,}5$, *and* $82{,}5\times82{,}5=6806\frac{1}{4}$ *feet*, $=1\frac{157}{528}$ *miles, Ans.*

60. *The Time given, To find the space fallen through.*

RULE. 1. *The square root of the feet, in the space fallen through, will ever be equal to four times the number of seconds the Body has been falling, therefore,*

2. *Multiply the Time by* 4, *and the square of the Product will be the Space fallen through in the given Time.*

How many feet will a body fall in 5 Seconds?

$5\times4=20$, *and* $20\times20=400$ *feet, Answer.*

61. A Bullet is dropped from the top of a Building, and found to reach the ground in $1\frac{3}{4}$ second; Required its height?

$1{,}75\times4=7$, *and* $7\times7=49$ *feet, Ans. Or,* $1\frac{3}{4}=7$ *qrs. and* $7\times7=49$.
Or, $1{,}75\times1{,}75\times16=49$ *feet, Ans.*

62. WHAT is the difference between the depth of two wells, into each of which should a Stone be dropped in the same Instant, one would reach the bottom in 5 seconds, and the other in 3?

5×4

* THE exact velocity *in Vacuo* is 16,1 feet in the Second; but in the Air it will be scarcely 16 feet.

$5 \times 4 = 20$, *and* $20 \times 20 = 400$ *feet.*
$3 \times 4 = 12$, *and* $12 \times 12 = 144$ *feet.*

Answer, 256 *feet.*

63. ASCENDING Bodies are retarded in the same Ratio that descending Bodies are accelerated; Therefore, If a Ball, discharged from a Gun, returned to the Earth in 12 seconds; How high did it ascend?

The Ball being half of the Time, or 6 *seconds in its ascent, therefore,* $6 \times 4 = 24$, *and* $24 \times 24 = 576$ *feet, Ans.*

64. *The Velocity* per *second given, To find the Time.*

RULE. 1. *Four times the number of Seconds, in which a Body has been falling, is equal to one eighth of the velocity, in feet* per *second, acquired at the end of the fall; Therefore,*

2. *Divide the given velocity by* 8, *and one fourth part of the Quotient will be the Answer.*

How long must a Bullet be falling to acquire a velocity of 160 feet *per* Second?

$160 \div 8 = 20$, *and* $20 \div 4 = 5$ *Seconds, Ans.*

65. *The space, through which a Body has fallen, given, to find the Time it has been falling.*

RULE. 1. *Four times the number of Seconds, in which the Body has been falling, will ever be equal to the Square Root of the Space, in feet through which it has fallen; therefore,*

2. *Divide the Square Root of the Space fallen through by* 4, *and the Quotient will be the time, in which it was falling.*

IN how many seconds will a Bullet fall through a space of 10125 feet?

$\sqrt{10125} = 100{,}6$, *and* $100{,}6 \div 4 = 25{,}15$ *Seconds*, $= 25'' \; 9'''$ *Answer.*

66. IN what time will a Musket-ball, dropped from the top of a Steeple 484 feet high, come to the ground?

$\sqrt{484} = 22$, *and* $22 \div 4 = 5\frac{1}{2}$ *Seconds, Answer.*

67. *To find the velocity* per *Second, with which a heavy body will begin to descend, at any distance from the Earth's surface.*

RULE. *As the square of the Earth's Semidiameters to* 16 *feet: so is the Square of any other distance from the Earth's Centre inversely, to the Velocity with which it begins to descend* per *Second.*

WITH what Velocity *per* Second will an Iron-ball begin to descend, if raised 3000 miles above the Earth's surface?

As $4000 \times 4000 : 16 :: \overline{4000 + 3000} \times \overline{4000 + 3000} : 5{,}22449$ *feet, Ans.*

68. How high must a Ball be raised above the Earth's Surface, to begin to descend with a Velocity of 5,22449 feet *per* Second?

As $16 : 4000 \times 4000 :: 5{,}22449 : 49000000$, *and* $\sqrt{49000000} = 7000$,
Wherefore, $7000 - 4000 = 3000$ *miles, Answer.*

69. *To find the mean velocity of a falling Body.*

RULE. *Divide the Space fallen through by the number of Seconds it was falling, and the Quotient will be the mean velocity.*

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A Musket-ball dropped from the top of a steeple 484 feet high in $5\frac{1}{2}$ Seconds; required its mean Velocity?

$484 \div 5{,}5 = 88$ feet *per* Second, *Ans.*

70. *To find the velocity acquired by a falling Body* per *Second (or by a stream of water, having the perpendicular descent given) at the end of any given period of Time.*

RULE. 1. The velocity acquired at the end of any Period is equal to twice the mean velocity, with which it passed during that period.

OR 2. Multiply the perpendicular space fallen through by 64, and the square root of the Product is the velocity required.

IF a Ball fall through a space of 484 feet in $5\frac{1}{2}$ seconds; with what velocity will it strike?

By the former part of the Rule.
$484 \div 5{,}5 = 88$, *and* $88 \times 2 = 176$, *Ans.*

By the latter part, without regarding the Time.
$\sqrt{484 \times 64} = 176$, *Answer.*

71. THERE is a sluice, (or Flume) one end of which is $2\frac{1}{2}$ feet lower than the other; what is the velocity of the stream *per* second? $2{,}5 \times 64 = 160$, *and* $\sqrt{160} = 12{,}649$ *feet, Ans.*

72. THE velocity, with which a falling body strikes, given, to find the space fallen through.

RULE. Divide the square of the velocity by 64, and the quotient will be the height required.

IF a Ball strike the ground with a velocity of 56 feet *per* Second; from what height did it fall?

$\overline{56 \times 56} \div 64 = 49$ *feet, Answer.*

73. THE mean velocity of a Fluid, or Stream, is 12,649 feet *per* Second; what is the perpendicular fall of the stream?

$\overline{12{,}649 \times 12{,}649} \div 64 = 2\frac{1}{2}$ *feet, Answer.*

74. THE weight of a Body, and the space fallen through, given, to find the force with which it will strike.

RULE. The Momentum, or force, with which a falling body strikes, is equal to its weight multiplied by its velocity, therefore, find the velocity by Problem 70th. and multiply it by the weight, which will produce the force required.

IF the Rammer, used for driving the Piles of *Charlestown* Bridge, weighed $2\frac{1}{2}$ Tons, or 4500lb. and fell through a space of 10 feet, with what force did it strike the Pile?

$\sqrt{10 \times 64} = 25{,}3 =$ *velocity* & $25{,}3 \times 4500 = 113850$lb. *momentum, Ans.*

75. THE weight and Momentum, or striking force, given, to find the space fallen through.

RULE. Divide the Momentum by the weight, and the Quotient will be the velocity, then divide the square of the velocity by 64, and the Quotient will be the space fallen through.

 If

If the aforementioned Rammer weighed 4500 ℔. and ſtruck with a force of 113850 ℔. From what height did it fall?

$113850 \div 4500 = 25,3$, *and* $25,3 \times 25,3 \div 64 = 10$ *feet, Anſwer.*

76. If it were required to know with what Quantity of Motion, Momentum, or Force, a Fluid, moving with a given velocity, ſtrikes upon a fixed Obſtacle.

Rule. By Problem 72d. find the Fall, which will produce the given velocity: Multiply that height by 62,5 ℔. avoird. for clean River water,—by 63 ℔. for dirty water, and by 64 for Sea water.

Suppose a Stream of clear water to move at the Rate of 5 feet *per* Second, and to meet with a fixed obſtacle (or Bulk-Head) 15 feet wide and 4 feet high; what is the momentary, inſtantaneous Preſſure of the ſtream?

$5 \times 5 \div 64 = \frac{25}{64}$ *and* $25 \div 64 = ,39$ *of a foot, for the perpendicular fall of the water. Now* $62,5 \times ,39 = 24,375$ ℔. *the Preſſure upon each ſquare foot, which, multiplied by* 60 (*the number of ſquare feet in the obſtacle*) *gives* 1462,5 ℔. *going with the given velocity of* 5 *feet* per *Second, therefore,* $1462,5 \times 5 = 7312,5$ ℔. *Anſwer.*†

77. The velocity of water, ſpouting through a ſluice, or aperture in a Reſervoir, or a Bulk-head, is the ſame that a Body would acquire by falling through a perpendicular ſpace equal to that between the Top of the water in the Reſervoir, and the aperture.

What is the velocity of water iſſuing from a head of 5 feet deep?

By Problem 70th. $64 \times 5 = 320$, *and* $\sqrt{320} = 18$ *feet, nearly.*

78. If the velocity of a Stream iſſuing through the Bulk-head of a Mill, be 16 feet *per* Second; what head of water is there?

$16 \times 16 \div 64 = 4$ *feet, Anſwer.*

79. The Quantity of water, diſcharged from a hole in a veſſel, is as the ſquare root of the height of water above the aperture.

A Miller has a head of water 4 feet above the ſluice; How high muſt the water be raiſed above the opening, ſo that half as much again water may be diſcharged from the ſluice in the ſame time?

$\sqrt{4} = 2$, *and half as much again as* 2, *is* $2 + 1 = 3$, *for the ſquare root of the required depth, therefore* $3 \times 3 = 9$ *feet high, Anſwer.*

Of Pendulums.

80. The Time of a vibration, in the Cycloid, is to the Time of a heavy body's deſcent through half its length, as the Circumference of a circle to its Diameter, that is, As 3,1416 to 1; Therefore, (as a Body deſcends freely, by gravity, through about 193,5 Inches in the firſt Second) To find the length of a Pendulum, vibrating Seconds.

Rule.

† Water, being a yielding ſubſtance, loſes $\frac{2}{3}$ of its power in producing effects.

RULE. *As* $3,1416\times3,1416$: 1×1 :: $193,5$: $19,6$ *Inches, the Half-length, and* $19,6\times2=39,2$ *Inches, the length.*

81. To find the length of a Pendulum, that will ſwing any given Time.

RULE. Multiply the ſquare of the ſeconds in any given time by 39,2, and the Product will be the length required, in Inches.

REQUIRED the lengths of ſeveral Pendulums, which will reſpectively ſwing $\frac{1}{4}$ ſeconds, $\frac{1}{2}$ ſeconds, ſeconds, minutes, and hours? $,25\times,25\times39,2=2,45$ *Inches for* $\frac{1}{4}$ *ſeconds.* $,5\times,5\times39,2=9,8$ *Inches for* $\frac{1}{2}$ *ſeconds.* $1\times1\times39,2=39,2$ *Inches for ſeconds, as above;* $60\times60\times39,2=$ *the Inches in 2 miles and 120 feet, for Minutes; and 1 hour*$=3600$ *ſeconds, therefore* $3600\times3600\times39,2=$ *the Inches in 8018 miles and 96 feet, for Hours, Anſ.*

82. WHAT is the difference between the length of a Pendulum, which vibrates Half-ſeconds, and one which ſwings 3 ſeconds?

$3\times3\times39,2-,5\times,5\times39,2=28\frac{7}{12}$ *feet, Anſ.*

83. To find the Time which a Pendulum of any given length will ſwing.

RULE. Divide the given length by 39,2, and the Quotient will be the ſquare of the time in ſeconds.

OR, As 6,2696 (the ſquare root of 39,2) is to the ſquare root of the given length; So is 1 ſecond, to the time of one oſcillation; that is, divide the ſquare root of the given length by 6,2696, and the Quotient will be the time of one vibration of that Pendulum.

How often will a Pendulum of 9,8 Inches vibrate in a ſecond?

By the former part of the Rule, $9,8\div39,2=,25$ *of a ſecond, and* $\surd,25=,5$ *of a ſecond, the time of one vibration, that is, it vibrates half-ſeconds, or* $60\div,5=120$ *times in a minute.*

By the latter part. $\surd9,8=3,13$, *and* $\surd39,2=6,2696$, *therefore* $3,13\div6,2696=,5$ *of a ſecond.*

84. I obſerved that while a Stone was falling from a Precipice, a String, (with a Bullet at the end) which meaſured 25 Inches, (to the middle of the Ball) had made 5 vibrations; what was the height of the Precipice?

$25\div39,2=,6377$, *and* $\surd,6377=,7985$ *of a ſecond, the Time of one vibration, and* $,7985\times5=4$ *ſeconds, nearly, the Time of the Stone's deſcent, then* $4\times4=16$, *and* $16\times16=256$ *feet, Anſwer.*

85. To find the true depth of a well, by dropping a ſtone into it, alſo the time of the ſtone's deſcent, and of the ſound's aſcent.

RULE. 1. TAKE a line of any length, and by the laſt Problem find the time from the dropping of the ſtone 'till you hear it ſtrike the bottom.

2. MULTIPLY 73088($=16\times4\times1142$; 1142 feet being the diſtance, which ſound moves in a ſecond) by the number of ſeconds 'till you hear the ſtone ſtrike the bottom.

3. To

3. To this Product add 1304164(=the ſquare of 1142) and from the ſquare root of the ſum take 1142.

4. Divide the ſquare of the Remainder by 64(=16×4) and the quotient will be the depth of the well in feet.

5. Divide the depth by 1142, and the Quotient will be the time of the ſound's aſcent, which, being taken from the whole time, will leave the time of the ſtone's deſcent in ſeconds.

Suppose I drop a ſtone into a well, and a ſtring with a Plummet, which meaſured to the middle of the Ball 25 Inches, made 5 vibrations before I heard the ſtone ſtrike the Bottom; Required the Depth, Time of the Stone's deſcent, and of the ſound's aſcent?

$25\div39,2=,6377$, *and* $\sqrt{,6377}=,7985$, *and* $,7985\times5=4$ *ſeconds to the hearing of it ſtrike, then* $\sqrt{73088\times4+1304164}-1142=121,53$ *and* $121,53\times121,53\div64=230,77$ *feet, the depth, and* $230,77\div1142=,2$ *of a ſecond, the time of the ſound's aſcent, and* $4-,2=3,8$ *ſeconds, the time of the Stone's deſcent.*

Of the Lever, or Steelyard.

86. It is a Principle in Mechanics, that the Power is to the weight, as the velocity of the weight, to the velocity of the Power. Therefore, To find what weight may be raiſed or balanced by any given Power, ſay;

As the diſtance between the body to be raiſed or balanced, and the Fulcrum, or Prop, is to the diſtance between the Prop and the Point where the Power is applied; So is the Power to the weight which it will balance.

If a Man, weighing 160℔. reſt on the end of a Lever 10 feet long; what weight will he balance on the other end, ſuppoſing the prop one foot from the weight?

The diſtance between the weight and prop being 1 *foot, the diſtance from the prop to the Power is* 10—1=9 *feet, Therefore,*

Ft. *Ft.* ℔. ℔.
As 1 : 9 :: 160 : 1440 *Anſ.*

87. If a weight of 1440℔. were to be raiſed with a Lever 10 feet long, and the prop fixed 1 foot from the weight; what Power, or weight, applied to the other end of the Lever, would balance it?

As 9 : 1 :: 1440 : 160℔. *Anſ.*

88. If a weight of 1440℔. be placed 1 foot from the prop; at what diſtance from the prop muſt a power of 160℔. be applied, to balance it?

As 160 : 1440 :: 1 : 9 *feet, Anſwer.*

89. At what diſtance from a weight of 1440℔. muſt a Prop be placed, ſo as that a Power of 160℔. applied 9 feet from the prop, may balance it?

As 1440 : 160 :: 9 : 1 *foot, Anſ.*

90. In giving directions for making a chaiſe, the length of the ſhafts, between the axle-tree and Backband, being ſettled at 9 feet,

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a diſpute aroſe whereabout on the ſhafts the centre of the body ſhould be fixed. The Chaiſe-maker adviſed to place it 30 Inches before the axle-tree; other's ſuppoſed 20 inches would be a ſufficient Incumbrance for the Horſe; Now, ſuppoſing two paſſengers to weigh 3 *Cwt.* and the Body of the Chaiſe $\frac{3}{4}$ *Cwt.* more; what will the Beaſt in both theſe caſes bear, more than his Harneſs?

Weight of the Chaiſe and Paſſengers $3\frac{3}{4}$ *Cwt.*=420*lb.* *and* 9 *feet*=108 *Inches.*

$$\textit{Then, As } \overset{In.}{108} : \overset{lb.}{420} :: \left\{ \begin{matrix} \overset{in.}{30} : \overset{lb.}{116\frac{2}{3}} \\ 20 : 77\frac{7}{9} \end{matrix} \right\} \textit{Anſ.}$$

Of the Wheel and Axle.

91. The Proportion for the wheel and axle (in which the Power is applied to the Circumference of the wheel, and the weight is raiſed by a rope, which coils about the axle as the wheel turns round) is,

As the Diameter of the axle is to the diameter of the wheel; So is the Power applied to the wheel, to the weight ſuſpended by the axle.

A Mechanic would make a windlaſs in ſuch a manner, as that 1 ℔. applied to the wheel, ſhould be equal to 10*lb.* ſuſpended from the axle; Now, ſuppoſing the axle to be 6 Inches diameter; Required the diameter of the wheel?

As $\overset{lb.}{10} : \overset{in.}{6} :: \overset{lb.}{1} : \overset{in.}{60}$ *Inverſely, the diameter required.*

92. Suppose the diameter of the wheel to be 60 Inches, required the diameter of the Axle, ſo as that 1 ℔. on the wheel may balance 10 ℔. on the Axle?

Inverſely, As $\overset{℔}{1} : \overset{in}{60} :: \overset{℔.}{10} : \overset{in.}{6}$ *Diameter required.*

93. Suppose the Diameter of the Axle 6 Inches, and that of the wheel 60 Inches; What power at the wheel will balance 10 ℔. at the Axle?

Inverſely, As $\overset{in.}{6} : \overset{℔.}{10} :: \overset{in.}{60} : \overset{℔.}{1}$ *Anſwer.*

94. Suppose the Diameter of the Wheel 60 inches, and that of the Axle 6 inches; what weight at the Axle will balance 1*lb.* at the wheel?

Inverſely, As $\overset{in.}{60} : \overset{lb.}{1} :: \overset{in.}{6} : \overset{℔.}{10}$, *Anſwer.*

Of the Screw.

95. The Power is to the Weight, which is to be raiſed, as the diſtance between two Threads of the Screw, is to the circumference of a Circle deſcribed by the Power applied at the end of the Lever.

Rule. Find the circumference of the circle deſcribed by the end of the Lever; then, As that Circumference is to the Diſtance between the ſpiral threads of the Screw; So is the weight to be raiſed,

to

to the Power which will raiſe it, abating the friction, which is not proportional to the quantity of ſurface; but, to the weight of the incumbent part; and, at a medium, $\frac{1}{3}$ part of the effect of the machine is deſtroyed by it, ſometimes more and ſometimes leſs.

THERE is a Screw, whoſe threads are an inch aſunder; the Lever by which it is turned 30 inches long, and the weight to be raiſed, a ton, or 2240℔; What power or force muſt be applied to the end of the Lever, ſufficient to turn the Screw—that is, to raiſe the weight?

THE Lever being the Semidiameter of the Circle, the Diameter is 60 inches; then $3,1416 \times 30 = 188,496$ inches, the circumference:

Therefore, *As* 188,496 *in.* : 1 *in.* :: 2240 ℔. : 11,88 ℔., *Anſ.*

96. LET the Lever be 30 inches, (the circumference of which is found to be 188,496) the Threads 1 inch aſunder, and the Power 11,88*lb*; Required the weight to be raiſed?

As 1 *in.* : 188,496 *in.* :: 11,88 *lb.* : 2240 *lb.* *nearly, Anſ.*

97. LET the Weight be 2240*lb.* the Power 11,88*lb.* and the Lever 30 inches; Required the diſtance between the threads?

As 2240 *lb.* : 11,88 *lb.* :: 188,496 *in.* : 1 *in.* *nearly, Anſ.*

98. LET the Power be 11,88*lb.* the Weight 2240*lb.* and the Threads an inch aſunder, to find the length of the Lever?

As 11,88 *lb.* : 2240 *lb.* :: 1 *in.* : 188,5 *in.*; *Then, As* 355 : 113 :: 188,5 : 60 *inches nearly, the diameter, and* $60 \div 2 = 30$ *inches, Anſ.*

99. SUPPOSE one of thoſe Meteors, called Fire-balls, to move parallel to the Earth's ſurface, and 50 miles from it at the rate of 20 miles *per* ſecond; in what time would it move round the Earth?

Suppoſe the Earth's Diameter 8000 *miles, then* $8000 + 50 \times 2 = 8100$ *the diameter of the Circle deſcribed by the Ball*; *Then, As* 113 : 355 :: 8100 : 25801 *miles nearly, its circumference, and* $25801 \div 20 = 1290\frac{1}{20}$ *Seconds*, $= 21' \; 30'' \; 3'''$ *Anſ.*

100. SOUND, uninterrupted, moves about 1142 feet in a ſecond; How long, then, after firing of a Cannon at Newbury-port, before it will be heard at Ipſwich, eſtimating the diſtance at 10 miles in a right line?

10 *miles* = 52800 *feet, and* $52800 \div 1142 = 46\frac{134}{571}$ *Seconds, Anſ.*

101. IN a Thunder Storm I obſerved by my Clock that it was 6 ſeconds between the Lightning and Thunder; at what diſtance was the exploſion? $1142 \times 6 = 6852$ *feet* $= 1\frac{13}{120}$ *mile, Anſ.*

102. TUBES may be made of Gold, weighing not more than at the rate of $\frac{1}{1625}$ of a Grain *per* foot; what would be the weight of ſuch a Tube, which would extend acroſs the Atlantic, from Boſton to London, eſtimating the diſtance at 1000 Leagues?

$1000 \times 3 = 3000$ *miles, and* $3000 \times 5280 = 15840000$ *feet, and* $15840000 \times \frac{1}{1625} = 9747\frac{69}{100}$ *grains*, = 2 *oz.* 6 *pwt.* $3\frac{69}{100}$ *gr. Anſ.*

103. THE

103. THE mean distances of the Planets from the Sun in English miles are as follow; viz. Mercury 36841468.—Venus 68891486. The Earth 95173000.—Mars 145014148.—Jupiter 494990976.—Saturn 907956130, and the Moon 240000 from the Earth; Now as a Cannon-ball at its first discharge flies about a mile in 8 seconds, and sound, 1142 feet *per* Second; How long would a Bullet, at the afore-mentioned rate, be in passing from the Earth to the Sun, and sound in moving from the Sun to Saturn?

$95173000 \times 8'' = 24$ *Years*, 52 *days*, 8 *hours*, 33 *minutes* 20 *seconds*, *for the passage of the Ball*; *and* $907956130 \times 5280 = 4794008366400$ *feet*, *and* $4794008366400 \div 1142 = 133$ *yrs.* 41 *d.* 20 *h.* 55 *m.* $49\frac{521}{571}$ *s.*; *So long would Sound be in passing from the Sun to Saturn.*

104. LIGHT passes from the Sun to the Earth in about 8 minutes; How long would it be in passing from the Sun to *Herschell's* Planet, or the *Georgium Sidus*, supposing it to be 5000000000 miles?

As 95173000 : 8′ :: 5000000000 : 7 *h.* 0 *m.* 17 *s.* 7‴. *Answer.*

105. THE Diameter of the Sun is 890000 miles.—Mercury's Diameter 3000, Venus' 7924, the Earth's 7970, Mars' 7338, Jupiter's 156446, Saturn's 114172, and the Moon's 2182; What is the comparative magnitude between the Sun and the Earth, and between the Earth and all the others?

The Sun $= 890000 \times 890000 \times 890000 \div 7970 \times 7970 \times 7970 = 1392499{,}52$ *times larger than the Earth.*—The Earth$=7970 \times 7970 \times 7970 \div 3000 \times 3000 \times 3000 = 18{,}75$ *larger than Mercury.* $=7970 \times 7970 \times 7970 \div 7924 \times 7924 \times 7924 = 1{,}0175$ *times larger than Venus.* $=7970 \times 7970 \times 7970 \div 5400 \times 5400 \times 5400 = 3{,}21$ *times larger than Mars.* $=7970 \times 7970 \times 7970 \div 2182 \times 2182 \times 2182 = 48{,}82$ *times larger than the Moon.* *Jupiter*$=94000 \times 94000 \times 94000 \div 7970 \times 7970 \times 7970 = 1640{,}62$ *times larger than the Earth.*—*Saturn*$=78000 \times 78000 \times 78000 \div 7970 \times 7970 \times 7970 = 937{,}36$ *times larger than the Earth.*

106. THE Density of the Moon is to that of the Earth, as 123,5 to 100; what is the proportion between the quantity of matter in the Earth and that of the Moon, allowing the Earth's diameter to be 7970, and the Moon's 2182 miles, and supposing the Earth to be a complete sphere, which however it is not?

There is $\dfrac{7970 \times 7970 \times 7970 \times 100}{2182 \times 2182 \times 2182 \times 123{,}5} = 39{,}534$ *times the quantity of matter in the Earth, than is in the Moon,—In other words, the Earth weighs so much more than the Moon.*

107. THE mean Diameter of the Earth's Orbit (or annual path round the Sun) is 191263000 miles; Required its mean motion, (or the space through which it moves,) *per* minute?

$191263000 \times 3{,}1416 = 600871840{,}8$ *miles*, *Circumference.* *Then,*

As 365,25 *Days* : 600871840,8 *Miles* :: 1′ : 1142,44 *miles*, *Answer.*

N. B. The Earth's diurnal motion round its Axis is $17\frac{1}{4}$ *miles* per *minute, at the Equator.*

OF

Of the Specific Gravities of Bodies.

The ſpecific gravities of Bodies are as their denſities, or weights, bulk for bulk; thus, a body is ſaid to have two or three times the ſpecific gravity of another, when it contains two or three times as much matter in the ſame ſpace.

A body, immerſed in a fluid, will ſink, if it be heavier than its bulk of the fluid. If it be ſuſpended therein, it will loſe ſo much of what it weighed in the air, as its bulk of the fluid weighs. Hence, all bodies of equal bulks, which will ſink in fluids, loſe equal weights when ſuſpended therein, and unequal bodies loſe in proportion to their bulks.

The *Hydroſtatic Balance* differs very little from a common balance that is nicely made; only it has a hook at the bottom of each ſcale, on which ſmall weights may be hung by horſe-hairs, ſo that a body, ſuſpended by the hair, may be immerſed in water without wetting the ſcales.

How to find the Specific Gravities of Bodies.

If the body thus ſuſpended under the Scale, at one end of the balance, be firſt counterpoized in air by weights in the oppoſite Scale, and then immerſed in water, the equilibrium will be immediately deſtroyed; then, if as much weight be put into the Scale to which the body is ſuſpended as will reſtore the equilibrium, (without altering the weights in the oppoſite Scale) that weight, which reſtores the equilibrium, will be equal to a quantity of water as big as the immerſed body; and if the weight of the body in air be divided by what it loſes in water, the quotient will ſhew how much that body is heavier than its bulk of water. Thus, if a Guinea, ſuſpended in air, be counterbalanced by 129 grains in the oppoſite Scale, and then, upon being immerſed in water, it becomes ſo much lighter, as to require $7\frac{1}{4}$ grains to be put into the Scale over it, to reſtore the equilibrium, it ſhews that a quantity of water, of equal bulk with the guinea, weighs 7,25 grains; by which divide 129 (the weight of the guinea in air) and the quotient will be 17,793; which ſhews that the guinea is 17,793 times as heavy as its bulk of water.

Thus may any piece of Gold be tried, by weighing it firſt in air, and then in water; and if, upon dividing the weight in air by the loſs in water, the quotient comes out 17,793, the gold is good: if the quotient be 18, or between 18 and 19, the gold is very fine; but, if it be leſs than 17, the gold is too much alloyed, by being mixed with ſome other metal.

If Silver be tried in this manner and found to be 11 times as heavy as water, it is very fine: if it be $10\frac{1}{2}$ times as heavy, it is ſtandard; but if it be of any leſs weight, compared with water, it is mixed with ſome lighter metal, ſuch as tin, &c.

If a piece of braſs, glaſs, lead, or ſilver, be immerſed and ſuſpended in different ſorts of fluids, the different loſſes of weight therein

therein will ſhew how much heavier it is than its bulk of the fluid; that fluid being lighteſt, in which the immerſed body loſes leaſt of its ærial weight.

COMMON clear water, for common uſes, is generally made a ſtandard for comparing bodies by, whoſe gravity may be repreſented by unity, or 1, or, in caſe great accuracy be required, by 1,000, where 3 cyphers are annexed to give room to expreſs the ratios of other gravities in larger numbers in the table. In doing this there is a twofold advantage; the firſt is, that, by this mean, the ſpecific gravities of bodies may be expreſſed to a much greater degree of accuracy. The ſecond is, that the numbers of the Table, conſidered as whole numbers, do alſo expreſs the ounces *Avoirdupois* contained in a cubic foot of every ſort of matter therein ſpecified, becauſe a cubic foot of common water is found by experiment to weigh very nearly 1000 ounces Avoirdupois, or $62\frac{1}{2}$ pounds.

A TABLE of the ſpecific gravities of ſeveral ſolid and fluid bodies, where, the ſecond column contains their abſolute weight, and the third, their relative weight in Avoirdupois ounces.

A cubic Foot of	*Abſo. wt.*	*Rela. wt.*	*A cubic Foot of*	*Abſo. wt.*	*Rela. wt.*
Platina rendered malleable and hammered	20170	20,170	Brick - - - - - -	2000	2,000
			Live Sulphur - - -	2000	2,000
Very fine Gold - - -	19637	19,637	Nitre - - - - - - -	1900	1,900
Standard Gold - - -	18888	18,888	Alabaſter - - - - -	1875	1,875
Guinea Gold - - -	17793	17,793	Dry Ivory - - - -	1825	1,825
Moidore Gold - - -	17140	17,140	Brimſtone - - - -	1800	1,800
Quickſilver - - - -	13600	13,600	Solid ſubſtance of gun-powder	1745	1,745
Lead - - - - - -	11325	11,325	Allum - - - - - - -	1714	1,714
Fine Silver - - - -	11087	11,087	Ebony - - - - - -	1117	1,117
Standard Silver - - -	10535	10,535	Human Blood - - -	1054	1,054
Roſe-Copper - - -	9000	9,000	Amber - - - - - - -	1030	1,030
Copper - - - -	8843	8,843	Cows Milk - - - - - -	1030	1,030
Plate Braſs - - -	8000	8,000	Sea-water - - - - - -	1030	1,030
Steel - - - -	7852	7,852	Pure water - - - - -	1000	1,000
Caſt Braſs - - - - -	7850	7,850	Red-wine - - - - -	993	0,993
Iron - - - - -	7645	7,645	Oil of Amber - - -	978	0,978
Block Tin - - - - -	7321	7,321	Proof Spirits* - - -	925	0,925
Caſt Iron - - - -	7135	7,135	Dry-Oak - - - - -	925	0,925
Lead-Ore - - - -	6800	6,800	Olive-Oil - - - - -	913	0,913
Copper-Ore - - - -	3775	3,775	Looſe gun-powder - -	872	0,872
Diamond - - - -	3400	3,400	Spirit of Turpentine -	864	0,864
Chryſtal Glaſs - - -	3150	3,150	Alcohol or pure Spirit - -	850	0,850
White Marble - - -	2707	2,707	Elm and Aſh - - - - -	800	0,800
Black ditto - - - -	2704	2,704	Oil of Turpentine - -	772	0,772
Rock Chryſtal - - -	2658	2,658	Dry Crab-Tree - -	765	0,765
Green Glaſs - - - -	2620	2,620	Æther - - - - -	732	0,732
Clear Glaſs - - - - -	2600	2,600	White-pine - - - -	569	0,569
Stone: Flint - - - -	2582	2,582	Saſſafras wood - - -	482	0,482
Stone: Paving - - -	2570	2,570	Cork - - - - - - -	240	0,240
Stone: Cornelian - -	2568	2,568	Common Air - - - - -	$1\frac{25}{100}$	0,00125
Stone: Free - - - -	2352	2,352	Inflammable Air - - - -	$0\frac{12}{100}$	0,00012

* ALTHOUGH the ſpecific gravity be 925 according to Theory, yet, in fact, it will amount to about 927.

Specific gravities of the Solar System.

A cubic Foot of	*Abso. wt.*	*Relative weight.*
The Sun	11333	11,333
Mercury	9166	9,166
Venus	5733	5,733
Earth	4500	4,500
Mars	3286	3,286
Moon	3092	3,092
Jupiter	1042	1,042
Saturn	406	0,406

The use of the Table of specific gravities will best appear by several Examples.

How to discover the quantity of adulteration in metals.

Suppose a body be compounded of Gold and Silver, and it be required to find the quantity of each metal in the compound.

First. Find the specific gravity of the compound, by weighing it in air and in water, and dividing its aerial weight by what it loses thereof in water, and the quotient will shew its specific gravity, or how many times it is heavier than its bulk of water Then, subtract the specific gravity of silver (found in the Table) from that of the compound, and the specific gravity of the compound from that of the Gold :—the first remainder will shew the bulk of Gold, and the latter, the bulk of silver in the whole compound : and if these remainders be multiplied by the respective specific gravities, the products will shew the proportional weights of each metal in the body.

Suppose the specific gravity of the compounded body be 14 ; that of standard Silver (by the Table) is 10,535, and that of standard Gold 18,888 ; therefore, 10,535 from 14, remains 3,465, the proportional *bulk* of the Gold in the compound : and 14 from 18,888, remains 4,888, the proportional *bulk* of Silver in the compound : Then 18,888, the specific gravity of Gold, multiplied by the first remainder 3,465, produces 65,447 for the proportional *weight* of Gold ; and 10,535, the specific gravity of Silver, multiplied by the last remainder, produces 51,495 for the proportional weight of Silver in the whole body: so that for every 65,447 ounces or pounds of Gold, there are 51,495 ounces or pounds of Silver in the body.

Hence, it is easy to know whether any suspected metal be genuine, or alloyed or counterfeit, by finding how much heavier it is than its bulk of water, and comparing the same with the table ; if they agree, the metal is good ; if they differ, it is alloyed or counterfeited.

How to try Spirituous Liquors.

A cubic inch of good brandy, rum or other proof spirits, weighs 234 grains ; therefore, if a true inch-cube of any metal weighs 234 grains less in spirits than in air, it shews the spirits are proof : if it lose less of its aerial weight in spirits, they are above proof ; if it lose more, they are under proof : for the better the spirits are, the lighter they are, and the worse, the heavier.

Or, let any solid, of sufficient specific gravity, be weighed first in air, then in water, and then in another liquid ; from its weight in the air take its weight in water, and the remainder is the weight of its bulk

bulk of water. From its weight in air take its weight in the other liquid, and the remainder is the weight of the ſame quantity of that liquid. Divide the weight of this quantity of liquid by the weight of the ſame quantity of water, and the quotient will be the ſpecific gravity of the liquid.

ALL bodies expand with heat, and contract with cold; but ſome more, and ſome leſs than others; therefore the ſpecific gravities of bodies are not preciſely the ſame in ſummer as in winter.

The four following Problems, relating to Spirituous Liquors, are wrought by Alligation.

108. WHAT proportion of rectified ſpirits of wine muſt be mixed with water, to make proof-ſpirit; the ſpecific gravity of the rectified ſpirits being 850, that of proof-ſpirit 925, and of water 1000?

$$925\left\{\begin{matrix}1000\\850\end{matrix}\right\}\begin{matrix}75\\75\end{matrix}\Big\}\ \textit{Or equal Measures.}$$

109. WHAT proportional *weight* of rectified ſpirits of wine and water muſt be mixed, to make proof-ſpirit, the ſpecific gravities as before?

$$\textit{Anſ.}\ \frac{1000}{850}=\frac{20}{17},\ \textit{or as 20 to 17.}$$

110. WHAT is the ſpecific gravity of beſt French Brandy, conſiſting of 5 parts, meaſure, of rectified ſpirits of wine, and 3 parts water?

$$\begin{array}{r}850\times5=4250\\1000\times3=3000\\\hline 5+3=8\ )7250\\\hline 906,25=\textit{Specific Gravity.}\end{array}$$

111. A Retailer has 30 gallons of Rum, whoſe ſpecific gravity is 900; How much water muſt he add, to reduce it to ſtandard proof?

$$925\left\{\begin{matrix}1000\\900\end{matrix}\right\}\begin{matrix}25\\75\end{matrix}\Big\}\ \begin{matrix}&\textit{G. Rum.}&&\textit{G. Wat.}&&\textit{G. Rum.}&&\textit{G. Wat.}\\\textit{As}&75&:&25&::&30&:&10\ \textit{to be added.}\end{matrix}$$

112. THE cubic inch of common Glaſs, weighs about 1,36*oz.* Troy; ditto of ſalt water ,5427 *oz.* ditto of Brandy ,48927 *oz.* Suppoſe then, a ſeaman has a gallon of Brandy in a bottle, which weighs $4\frac{1}{2}$ ℔. Troy, out of water, and, to conceal it, throws it overboard into ſalt water; Pray, will it ſink or ſwim, and by how much is it heavier or lighter than the ſame bulk of ſalt water?

$$4\tfrac{1}{2}\text{℔}.=54\ oz.=\textit{weight of the bottle.}\ \frac{54}{1,36}=39,7059\ \textit{Cu. In. in the bot.}$$
$$\textit{Add}\ 231\ =\textit{do. in the Brandy.}$$
$$270,7059=\textit{ditto in both.}$$

Then, $270,7059\times,5427=146,912$ *oz.* = Weight of ſalt water occupied by the bottle and brandy. And ,48927 (= weight of a cubic inch of brandy) $\times231=113,02$ *oz.* and $113,02+54=167,02$ *oz.* = weight

=weight of the bottle and brandy. From this take the weight of the salt water, viz. 146,912 *oz.* and it leaves 20,11 *oz.*—*Answer. Supposing the bottle full, it is* 20,11 *oz. heavier than the same bulk of salt water, and therefore will sink.*

Given the Weight to be raised by a Balloon, to find its Diameter.

RULE.

1. As the specific difference between common and inflammable air, is to 1 cubic foot; so is any weight to be raised, to the cubic feet contained in the balloon.

2. DIVIDE the cubic feet by ,5236, and the Cube Root of the quotient will be the diameter required, to balance it with common air; but, to raise it, the diameter must be somewhat greater, or the weight somewhat less.

113. I would construct a spherical Balloon of sufficient capacity to ascend with 4 persons weighing one with another 160℔. and the Balloon and a bag of sand weighing 60℔; Required the diameter of the Balloon?

By the Table of Specific Gravities, Page 369th. I find a cubic foot of common air weighs 1,25 ounce Avoirdupois, and a cubic foot of inflammable air ,12 of an ounce Avoirdupois; therefore,

1,25—,12=1,13 *oz. difference.* *And* 160 ℔. × 4 + 60 ℔. = 700 ℔. = 11200 oz.

As 1,13 oz. : 1 Cub. foot :: 11200 oz. : 9911,5044 Cub. feet. *And* $\sqrt[3]{\frac{9911,5044}{,5236}} = 26,65$ (*feet, Ans.*

Given the Diameter of a Balloon, to find what weight it is capable of raising.

RULE.

1. MULTIPLY the Cube of the Diameter by ,5236, and the product will be the content in cubic feet.

2. As one cubic foot is to the specific difference between common and inflammable air; so is the content of the Balloon to the weight it will raise.

114. THE Diameter of a Balloon is 26,65 feet; what weight is it capable of raising?

26,65 × 26,65 × 26,65 × 5236 = 9911,4+ *Cubic feet. And*

As 1 Cub. foot : 1,13 oz. :: 9911,4+ Cub. feet : 11199,882 oz. = 700℔. *nearly.*

If the Magnitude of any body be multiplied by its specific gravity, the product will be its absolute weight.

115. WHAT weight of Lead will cover a house, the Area of whose roof is 6000 feet, and the thickness of the lead $\frac{1}{120}$ of a foot?

6000 × $\frac{1}{120}$ = 50 *cubic feet, and its specific gravity* 11325 × 50 = 566250

	T.	Cwt.	qrs.	℔.	oz.	
ounces =	15	5	0	10	10	*Ans.*

To

To find the magnitude of any thing, when the weight is known.

DIVIDE the weight by the ſpecific gravity in the Table, and the Quotient will be the magnitude ſought.

116. WHAT is the magnitude of ſeveral fragments of clear glaſs, whoſe weight is 13 ounces?

13÷3150=,004127 *of a cubic foot, and* ,004127×1728=7,13 *cubic Inches.*

Having the magnitude and weight of any body given, to find its ſpecific Gravity.

DIVIDE the weight by the magnitude, and the quotient will be the ſpecific gravity.

117. SUPPOSE a piece of marble contains 8 cubic feet, and weighs $1353\frac{1}{2}$ lb. or 21656 ounces; what is the ſpecific gravity?

21656÷8=2707, *the ſpecific gravity required, as by the Table.*

To find the quantity of Preſſure againſt the ſluice or bank, which pens water.

MULTIPLY the area of the ſluice, under water, by the depth of the centre of Gravity, (which is equal to half the depth of the water) in feet, and that product again by $62\frac{1}{2}$ (the number of pounds avoirdupois in a cubic foot of freſh water) or by 64,4*lb.* (the avoirdupois weight of a cubic foot of ſalt water) and the product will be the number of pounds required.

118. SUPPOSE the length of a ſluice or flume be 30 feet, and the depth of the water 4 feet; what is the preſſure againſt the ſide of the ſluice?

30×4=120 *feet, the area of the ſide, and* 120×2 (*the depth of the centre of gravity*) *gives* 240 *cubic feet, and* 240×62,5=15000*lb.*

T.	*Cwt.*	*qrs.*	*lb.*
=6	13	3	20

The perpendicular preſſure of fluids on the bottoms of veſſels is eſtimated by the area of the bottom multiplied by the altitude of the fluid.

119. SUPPOSE a veſſel 3 feet wide, 5 feet long, and 4 feet high; what is the preſſure on the bottom, it being filled with water to the brim?

3×5=15 *ſquare feet, the area of the bottom, and* 15×4=60 *cubic feet,*

	Cwt.	*qrs.*	*lb.*
and 60×62,5=3750*lb.*=	33	1	26

THE USE OF THE BAROMETER.

THE Barometer is ſo formed, that a column of quick-ſilver is ſupported within it, to ſuch a height as to counter-balance the weight of a column of air, of an equal Diameter, extending from the Barometer to the top of the Atmoſphere.

120. AT the ſurface of the Earth, the height of this column of quick-ſilver is, at an average, almoſt 30 inches; when the Barometer is at that height; what is the preſſure of Atmoſphere on a ſquare foot,—and on the ſurface of a man's body, eſtimated at 14 ſquare feet?

At

As the cubic foot of quick-silver is 13600 ounces, avoirdupois, and as the height in the Barometer is 2,5 feet, therefore $13600 \times 2,5 = 34000$ ounces, $= 2125$ pounds on a square foot; and $2125 \times 14 = 29750$ pounds on a Man's body.

121. If the Mercury in a Barometer, at the bottom of a Tower, be observed to stand at 30 Inches, and, on being carried to the top of it, be observed at 29,9 inches; what is the height of the Tower?

Divide 13600, *the specific gravity of quicksilver, by* 1,25, *the specific gravity of air, and the quotient will be the height of the Tower, in* tenths *of an inch.*

$\frac{13600}{1,25} = 10880$ *tenths, and* $\frac{10880}{10} = 1088$ *inches* $= 90\frac{2}{3}$ *feet, Ans.*

The number of feet, in height, of the atmosphere, corresponding with $\frac{1}{10}$ of an Inch on the Barometer is variable, depending on the temperature and density of the atmosphere.

The variation, depending on the temperature, is shewn in the following Table, calculated for every 5 degrees, from 32 to 80, Farenheit's Thermometer, from whence it may be easily calculated for the intermediate degrees, by allowing $\frac{21}{100}$ of a foot for each degree.

TABLE.

Thermo.	*Feet.*
32°	86,86
35	87,49
40	88,54
45	89,60
50	90,66
55	91,72
60	92,77
65	93,82
70	94,88
75	95,93
80	96,99

The Altitude, thus found, will be to the altitude corrected for the density of the air, inversely, as the mean height of the Barometer, at the two Stations, is to 30 inches; therefore,

Rule. Multiply the mean temperature of the two Barometers (found in the Table) by the *tenths* of an Inch in the difference of the two Barometers, and this product by 30;—divide this last product by the mean height of the two Barometers, and the quotient will be the Answer, or height required, with the error of a few feet only, if the height be less than a mile.*

122. At the 1st. Station, suppose the Barometer to stand at 29. and the Thermometer at 60; at the 2d station, the Barometer at 28, and the Thermometer at 40; what is the height of the 2d. station, or the distance between the two places of observation?

Barometer.

* Let b=mean height of the Barometer at its two stations, (or of two Barometers, one at each station) in Inches. d= Difference of the two Barometers in *tenths* of an Inch: and n= number from the Table answering to the mean Temperature of the two Thermometers accompanying the Barometer, then $\frac{30dn}{b}$= the Altitude required nearly.

Barometer.

Add { *First station* = 29
{ *Second station* = 28

$\frac{1}{2}$)57

$\frac{1}{2}$ *Sum* = 28,5 = *mean height of the two Barometers.*

29
28

Difference = 1 = 10 *tenths of an Inch.*

Thermometer.

First Station = 60
Second Station = 40

$\frac{1}{2}$)100

50 = *mean height of the two Thermometers, against which, in the Table, you will find* 90,66, *the mean Temperature of the two Barometers. Now according to the rule* 90,66 × 10 × 30 ÷ 28,5 = 954,3 *feet, the Answer nearly.*

By a late Regulation of the Assembly of Massachusetts An English or French Crown is to pass at 6/8,—A Spanish milled Dollar at 6*s.*—An English Guinea, weighing 5 *pwt.* 6 *gr.* at £1 8*s.*—A French Guinea of 5 *pwt.* 6 *gr.* at £1 7/4,—A Johannes of 18 *pwt.* at £4 16*s.* A Moidore of 6 *pwt.* 18 *gr.* at £1 16*s.* A Doubloon, or 4 Pistole-piece of 16 *pwt.* 12 *gr.* at £4 8*s.*—It is also enacted that all pieces of Gold Coin before enumerated, and which shall weigh more or less than is, by the said act, established, as their current weight, shall be received at the Treasury, and in all debts for a sum proportioned to the value of Gold, as thereby stated, *viz.* at £5 6/8 *per oz.*

Also, By a late Regulation of the Assembly of South-Carolina, the Standard weight of English Guineas is fixed 5 *pwt.* 7 *gr.*—Pistole 4 *pwt.* 6 *gr.*—and Moidores 6 *pwt.* 16 *gr.*

		pwt.	*gr.*		£.	*s.*	*d.*		£.	*s.*	*d.*
A Johannes	weighing	18	$10\frac{1}{2}$	Current in *England* at	3	12	0	Current in *Ireland* at	3	17	8
Half ditto		9	$5\frac{1}{4}$		1	16	0		1	18	10
A Moidore		6	22		1	7	0		1	9	3
A Guinea					1	1	0		1	2	9
A Louis d'or or French Pist.		4	8		0	10	6		0	11	$4\frac{1}{4}$
A Crown-piece					0	5	0		0	5	5
An English Shill.					0	1	0		0	1	1
Half-ditto.					0	0	6		0	0	$6\frac{1}{2}$

Federal Money.	N. Hampshire, Massachus. R. Island. Connectic. & Virgin.	New-York and N. Carolina.	N. Jersey, Pennsylvania Delaware & Maryland.	S. Carolina and Georgia.	Canada and Nova-Scotia	French.
Dol. d. c.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	Liv. Tour. } Sous
0,0 1	18/25	24/25	9/10	14/25	3/5	1 1/20
0,0 2	1 11/25	1 23/25	1 8/10	1 3/25	1 1/5	2 1/10
0,0 3	2 4/25	2 22/25	2 7/10	17/25	1 4/5	3 3/20
0,0 4	2 22/25	3 21/25	3 6/10	2 6/25	2 2/5	4 1/5
0,0 5	3 3/5	4 4/5	4 5/10	2 4/25	3	5 1/4
0,0 6	4 8/25	5 19/25	5 4/10	3 9/25	3 3/5	6 3/10
0,0 7	5 1/25	6 18/25	6 3/10	3 23/25	4 1/5	7 7/20
0,0 8	5 19/25	7 17/25	7 2/10	4 12/25	4 4/5	8 3/5
0,0 9	6 12/25	8 16/25	8 1/10	5 1/25	5 2/5	9 9/20
0,1 0	7 1/5	9 3/5	9	5 3/5	6	10 1/2
0,2 0	1 2 2/5	1 7 1/5	1 6	11 1/5	1 0	1 1
0,3 0	1 9 3/5	2 4 4/5	2 3	1 4 4/5	1 6	1 11 1/2
0,4 0	2 4 4/5	3 2 2/5	3 0	1 10 2/5	2 0	2 2
0,5 0	3 0	4 0	3 9	2 4	2 6	2 12 1/2
0,6 0	3 7 1/5	4 9 1/5	4 6	2 9 3/5	3 0	3 3
0,7 0	4 2 2/5	5 7 2/5	5 3	3 3 1/5	3 6	3 13 1/2
0,8 0	4 9 3/5	6 4 3/5	6 0	3 8 4/5	4 0	4 4
0,9 0	5 4 4/5	7 2 4/5	6 9	4 2 2/5	4 6	4 14 1/2
1,0 0	6 0	8 0	7 6	4 8	5 0	5 5
2,0 0	12 0	16 0	15 0	9 4	10 0	10 10
3,0 0	18 0	1 4 0	1 2 6	14 0	15 0	15 15
4,0 0	1 4 0	1 12 0	1 10 0	18 8	1 0 0	21 0
5,0 0	1 10 0	2 0 0	1 17 6	1 3 4	1 5 0	26 5
6,0 0	1 16 0	2 8 0	2 5 0	1 8 0	1 10 0	31 10
7,0 0	2 2 0	2 16 0	2 12 6	1 12 8	1 15 0	36 15
8,0 0	2 8 0	3 4 0	3 0 0	1 17 4	2 0 0	42 0
9,0 0	2 14 0	3 12 0	3 7 6	2 2 0	2 5 0	47 5
10,0 0	3 0 0	4 0 0	3 15 0	2 6 8	2 10 0	52 10
20,0 0	6 0 0	8 0 0	7 10 0	4 13 4	5 0 0	105 0
30,0 0	9 0 0	12 0 0	11 5 0	7 0 0	7 10 0	157 10
40,0 0	12 0 0	16 0 0	15 0 0	9 6 8	10 0 0	210
50,0 0	15 0 0	20 0 0	18 15 0	11 13 4	12 10 0	262 10
60,0 0	18 0 0	24 0 0	22 10 0	14 0 0	15 0 0	315
70,0 0	21 0 0	28 0 0	26 5 0	16 6 8	17 10 0	367 10
80,0 0	24 0 0	32 0 0	30 0 0	18 13 4	20 0 0	420
90,0 0	27 0 0	36 0 0	33 15 0	21 0 0	22 10 0	472 10
100,0 0	30 0 0	40 0 0	37 10 0	23 6 8	25 0 0	525
200,0 0	60 0 0	80 0 0	75 0 0	46 13 4	50 0 0	1050
300,0 0	90 0 0	120 0 0	112 10 0	70 0 0	75 0 0	1575
400,0 0	120 0 0	160 0 0	150 0 0	93 6 8	100 0 0	2100
500,0 0	150 0 0	200 0 0	187 10 0	116 13 4	125 0 0	2625
600,0 0	180 0 0	240 0 0	225 0 0	140 0 0	150 0 0	3150
700,0 0	210 0 0	280 0 0	262 10 0	163 6 8	175 0 0	3675
800,0 0	240 0 0	320 0 0	300 0 0	186 13 4	200 0 0	4200
900,0 0	270 0 0	360 0 0	337 10 0	210 0 0	225 0 0	4725
1000,0 0	300 0 0	400 0 0	375 0 0	233 6 8	250 0 0	5250

N. Hamp. Maſſach. R. Iſland, Connecticut. and Virginia.	*Federal Coin.*	*New-York, and North-Carolina.*	*New-Jerſey Pennſylvania Delaware & Maryland.*	*S. Carolina and Georgia.*	*Eng. Money.*	*French Money.*
£. s. d.	*Dol. d. c.*	*£. s. d.*	*£. s. d.*	*£. s. d.*	*£. s. d.*	*Livr. Tour.* } *Sous.*
1	0,01$\frac{7}{18}$	1$\frac{1}{3}$	1$\frac{1}{4}$	$\frac{7}{9}$	$\frac{3}{4}$	1$\frac{11}{24}$
2	0,02$\frac{7}{9}$	2$\frac{2}{3}$	2$\frac{1}{2}$	1$\frac{5}{9}$	1$\frac{1}{2}$	2$\frac{11}{12}$
3	0,04$\frac{1}{6}$	4	3$\frac{3}{4}$	2$\frac{3}{9}$	2$\frac{1}{4}$	4 $\frac{3}{8}$
4	0,05$\frac{5}{9}$	5$\frac{1}{3}$	5	3$\frac{1}{9}$	3	5 $\frac{5}{6}$
5	0,06$\frac{8}{9}$	6$\frac{2}{3}$	6$\frac{1}{4}$	3$\frac{8}{9}$	3$\frac{3}{4}$	7$\frac{7}{24}$
6	0,08$\frac{1}{3}$	8	7$\frac{1}{2}$	4$\frac{6}{9}$	4$\frac{1}{2}$	8 $\frac{3}{4}$
7	0,09$\frac{2}{3}$	9$\frac{1}{3}$	8$\frac{3}{4}$	5$\frac{4}{9}$	5$\frac{1}{4}$	10$\frac{5}{24}$
8	0,11$\frac{1}{9}$	10$\frac{2}{3}$	10	6$\frac{2}{9}$	6	11 $\frac{2}{3}$
9	0,12$\frac{1}{2}$	1 0	11$\frac{1}{4}$	7	6$\frac{3}{4}$	13 $\frac{1}{8}$
10	0,13$\frac{8}{9}$	1 1$\frac{1}{3}$	1 0$\frac{1}{2}$	7$\frac{7}{9}$	7$\frac{1}{2}$	14$\frac{7}{12}$
11	0,15$\frac{1}{3}$	1 2$\frac{2}{3}$	1 1$\frac{3}{4}$	8$\frac{5}{9}$	8$\frac{1}{4}$	16$\frac{1}{24}$
1 0	0,16$\frac{2}{3}$	1 4	1 3	9$\frac{3}{9}$	9	17 $\frac{1}{2}$
2 0	0,33$\frac{1}{3}$	2 8	2 6	1 6$\frac{6}{9}$	1 6	1 15
3 0	0,50	4 0	3 9	2 4	2 3	2 12 $\frac{1}{2}$
4 0	0,66$\frac{2}{3}$	5 4	5 0	3 1$\frac{3}{9}$	3 0	3 10
5 0	0,83$\frac{1}{3}$	6 8	6 3	3 10$\frac{6}{9}$	3 9	4 7 $\frac{1}{2}$
6 0	1,00	8 0	7 6	4 8	4 6	5 5
7 0	1,16$\frac{2}{3}$	9 4	8 9	5 4$\frac{3}{9}$	5 3	6 2 $\frac{1}{2}$
8 0	1,33$\frac{1}{3}$	10 8	10 0	6 2$\frac{6}{9}$	6 0	7 0
9 0	1,50	12 0	11 3	7 0	6 9	7 17 $\frac{1}{2}$
10 0	1,66$\frac{2}{3}$	13 4	12 6	7 9$\frac{3}{9}$	7 6	8 15
1 0 0	3,33$\frac{1}{3}$	1 6 8	1 5 0	15 6$\frac{6}{9}$	15 0	17 10
2 0 0	6,66$\frac{2}{3}$	2 13 4	2 10 0	1 11 1$\frac{3}{9}$	1 10 0	35
3 0 0	10,00	4 0 0	3 15 0	2 6 8	2 5 0	52 10
4 0 0	13,33$\frac{1}{3}$	5 6 8	5 0 0	3 2 2$\frac{6}{9}$	3 0 0	70
5 0 0	16,66$\frac{2}{3}$	6 13 4	6 5 0	3 17 9$\frac{3}{9}$	3 15 0	87 10
6 0 0	20,00	8 0 0	7 10 0	4 13 4	4 10 0	105
7 0 0	23,33$\frac{1}{3}$	9 6 8	8 15 0	5 8 10$\frac{6}{9}$	5 5 0	122 10
8 0 0	26,66$\frac{2}{3}$	10 13 4	10 0 0	6 4 5$\frac{3}{9}$	6 0 0	140
9 0 0	30,00	12 0 0	11 5 0	7 0 0	6 15 0	157 10
10 0 0	33,33$\frac{1}{3}$	13 6 8	12 10 0	7 15 6$\frac{6}{9}$	7 10 0	175
20 0 0	66,66$\frac{2}{3}$	26 13 4	25 0 0	15 11 1$\frac{3}{9}$	15 0 0	350
30 0 0	100,00	40 0 0	37 10 0	23 6 8	22 10 0	525
40 0 0	133,33$\frac{1}{3}$	53 6 8	50 0 0	31 2 2$\frac{6}{9}$	30 0 0	700
50 0 0	166,66$\frac{2}{3}$	66 13 4	62 10 0	38 17 9$\frac{3}{9}$	37 10 0	875
60 0 0	200,00	80 0 0	75 0 0	46 13 4	45 0 0	1050
70 0 0	233,33$\frac{1}{3}$	93 6 8	87 10 0	54 8 10$\frac{6}{9}$	52 10 0	1225
80 0 0	266,66$\frac{2}{3}$	106 13 4	100 0 0	62 4 5$\frac{3}{9}$	60 0 0	1400
90 0 0	300,00	120 0 0	112 10 0	70 0 0	67 10 0	1575
100 0 0	333,33$\frac{1}{3}$	133 6 8	125 0 0	77 15 6$\frac{6}{9}$	75 0 0	1750
200 0 0	666,66$\frac{2}{3}$	266 13 4	250 0 0	155 11 1$\frac{3}{9}$	150 0 0	3500
300 0 0	1000,00	400 0 0	375 0 0	233 6 8	225 0 0	5250
400 0 0	1333,33	533 6 8	500 0 0	311 2 2$\frac{6}{9}$	300 0 0	7000
500 0 0	1666,66	666 13 4	625 0 0	388 17 9$\frac{3}{9}$	375 0 0	8736

TABLE *of the Value of several Pieces of Coin, in the Federal Coin, and the several currencies of the United States.*

	pw.	gr.	*Federal Coin.* Cents.	*N. Hampshire, Massachusetts, Rh. Island, Connecticut, Virginia.* £. s. d.	*New-York and North-Carolina.* £. s. d.	*N. Jersey, Pennsylvania, Delaware and Maryland.* £. s. d.	*S. Carolina & Georgia* £. s. d.
$\frac{1}{16}$ of a Dollar			0,06 $\frac{1}{4}$	$4\frac{1}{2}$	6	$5\frac{5}{8}$	$3\frac{1}{2}$
$\frac{1}{2}$ a Pistareen			0,10	$7\frac{1}{5}$	$9\frac{3}{5}$	9	$5\frac{3}{5}$
				Vir. 8			
$\frac{1}{9}$ of a Dollar			0,11 $\frac{1}{9}$	8	$10\frac{2}{3}$	10	$6\frac{2}{9}$
$\frac{1}{8}$ of ditto			0,12 $\frac{1}{2}$	9	1 0	$11\frac{1}{5}$	7
A Pistareen			0,20	1 $2\frac{2}{5}$	1 $4\frac{1}{2}$	1 6	$11\frac{1}{5}$
				Vir. 1 4			
An Eng. Shill.			0,22 $\frac{2}{9}$	1 4	1 $7\frac{2}{3}$	1 8	1 $0\frac{4}{9}$
$\frac{1}{4}$ of a Dollar			0,25	1 6	2 0	1 $10\frac{1}{2}$	1 2
Half ditto			0,50	3 0	4 0	3 9	2 4
A Dollar			1,00	6 0	8 0	7 6	4 8
En. or Fr. Crown			1,11 $\frac{1}{9}$	6 8	*N.York* 9 0 *N Caro.* 8 9	8 4	5 $2\frac{2}{9}$
Fr. Guin.	5	5	4,62 $\frac{26}{27}$	1 7 6	1 16 0	1 14 6	1 1 5
In Massa.	5	6	4,55 $\frac{1}{9}$	1 7 4			
En. Guin.	5	6	4,66 $\frac{2}{3}$	1 8 0	1 17 0	1 15 0	1 1 9
In S Caro.	5	7					1 1 10
$\frac{1}{2}$ Johann.	9	0	4,00	2 8 0	3 4 0	3 0 0	1 17 4
Pistole In Massa.	4 4	5 3	3,66 $\frac{2}{3}$	1 2 0	1 8 0	1 7 0	17 6
Moidore	6	18	6,00	1 16 0	2 6 0	2 5 0	1 8 0
Doubloon	17	0	14,66 $\frac{2}{3}$	4 8 0	5 16 0	5 12 0	3 10 0

THE standard weight of an Eagle 11 *pwt.* $4\frac{2}{3}$ *gr.*—Half ditto 5 *pwt.* $14\frac{1}{3}$ *gr.*—A Dollar 17 *pwt.* $1\frac{3}{4}$ *gr.*—Half ditto 8 *pwt.* $12\frac{7}{8}$ *gr.*—A Double Dime 3 *pwt.* $9\frac{4}{5}$ *gr.*—A Dime 1 *pwt.* $16\frac{9}{10}$ *gr.*

TABLE *of Refiner's Weight.*

Blanks.

24 = 1 *Perrot.*
480 = 20 = 1 *Mite.*
9600 = 400 = 20 = 1 *Grain.*

Note—*What they denominate a* carat, *is the* $\frac{1}{24}$ *of a lb. an oz. or any other weight.*

DUTCH WEIGHTS for GOLD and SILVER.

Note, 32 *aces*=1 *engel*, 20 *engels*=1 *ounce*, 8 *ounces*=1 *mark, for gross Gold.*——*Also*, 24 *parts*=1 *grain*, 12 *grains*=1 *carat*, 24 *carats*=1 *mark, for fine Gold.*

The mark-weights are 1 per cent. *lighter than our Troy weight.*

A

A Table of Commission or Brokage.

Goods or stock sold	at ½ per Cent.			at 1 per Cent.			at 1½ per Cent.			at 2 per Cent.			at 2½ per Cent.			at 3 per Cent.		
Shill. 1	£.	s.	d.	0	0	0	0	0	0	0	0	0	0	0	0¼	0	0	0¼
2	0	0	0	0	0	0¼	0	0	0¼	0	0	0¼	0	0	0½	0	0	0½
3	0	0	0¼	0	0	0¼	0	0	0½	0	0	0½	0	0	0¾	0	0	1
4	0	0	0¼	0	0	0½	0	0	0½	0	0	0¾	0	0	1	0	0	1¼
5	0	0	0¼	0	0	0½	0	0	0¾	0	0	1	0	0	1½	0	0	1¾
6	0	0	0¼	0	0	0¾	0	0	1	0	0	1¼	0	0	1¾	0	0	2
7	0	0	0½	0	0	0¾	0	0	1¼	0	0	1½	0	0	2	0	0	2½
8	0	0	0½	0	0	1	0	0	1¼	0	0	1¾	0	0	2¼	0	0	2¾
9	0	0	0½	0	0	1	0	0	1½	0	0	2	0	0	2½	0	0	3
10	0	0	0½	0	0	1¼	0	0	1¾	0	0	2¼	0	0	3	0	0	3½
11	0	0	0½	0	0	1½	0	0	1¾	0	0	2½	0	0	3¼	0	0	3¾
12	0	0	0½	0	0	1½	0	0	2	0	0	2¾	0	0	3½	0	0	4¼
13	0	0	0¾	0	0	1½	0	0	2¼	0	0	3	0	0	3¾	0	0	4½
14	0	0	0¾	0	0	1¾	0	0	2½	0	0	3¼	0	0	4	0	0	5
15	0	0	0¾	0	0	1¾	0	0	2½	0	0	3½	0	0	4½	0	0	5¼
16	0	0	0¾	0	0	2	0	0	2¾	0	0	3¾	0	0	4¾	0	0	5¾
17	0	0	1	0	0	2	0	0	3	0	0	4	0	0	5	0	0	6
18	0	0	1	0	0	2¼	0	0	3	0	0	4¼	0	0	5¼	0	0	6¼
19	0	0	1	0	0	2¼	0	0	3¼	0	0	4½	0	0	5½	0	0	6¾
Pounds 1	0	0	1¼	0	0	2½	0	0	3½	0	0	4¾	0	0	6	0	0	7
2	0	0	2½	0	0	5	0	0	7	0	0	9½	0	1	0	0	1	2½
3	0	0	3¾	0	0	7	0	0	10¾	0	1	2¼	0	1	6	0	1	9½
4	0	0	5	0	0	9½	0	1	2¼	0	1	7	0	2	0	0	2	4¾
5	0	0	6	0	1	0	0	1	6	0	2	0	0	2	6	0	3	0
6	0	0	7¼	0	1	2¼	0	1	9½	0	2	4¾	0	3	0	0	3	7
7	0	0	8½	0	1	4¾	0	2	1	0	2	9½	0	3	6	0	4	2¼
8	0	0	9¾	0	1	7	0	2	4¾	0	3	2¼	0	4	0	0	4	9½
9	0	0	10¾	0	1	9½	0	2	8¼	0	3	7	0	4	6	0	5	4¾
10	0	1	0	0	2	0	0	3	0	0	4	0	0	5	0	0	6	0
20	0	2	0	0	4	0	0	6	0	0	8	0	0	10	0	0	12	0
30	0	3	0	0	6	0	0	9	0	0	12	0	0	15	0	0	18	0
40	0	4	0	0	8	0	0	12	0	0	16	0	1	0	0	1	4	0
50	0	5	0	0	10	0	0	15	0	1	0	0	1	5	0	1	10	0
60	0	6	0	0	12	0	0	18	0	1	4	0	1	10	0	1	16	0
70	0	7	0	0	14	0	1	1	0	1	8	0	1	15	0	2	2	0
80	0	8	0	0	16	0	1	4	0	1	12	0	2	0	0	2	8	0
90	0	9	0	0	18	0	1	7	0	1	16	0	2	5	0	2	14	0
100	0	10	0	1	0	0	1	10	0	2	0	0	2	10	0	3	0	0
200	1	0	0	2	0	0	3	0	0	4	0	0	5	0	0	6	0	0
300	1	10	0	3	0	0	4	10	0	6	0	0	7	10	0	9	0	0
400	2	0	0	4	0	0	6	0	0	8	0	0	10	0	0	12	0	0
500	2	10	0	5	0	0	7	10	0	10	0	0	12	10	0	15	0	0
600	3	0	0	6	0	0	9	0	0	12	0	0	15	0	0	18	0	0
700	3	10	0	7	0	0	10	10	0	14	0	0	17	10	0	21	0	0
800	4	0	0	8	0	0	12	0	0	16	0	0	20	0	0	24	0	0
900	4	10	0	9	0	0	13	10	0	18	0	0	22	10	0	27	0	0
1000	5	0	0	10	0	0	15	0	0	20	0	0	25	0	0	30	0	0

A TABLE *of the Returns of the Neat Proceeds of an Account of Sales from a Factor to his Employer, reserving his Commissions for Remittance.*

Neat Proceeds.			*Sum to be remitted, reserving 2½ pr cent. Commission.*			*Sum to be remitted, reserving 5 pr cent. Commission.*			*Neat Proceeds.*			*Sum to be remitted, reserving 2½ per cent. Commission.*			*Sum to be remitted, reserving 5 per cent. Commission.*		
£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
		3			3			2¾	6	0	0	5	17	0¾	5	14	5½
		4			4			3¾	7	0	0	6	16	7	6	13	4
		5			5			4¾	8	0	0	7	16	1¼	7	12	4½
		6			5¾			5¾	9	0	0	8	15	7¼	8	11	5¼
		7			6¾			6¾	10	0	0	9	15	1½	9	10	5¾
		8			7¾			7½	20	0	0	19	10	3	19	0	11½
		9			8¾			8½	30	0	0	29	5	4¼	28	11	5¼
		10			9¾			9½	40	0	0	39	0	5¾	38	1	10¾
		11			10¾			10¼	50	0	0	48	15	7¼	47	12	4½
	1	0			11¾			11¼	60	0	0	58	10	8¾	57	2	10¼
	2	0		1	11½		1	10¾	70	0	0	68	5	10	66	13	4
	3	0		2	11¼		2	10¼	80	0	0	78	0	11½	76	3	9¾
	4	0		3	10¾		3	9¾	90	0	0	87	16	1	85	14	3½
	5	0		4	10½		4	9¼	100	0	0	97	11	2¾	95	4	9
	6	0		5	10¼		5	8½	200	0	0	195	2	5¼	190	9	6¼
	7	0		6	10		6	8	300	0	0	292	13	8	285	14	3¼
	8	0		7	9¾		7	7½	400	0	0	390	4	10½	380	19	0½
	9	0		8	9¼		8	6¾	500	0	0	487	16	1¼	476	3	9½
	10	0		9	9		9	6	600	0	0	585	7	3¾	571	8	6¾
1	0	0		19	6¼		19	0½	700	0	0	682	18	4¾	666	13	4
2	0	0	1	19	0¼	1	18	1¼	800	0	0	780	9	9	761	18	1
3	0	0	2	18	6½	2	17	1¾	900	0	0	878	0	11¾	857	2	10
4	0	0	3	18	0½	3	16	2¼	1000	0	0	975	12	2¼	952	7	7¼
5	0	0	4	17	6¾	4	15	2¾									

SUPPOSE I have the Neat Proceeds, or Balance of an Account of Sales £325 17/9 in my hands, and would make Remittance to my Employer, reserving my Commission at 2½ *per Cent.* What sum must be remitted, so that my Employer's account may be closed?

	£.				£.		
Against	300	0	0	*Stands*	292	13	8
	20	0	0		19	10	3
	5	0	0		4	17	6¾
		10	0			9	9
		7	0			6	10
			9				8¾

To be remitted £317 18 9½ *Answer.*

A

A T A B L E, *shewing the number of Days from any Day in any Month to the same Day in any other Month through the Year.*

	From January	*February*	*March*	*April*	*May*	*June*	*July*	*August*	*September*	*October*	*November*	*December*
To	*Feb.* 31	*Mar.* 28	*April* 31	*May* 30	*June* 31	*July* 30	*Aug.* 31	*Sept.* 31	*Oct.* 30	*Nov.* 30	*Dec.* 30	*Jan.* 31
	Mar. 59	*April* 59	*May* 61	*June* 61	*July* 61	*Aug.* 61	*Sept.* 62	*Oct.* 61	*Nov.* 61	*Dec.* 61	*Jan.* 61	*Feb.* 62
	April 90	*May* 89	*June* 92	*July* 91	*Aug.* 92	*Sept.* 92	*Oct.* 92	*Nov.* 92	*Dec.* 91	*Jan.* 91	*Feb.* 92	*Mar.* 90
	May 120	*June* 120	*July* 122	*Aug.* 122	*Sept.* 123	*Oct.* 122	*Nov.* 123	*Dec.* 122	*Jan.* 122	*Feb.* 122	*Mar.* 120	*April* 121
	June 151	*July* 150	*Aug.* 153	*Sept.* 153	*Oct.* 153	*Nov.* 153	*Dec.* 153	*Jan.* 153	*Feb.* 153	*Mar.* 153	*April* 151	*May* 151
	July 181	*Aug.* 181	*Sept.* 184	*Oct.* 183	*Nov.* 184	*Dec.* 183	*Jan.* 184	*Feb.* 184	*Mar.* 181	*April* 181	*May* 181	*June* 182
	Aug. 212	*Sept.* 212	*Oct.* 214	*Nov.* 214	*Dec.* 214	*Jan.* 214	*Feb.* 215	*Mar.* 212	*April* 212	*May* 212	*June* 212	*July* 212
	Sept. 243	*Oct.* 242	*Nov.* 245	*Dec.* 244	*Jan.* 245	*Feb.* 245	*Mar.* 243	*April* 243	*May* 242	*June* 242	*July* 242	*Aug.* 243
	Oct. 273	*Nov.* 273	*Dec.* 275	*Jan.* 275	*Feb.* 276	*Mar.* 273	*April* 274	*May* 273	*June* 273	*July* 273	*Aug.* 273	*Sept.* 274
	Nov. 304	*Dec.* 303	*Jan.* 306	*Feb.* 306	*Mar.* 304	*April* 304	*May* 304	*June* 304	*July* 303	*Aug.* 303	*Sept.* 304	*Oct.* 304
	Dec. 334	*Jan.* 334	*Feb.* 337	*Mar.* 334	*April* 335	*May* 334	*June* 335	*July* 334	*Aug.* 334	*Sept.* 334	*Oct.* 334	*Nov.* 335
	Jan. 365	*Feb.* 365	*Mar.* 365	*April* 365	*May* 365	*June* 365	*July* 365	*Aug.* 365	*Sept.* 365	*Oct.* 365	*Nov.* 365	*Dec.* 365

The use of the preceding Table of number of Days will easily appear from the following Examples.

SUPPOSE the number of days between the 1st. or 10th. or 30th, &c. of *January*, and the 1st. or 10th. or 30th. &c. of *October*, were required?—Look in the column under *January* for *October*, and against that month you will find 273, which is the number of days between the said times; and so for the days between any other two months.

IF the *given days* be *different*, it is only adding or subtracting their inequality to or from the *tabular number*.

How many days from the 6th. of April to the 12th. of January?——From the 6th. of April to the 6th. of January is 275, and adding the 6 overplus days, it makes 281 days. And from the 5th. of June to the 1st. of February is 240 days.

NOTE, after *February* 31, (in leap years) increase each number with an unit or 1.

A TABLE of the Meaſure of Length of the principal places in Europe compared with the American yard.

100 *Aunes or Ells of England*	— — —	= 125
100 —— *of Holland or Amſterdam, Hærlem, Leyden, the Hague, Rotterdam, Nuremburg, and other cities of Holland*	— —	= 75
100 —— *of Brabant or Antwerp*	—— ——	= 76
100 —— *of France and Oznaburg*	— —	= $128\frac{1}{2}$
100 —— *of Hamburg, Francfort, Leipſic, Bern aud Baſil*		= $62\frac{1}{2}$
100 —— *of Breſlau*	—— —— —	= 60
100 —— *of Dantzick*	—— ——	= $66\frac{3}{4}$
100 —— *of Bergen and Drontheim*	— —	= $68\frac{1}{4}$
100 —— *of Sweden and Stockholm*	—— ——	= $65\frac{3}{4}$
100 —— *of St. Gall. for Linens*	—— ——	= $87\frac{1}{2}$
100 —— *of ditto. for Cloths*	—— ——	= 67
100 —— *of Geneva*	—— ——	= $124\frac{3}{4}$
100 *Canes of Marſeilles and Montpelier*	— —	= $214\frac{1}{2}$
100 —— *of Thoulouſe and high Languedoc*	——	= 200
100 —— *of Genoa, of 9 palms*	—— ——	= $245\frac{1}{4}$
100 —— *of Rome*	—— ——	= $227\frac{1}{4}$
100 *Varas of Spain*	—— ——	= $93\frac{3}{4}$
100 —— *of Portugal*	—— ——	= 123
100 *Cavidos of Portugal*	—— ——	= 75
100 *Braſſes of Venice*	—— ——	= $73\frac{1}{2}$
100 —— *of Bergamo*	—— ——	= $71\frac{1}{4}$
100 —— *of Florence and Leghorn*	— —	= 64
100 —— *of Milan*	—— ——	= $58\frac{1}{2}$

THE uſe of the following Table, directing how to buy and ſell by the hundred.

IF you buy or ſell any thing by the great hundred (112℔.) and deſire to know, by the pound, what the hundred is valued at, obſerve the following Examples.

1. IF you buy Sugar at $6\frac{3}{4}d.$ *per* ℔; look for $6\frac{3}{4}d.$ in the left hand column of the Table, and againſt it, in the ſecond column you will find £3 3*s.* which is the value of 1*Cwt.* at that rate.

2. IF 1*Cwt.* (112℔.) coſt £9 4ſ 4*d.* To know how much it is *per* ℔. look £9 4 4 in the fourth column, and againſt it, in the next left-hand column, you will find 1ſ $7\frac{3}{4}d.$ which is the price *per* ℔.

AGAIN, If you buy one hundred weight of Goods for £9 4 4, and retail it at 1ſ $9\frac{3}{4}d.$ *per* ℔. it comes at that rate, to £10 3ſ; then take £9 4 4 from £10 3*s.* and by the remainder, you will find that you have gained £0 18ſ 8*d.*

AND in this manner, you may, with eaſe, calculate any quantity by the following Table.

A

A Table directing how to buy and sell by the *Hundred.*

d.	£. s. d.	s. d	£. s. d.	s. d.	£. s. d.
¼	0 2 4	1 0 ¼	5 14 4	2 0 ¼	11 6 4
½	0 4 8	1 0 ½	5 16 8	2 0 ½	11 8 8
¾	0 7 0	1 0 ¾	5 19 0	2 0 ¾	11 11 0
1	0 9 4	1 1	6 1 4	2 1	11 13 4
1¼	0 11 8	1 1 ¼	6 3 8	2 1 ¼	11 15 8
1½	0 14 0	1 1 ½	6 6 0	2 1 ½	11 18 0
1¾	0 16 4	1 1 ¾	6 8 4	2 1 ¾	12 0 4
2	0 18 8	1 2	6 10 8	2 2	12 2 8
2¼	1 1 0	1 2 ¼	6 13 0	2 2 ¼	12 5 0
2½	1 3 4	1 2 ½	6 15 4	2 2 ½	12 7 4
2¾	1 5 8	1 2 ¾	6 17 8	2 2 ¾	12 9 8
3	1 8 0	1 3	7 0 0	2 3	12 12 0
3¼	1 10 4	1 3 ¼	7 2 4	2 3 ¼	12 14 4
3½	1 12 8	1 3 ½	7 4 8	2 3 ½	12 16 8
3¾	1 15 0	1 3 ¾	7 7 0	2 3 ¾	12 19 0
4	1 17 4	1 4	7 9 4	2 4	13 1 4
4¼	1 19 8	1 4 ¼	7 11 8	2 4 ¼	13 3 8
4½	2 2 0	1 4 ½	7 14 0	2 4 ½	13 6 0
4¾	2 4 4	1 4 ¾	7 16 4	2 4 ¾	13 8 4
5	2 6 8	1 5	7 18 8	2 5	13 10 8
5¼	2 9 0	1 5 ¼	8 1 0	2 5 ¼	13 13 0
5½	2 11 4	1 5 ½	8 3 4	2 5 ½	13 15 4
5¾	2 13 8	1 5 ¾	8 5 8	2 5 ¾	13 17 8
6	2 16 0	1 6	8 8 0	2 6	14 0 0
6¼	2 18 4	1 6 ¼	8 10 4	2 6 ¼	14 2 4
6½	3 0 8	1 6 ½	8 12 8	2 6 ½	14 4 8
6¾	3 3 0	1 6 ¾	8 15 0	2 6 ¾	14 7 0
7	3 5 4	1 7	8 17 4	2 7	14 9 4
7¼	3 7 8	1 7 ¼	8 19 8	2 7 ¼	14 11 8
7½	3 10 0	1 7 ½	9 2 0	2 7 ½	14 14 0
7¾	3 12 4	1 7 ¾	9 4 4	2 7 ¾	14 16 4
8	3 14 8	1 8	9 6 8	2 8	14 18 8
8¼	3 17 0	1 8 ¼	9 9 0	2 8 ¼	15 1 0
8½	3 19 4	1 8 ½	9 11 4	2 8 ½	15 3 4
8¾	4 1 8	1 8 ¾	9 13 8	2 8 ¾	15 5 8
9	4 4 0	1 9	9 16 0	2 9	15 8 0
9¼	4 6 4	1 9 ¼	9 18 4	2 9 ¼	15 10 4
9½	4 8 8	1 9 ½	10 0 8	2 9 ½	15 12 8
9¾	4 11 0	1 9 ¾	10 3 0	2 9 ¾	15 15 0
10	4 13 4	1 10	10 5 4	2 10	15 17 4
10¼	4 15 8	1 10 ¼	10 7 8	2 10 ¼	15 19 8
10½	4 18 0	1 10 ½	10 10 0	2 10 ½	16 2 0
10¾	5 0 4	1 10 ¾	10 12 4	2 10 ¾	16 4 4
11	5 2 8	1 11	10 14 8	2 11	16 6 8
11¼	5 5 0	1 11 ¼	10 17 0	2 11 ¼	16 9 0
11½	5 7 4	1 11 ½	10 19 4	2 11 ½	16 11 4
11¾	5 9 8	1 11 ¾	11 1 8	2 11 ¾	16 13 8
12	5 12 0	2 0	11 4 0	3 0	16 16 0

A Comparison of the American foot with the feet of other Countries.

The American foot being divided into 1000 parts, or into 12 inches, the feet of several other Countries will be as follow.

	Parts.	Inch.	lin.	points.
America	1000	12	0	0 *dec.*
London	1000	12	0	0
Antwerp	946	11	4	1,32
Bologna	1204	14	5	2,25
Bremen	964	11	6	4,89
Cologne	954	11	5	2,25
Copenhagen	965	11	6	5,76
Amsterdam	942	11	3	3,88
Dantzick	944	11	3	5,61
Dort	1184	14	2	2,97
Frankfort on the main	948	11	4	3,07
The Greek	1007	12	1	0,04
Lorrain	958	11	5	5,71
Mantua	1569	18	9	5,61
Mecklin	919	11	0	2,01
Middleburg	991	11	10	4,22
France	938	11	3	0,43
Prague	1026	12	3	4,46
Rhyneland or Leyden	1033	12	4	4,51
Riga	1831	21	11	3,98
Roman	967	11	7	1,48
Old Roman	970	11	8	0
Scotch	1005	12	0	4,32
Strasburgh	920	11	0	2,88
Toledo	899	10	9	2,73
Turin	1062	12	8	5,66
Venice	1162	13	11	1,96

A Table representing the conformity of the weights of the principal trading Cities of Europe with those of America.

℔			of *America.*	
100	of *England, Scotland and Ireland*	Equal	100*lb.*	*oz.*
100	of *Amsterdam, Paris, Bourdeaux, &c.*		109	8
100	of *Antwerp, or Brabant*		103	12
100	of *Roüen, the Viscounty*		113	14
100	of *Lyons, the City*		94	3
100	of *Rochelle*		110	9
100	of *Toulouse, and upper Languedoc*		92	6
100	of *Marseilles and Provence*		88	11
100	of *Geneva*		123	
100	of *Hamburg*		107	5
100	of *Francfort*		111	11
100	of *Leipsic*		104	5

A

A Table representing the conformity of the weights of the principal trading Cities of Europe with those of America.

lb.		*Of America.*	
100	*of Genoa*	73	
100	*of Leghorn*	75	8
100	*of Milan*	65	3
100	*of Venice*	65	11
100	*of Naples*	64	10
100	*of Seville, Cadiz, &c.*	103	7
100	*of Portugal*	95	4
100	*of Leige*	104	
100	*of Spain*	97	*dr*
Note.	The Spanish Arrobe is 25 Span. pounds	25	12 6

A Table to cast up wages, or Expences for a year, at so much *per* day, week, or month.

Day. s. d.	*by week.* £. s. d.	*By mon.* £. s. d.	*By Yr.* £. s. d.
0 1	0 0 7	0 2 4	1 10 5
0 2	0 1 2	0 4 8	3 0 10
0 3	0 1 9	0 7 0	4 11 3
0 4	0 2 4	0 9 4	6 1 8
0 5	0 2 11	0 11 8	7 12 1
0 6	0 3 6	0 14 0	9 2 6
0 7	0 4 1	0 16 4	10 12 11
0 8	0 4 8	0 18 8	12 3 4
0 9	0 5 3	1 1 0	13 13 9
0 10	0 5 10	1 3 4	15 4 2
0 11	0 6 5	1 5 8	16 14 7
1 0	0 7 0	1 8 0	18 5 0
2 0	0 14 0	2 16 0	36 10 0
3 0	1 1 0	4 4 0	54 15 0
4 0	1 8 0	5 12 0	73 0 0
5 0	1 15 0	7 0 0	91 5 0
6 0	2 2 0	8 8 0	109 10 0
7 0	2 9 0	9 16 0	127 15 0
8 0	2 16 0	11 4 0	146 0 0
9 0	3 3 0	12 12 0	164 5 0
10 0	3 10 0	14 0 0	182 10 0
11 0	3 17 0	15 8 0	200 15 0
12 0	4 4 0	16 16 0	219 0 0
13 0	4 11 0	18 4 0	237 5 0
14 0	4 18 0	19 12 0	255 10 0
15 0	5 5 0	21 0 0	273 15 0
16 0	5 12 0	22 8 0	292 0 0
17 0	5 19 0	23 16 0	310 5 0
18 0	6 6 0	25 4 0	328 10 0
19 0	6 13 0	26 12 0	346 15 0
20 0	7 0 0	28 0 0	365 0 0

Note, in these two Tables the month is only 28 days.

A Table to find wages, or expences, for a month, week, or day, at so much by the year.

by Yr. £.	*By Mon.* £. s. d.	*By Week.* £. s. d.	*By Day.* £. s. d.
1	0 1 6½	0 0 4½	0 0 0¾
2	0 3 0¾	0 0 9¼	0 0 1¼
3	0 4 7¼	0 1 1¾	0 0 2
4	0 6 1¾	0 1 6½	0 0 2¾
5	0 7 8	0 1 11	0 0 3¼
6	0 9 2½	0 2 3½	0 0 4
7	0 10 9	0 2 8¼	0 0 4½
8	0 12 3¼	0 3 0¾	0 0 5¼
9	0 13 9¾	0 3 5½	0 0 6
10	0 15 4	0 3 10	0 0 6½
11	0 16 10½	0 4 2¾	0 0 7¼
12	0 18 5	0 4 7¼	0 0 8
13	0 19 11¾	0 4 11¾	0 0 8½
14	1 1 5¾	0 5 4½	0 0 9¼
15	1 3 0¼	0 5 9	0 0 9¾
16	1 4 6½	0 6 1¾	0 0 10½
17	1 6 1	0 6 6¼	0 0 11¼
18	1 7 7½	0 6 10¾	0 0 11¾
19	1 9 1¾	0 7 3½	0 1 0½
20	1 10 8¼	0 7 8	0 1 1¼
30	2 6 0¼	0 11 6	0 1 7¾
40	3 1 4½	0 15 4	0 2 2¼
50	3 16 8½	0 19 2¼	0 2 9
60	4 12 0¾	1 3 0¼	0 3 3½
70	5 7 4¾	1 6 10¼	0 3 10
80	6 2 9	1 10 8¼	0 4 4½
90	6 18 1	1 14 6¼	0 4 11¼
100	7 13 5	1 18 4¼	0 5 5¾
200	15 6 10¼	3 16 8½	0 10 11½
300	23 0 3¼	5 15 0¾	0 16 5¼
400	30 13 8½	7 13 5	1 1 11
500	38 7 1½	9 11 9¼	1 7 4½
1000	76 14 3	10 3 6½	2 14 9½

Tables of Simple Interest, at 6 *per cent.* from 1s. to £1000, *and from* 1 *Day to* 1 *Year.*

Princ. Money	1 day.			2 days.			3 days.			4 days.			5 days.			6 days.				7 days.			
	s.	d.	q.	s.	d.	q.	s.	d.	q	s.	d.	q.	s.	d.	q		s.	d.	q.		s.	d.	q.
s. 1																							
2																							
3																							
4																							
5																							
6																							
7																							
8																							
9																							
10																							
11																							
12																							
13																							
14																							
15		—			—			—			—			—				—				—	
16		—			—			—			—			—				—				—	
17		—			—			—			—			—				—				—	
18		—			—			—			—			—				—				—	
19		—			—			—			—			—				—		0	0	0	1
£. 1		—			—			—			—			—				—		0	0	0	1
2		—			—			—		0	0	1	0	0	1	0	0	0	1	0	0	0	2
3		—			—		0	0	1	0	0	1	0	0	2	0	0	0	2	0	0	0	3
4		—		0	0	1	0	0	1	0	0	2	0	0	3	0	0	0	3	0	0	1	0
5		—		0	0	1	0	0	2	0	0	3	0	0	3	0	0	1	0	0	0	1	1
6		—		0	0	1	0	0	2	0	0	3	0	1	0	0	0	1	1	0	0	1	2
7	0	0	1	0	0	2	0	0	3	0	1	0	0	1	1	0	0	1	2	0	0	1	3
8	0	0	1	0	0	2	0	0	3	0	1	1	0	1	2	0	0	1	3	0	0	2	0
9	0	0	1	0	0	2	0	1	0	0	1	1	0	1	3	0	0	2	0	0	0	2	1
10	0	0	1	0	0	3	0	1	0	0	1	2	0	1	3	0	0	2	1	0	0	2	3
20	0	0	3	0	1	2	0	2	1	0	3	0	0	3	3	0	0	4	2	0	0	5	2
30	0	1	0	0	2	1	0	3	2	0	4	2	0	5	2	0	0	7	0	0	0	8	1
40	0	1	2	0	3	0	0	4	2	0	6	1	0	7	3	0	0	9	1	0	0	11	0
50	0	1	3	0	3	3	0	5	3	0	7	3	0	9	3	0	0	11	3	0	1	1	3
60	0	2	1	0	4	2	0	7	0	0	9	1	0	11	3	0	1	2	0	0	1	4	2
70	0	2	3	0	5	2	0	8	1	0	11	0	1	1	3	0	1	4	2	0	1	7	1
80	0	3	0	0	6	1	0	9	1	1	0	2	1	3	3	0	1	6	2	0	1	10	0
90	0	3	2	0	7	0	0	10	2	1	2	0	1	5	2	0	1	9	1	0	2	0	3
100	0	3	3	0	7	3	0	11	3	1	3	3	1	7	2	0	1	11	2	0	2	3	2
200	0	7	3	1	3	3	1	11	2	2	7	2	3	3	1	0	3	11	1	0	4	7	0
300	0	11	3	1	11	2	2	11	1	3	11	1	4	11	0	0	5	10	3	0	6	10	3
400	1	3	3	2	7	2	3	11	1	5	3	0	6	6	3	0	7	10	2	0	9	2	1
500	1	7	2	3	3	1	4	11	0	6	6	3	8	2	2	0	9	10	1	0	11	6	0
600	1	11	2	3	11	1	5	10	3	7	10	2	9	10	1	0	11	9	3	0	13	9	2
700	2	3	2	4	7	0	6	10	3	9	2	1	11	5	3	0	13	9	2	0	16	1	0
800	2	7	2	5	3	0	7	10	2	10	6	0	13	1	2	0	15	9	1	0	18	4	3
900	2	11	1	5	10	3	8	10	1	11	9	0	14	9	1	0	17	8	3	1	0	8	1
1000	3	3	1	6	6	2	9	10	1	13	1	0	16	5	0	0	19	10	2	1	3	0	0

SIMPLE INTEREST *at £6 per cent. from 1s. to £1000, & from 1 day to 1 year*

Princ. Money	8 days.				10 days.				15 days.				30 days, or 1 month.				Two such months.			
		s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.
s. 1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	1
4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	1
5	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	1	0	0	0	2
6	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	1	0	0	0	2
7	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	1	0	0	0	3
8	—	—	—	—	—	—	—	—	—	—	—	—	0	0	0	1	0	0	0	3
9	—	—	—	—	—	—	—	—	0	0	0	1	0	0	0	2	0	0	1	0
10	—	—	—	—	—	—	—	—	0	0	0	1	0	0	0	2	0	0	1	0
11	—	—	—	—	—	—	—	—	0	0	0	1	0	0	0	2	0	0	1	1
12	—	—	—	—	—	—	—	—	0	0	0	1	0	0	0	2	0	0	1	1
13	—	—	—	—	0	0	0	1	0	0	0	1	0	0	0	3	0	0	1	2
14	—	—	—	—	0	0	0	1	0	0	0	1	0	0	0	3	0	0	1	2
15	—	—	—	—	0	0	0	1	0	0	0	1	0	0	0	3	0	0	1	3
16	—	—	—	—	0	0	0	1	0	0	0	1	0	0	0	3	0	0	1	3
17	0	0	0	1	0	0	0	1	0	0	0	2	0	0	0	3	0	0	2	0
18	0	0	0	1	0	0	0	1	0	0	0	2	0	0	1	0	0	0	2	0
19	0	0	0	1	0	0	0	1	0	0	0	2	0	0	1	0	0	0	2	0
£. 1	0	0	0	1	0	0	0	1	0	0	0	2	0	0	1	0	0	0	2	1
2	0	0	0	2	0	0	0	3	0	0	1	0	0	0	2	1	0	0	4	2
3	0	0	0	3	0	0	1	0	0	0	1	3	0	0	3	2	0	0	7	0
4	0	0	1	1	0	0	1	2	0	0	2	1	0	0	4	2	0	0	9	1
5	0	0	1	2	0	0	1	3	0	0	2	3	0	0	5	3	0	0	11	3
6	0	0	1	3	0	0	2	1	0	0	3	2	0	0	7	0	0	1	2	0
7	0	0	2	0	0	0	2	3	0	0	4	0	0	0	8	1	0	1	4	2
8	0	0	2	2	0	0	3	0	0	0	4	2	0	0	9	1	0	1	6	3
9	0	0	2	3	0	0	3	2	0	0	5	1	0	0	10	2	0	1	9	1
10	0	0	3	0	0	0	3	3	0	0	5	3	0	0	11	3	0	1	11	2
20	0	0	6	1	0	0	7	3	0	0	11	3	0	1	11	2	0	3	11	1
30	0	0	9	1	0	0	11	3	0	1	5	2	0	2	11	1	0	5	11	0
40	0	1	0	2	0	1	3	3	0	1	11	2	0	3	11	1	0	7	10	2
50	0	1	3	2	0	1	7	2	0	2	5	2	0	4	11	0	0	9	10	1
60	0	1	6	3	0	1	11	2	0	2	11	1	0	5	10	3	0	11	0	0
70	0	1	10	0	0	2	3	2	0	3	5	1	0	6	10	3	0	13	9	2
80	0	2	1	0	0	2	7	2	0	3	11	0	0	7	10	2	0	15	9	1
90	0	2	4	1	0	2	11	1	0	4	5	0	0	8	10	1	0	17	9	0
100	0	2	7	1	0	3	3	1	0	4	11	0	0	9	10	1	0	19	8	2
200	0	5	2	3	0	6	6	3	0	9	10	0	0	19	8	2	1	19	5	1
300	0	7	10	1	0	9	10	1	0	14	9	0	1	9	6	3	2	19	2	0
400	0	10	5	3	0	13	1	2	0	19	8	0	1	19	5	1	3	18	10	2
500	0	13	1	0	0	16	5	0	1	4	7	0	2	9	3	2	4	18	7	2
600	0	15	8	2	0	19	8	2	1	9	6	0	2	19	1	3	5	18	4	1
700	0	18	4	0	1	3	0	0	1	14	5	1	3	9	0	0	6	18	0	3
800	1	0	11	2	1	6	3	1	1	19	4	1	3	18	10	2	7	17	9	2
900	1	3	6	3	1	9	6	3	2	4	4	2	4	8	9	0	8	17	6	1
1000	1	6	2	1	1	12	1	0	2	9	2	1	4	18	7	0	9	17	3	0

SIMPLE INTEREST *at £6 per cent. from 1s. to £1000, & from 1 day to 1 year.*

	Three months.				*Four months.*				*Five months.*				*Six months.*				*Seven months.*			
	l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
s. 1									0	0	0	1	0	0	0	1	0	0	0	1
2	0	0	0	1	0	0	0	1	0	0	0	2	0	0	0	2	0	0	0	3
3	0	0	0	2	0	0	0	2	0	0	0	3	0	0	1	0	0	0	1	0
4	0	0	0	2	0	0	0	3	0	0	1	0	0	0	1	1	0	0	1	2
5	0	0	0	3	0	0	1	0	0	0	1	1	0	0	1	3	0	0	2	0
6	0	0	1	0	0	0	1	1	0	0	1	3	0	0	2	0	0	0	2	1
7	0	0	1	0	0	0	1	2	0	0	2	0	0	0	2	1	0	0	2	3
8	0	0	1	1	0	0	1	3	0	0	2	1	0	0	2	3	0	0	3	1
9	0	0	1	2	0	0	2	0	0	0	2	2	0	0	3	0	0	0	3	2
10	0	0	1	3	0	0	2	1	0	0	2	3	0	0	3	2	0	0	4	0
11	0	0	1	3	0	0	2	2	0	0	3	1	0	0	3	3	0	0	4	2
12	0	0	2	0	0	0	2	3	0	0	3	2	0	0	4	1	0	0	4	3
13	0	0	2	1	0	0	3	0	0	0	3	3	0	0	4	2	0	0	5	1
14	0	0	2	1	0	0	3	1	0	0	4	0	0	0	4	3	0	0	5	3
15	0	0	2	2	0	0	3	2	0	0	4	1	0	0	5	1	0	0	6	0
16	0	0	2	3	0	0	3	3	0	0	4	2	0	0	5	2	0	0	6	2
17	0	0	2	3	0	0	4	0	0	0	5	0	0	0	6	0	0	0	7	0
18	0	0	3	0	0	0	4	1	0	0	5	1	0	0	6	1	0	0	7	1
19	0	0	3	1	0	0	4	1	0	0	5	2	0	0	6	2	0	0	7	3
£. 1	0	0	3	2	0	0	4	2	0	0	5	3	0	0	7	0	0	0	8	1
2	0	0	7	0	0	0	9	1	0	0	11	3	0	1	2	0	0	1	4	0
3	0	0	10	2	0	1	2	0	0	1	5	2	0	1	9	0	0	2	0	0
4	0	1	2	0	0	1	6	3	0	1	11	2	0	2	4	0	0	2	9	0
5	0	1	5	2	0	1	11	2	0	2	5	2	0	2	11	0	0	3	5	1
6	0	1	9	0	0	2	4	1	0	2	11	1	0	3	6	0	0	4	1	2
7	0	2	0	2	0	2	9	0	0	3	5	1	0	4	1	0	0	4	9	3
8	0	2	4	0	0	3	1	3	0	3	11	1	0	4	8	0	0	5	6	1
9	0	2	7	2	0	3	6	2	0	4	5	0	0	5	3	0	0	6	2	2
10	0	2	11	0	0	3	11	1	0	4	11	0	0	5	11	0	0	6	10	3
20	0	5	10	0	0	7	10	2	0	9	10	1	0	11	10	0	0	13	9	2
30	0	8	9	0	0	11	9	3	0	14	9	1	0	17	9	0	1	0	8	2
40	0	11	8	0	0	15	9	1	0	19	8	1	1	3	8	0	1	7	7	1
50	0	14	7	0	0	19	8	2	1	4	7	2	1	9	7	0	1	14	6	0
60	0	17	6	0	1	3	7	1	1	9	6	3	1	15	6	0	2	1	5	0
70	1	0	5	0	1	7	7	1	1	14	5	0	2	1	5	0	2	8	3	3
80	1	3	4	0	1	11	6	2	1	19	5	0	2	8	4	0	2	15	2	3
90	1	6	3	0	1	15	5	3	2	4	4	0	2	13	3	0	3	2	1	2
100	1	9	2	0	1	19	5	1	2	9	3	1	2	19	2	0	3	9	0	1
200	2	18	4	0	3	18	10	2	4	18	8	2	5	18	4	0	6	18	0	3
300	4	7	6	0	5	18	3	3	7	8	0	[illegible]	8	17	6	0	10	7	1	1
400	5	16	8	0	7	17	9	0	9	17	5	1	11	16	8	0	13	16	1	3
500	7	5	10	0	9	17	2	1	12	6	9	3	14	15	10	0	17	5	2	1
600	8	15	0	0	11	16	7	2	14	16	2	0	17	15	0	0	20	14	2	3
700	10	4	2	0	13	16	0	3	17	5	6	2	20	14	2	0	24	3	3	1
800	11	13	4	0	15	15	6	0	19	15	10	[illegible]	23	13	4	0	27	12	3	3
900	13	2	6	0	17	14	11	2	22	4	3	0	26	12	6	0	31	1	4	1
1000	14	11	8	0	19	14	4	3	24	13	7	2	29	11	8	0	34	10	4	3

SIMPLE INTEREST *at £6 per cent. from 1s. to £1000, & from 1 day to 1 year*

Princ. Money	8 months.				9 months.				10 months.				11 months. or 330 days.				1 Year, or 365 days.			
	l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
1	0	0	0	1	0	0	0	2	0	0	0	2	0	0	1	2	0	0	0	2
2	0	0	0	3	0	0	1	0	0	0	1	0	0	0	1	1	0	0	1	1
3	0	0	1	1	0	0	1	2	0	0	1	3	0	0	1	3	0	0	2	0
4	0	0	1	3	0	0	2	0	0	0	2	1	0	0	2	2	0	0	2	3
5	0	0	2	1	0	0	2	2	0	0	2	3	0	0	3	1	0	0	3	2
6	0	0	2	3	0	0	3	0	0	0	3	2	0	0	3	3	0	0	4	1
7	0	0	3	1	0	0	3	2	0	0	4	0	0	0	4	2	0	0	5	0
8	0	0	3	3	0	0	4	1	0	0	4	2	0	0	5	0	0	0	5	3
9	0	0	4	1	0	0	4	3	0	0	5	1	0	0	5	3	0	0	6	1
10	0	0	4	2	0	0	5	1	0	0	5	3	0	0	6	2	0	0	7	0
11	0	0	5	0	0	0	5	3	0	0	6	1	0	0	7	0	0	0	7	2
12	0	0	5	2	0	0	6	1	0	0	7	0	0	0	7	3	0	0	8	0
13	0	0	6	0	0	0	6	3	0	0	7	2	0	0	8	1	0	0	9	1
14	0	0	6	2	0	0	7	1	0	0	8	1	0	0	9	0	0	0	10	0
15	0	0	7	0	0	0	7	3	0	0	8	3	0	0	9	3	0	0	10	3
16	0	0	7	2	0	0	8	2	0	0	9	1	0	0	10	1	0	0	11	2
17	0	0	8	0	0	0	9	0	0	0	10	0	0	0	11	0	0	1	0	0
18	0	0	8	2	0	0	9	3	0	0	10	2	0	0	11	2	0	1	0	3
19	0	0	8	3	0	0	10	0	0	0	11	0	0	1	0	1	0	1	1	2
£. 1	0	0	9	1	0	0	10	2	0	0	11	3	0	1	1	0	0	1	2	1
2	0	1	6	3	0	1	9	1	0	1	11	2	0	2	2	0	0	2	4	3
3	0	2	4	1	0	2	7	3	0	2	11	1	0	3	3	0	0	3	7	0
4	0	3	1	3	0	3	6	2	0	3	11	0	0	4	4	0	0	4	9	2
5	0	3	11	1	0	4	2	0	0	4	11	0	0	5	5	0	0	6	0	0
6	0	4	8	2	0	5	3	3	0	5	10	3	0	6	6	0	0	7	2	1
7	0	5	6	0	0	6	2	2	0	6	10	2	0	7	7	0	0	8	4	3
8	0	6	3	2	0	7	1	0	0	7	8	3	0	8	8	0	0	9	7	0
9	0	7	1	0	0	7	11	3	0	8	10	0	0	9	9	0	0	10	9	2
10	0	7	10	2	0	8	10	2	0	9	10	0	0	10	10	0	0	12	0	0
20	0	15	9	0	0	17	9	0	0	19	8	0	1	1	8	0	1	4	0	0
30	1	3	7	2	1	6	7	2	1	9	6	0	1	12	6	0	1	16	0	0
40	1	11	6	0	1	15	6	0	1	19	4	1	2	3	4	0	2	8	0	0
50	1	19	4	3	2	4	4	2	2	9	2	1	2	14	2	0	3	0	0	0
60	2	7	3	1	2	13	3	0	2	19	0	1	3	5	0	0	3	12	0	0
70	2	15	1	3	3	2	1	2	3	8	10	2	3	15	10	0	4	4	0	0
80	3	3	0	1	3	11	0	0	3	18	8	2	4	6	8	0	4	16	0	0
90	3	10	11	0	3	19	10	2	4	8	6	2	4	17	6	0	5	8	0	0
100	3	18	9	2	4	8	11	0	4	18	4	3	5	8	4	0	6	0	0	0
200	17	17	7	0	8	17	10	0	9	16	9	2	10	6	8	0	12	0	0	0
300	11	16	4		13	6	9	0	14	15	2	1	16	5	0	0	18	0	0	0
400	15	15	2	1	17	15	8	0	19	13	7	0	21	13	4	0	24	0	0	0
500	29	14	0	0	22	4	7	0	24	12	0	0	27	1	8	0	30	0	0	0
600	23	12	9	2	26	13	6	0	29	10	4	3	32	10	0	0	36	0	0	0
700	37	11	7	0	31	2	5	0	34	8	5	2	37	18	4	0	42	0	0	0
800	31	10	4	3	35	11	4	0	39	7	2	1	43	6	8	0	48	0	0	0
900	35	9	2	1	40	0	4	0	44	5	7	0	48	15	0	0	54	0	0	0
1000	39	8	0	0	44	9	3	2	49	4	0	0	54	3	4	0	60	0	0	0

PERPETUAL ALMANACK.

February March November	*February* August*	*May*	*January October*	*January* April July*	*September December*	*June*
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

1787	Thursday	1820	*Wednesday* †
1788	*Saturday* †	1821	Thursday
1789	Sunday	1822	Friday
1790	Monday	1823	Saturday
1791	Tuesday	1824	*Monday* †
1792	*Thursday* †	1825	Tuesday
1793	Friday	1826	Wednesday
1794	Saturday	1827	Thursday
1795	Sunday	1828	*Saturday* †
1796	*Tuesday* †	1829	Sunday
1797	Wednesday	1830	Monday
1798	Thursday	1831	Tuesday
1799	Friday	1832	*Thursday* †
1800	Saturday	1833	Friday
1801	Sunday	1834	Saturday
1802	Monday	1835	Sunday
1803	Tuesday	1836	*Tuesday* †
1804	*Thursday* †	1837	Wednesday
1805	Friday	1838	Thursday
1806	Saturday	1839	Friday
1807	Sunday	1840	*Sunday* †
1808	*Tuesday* †	1841	Monday
1809	Wednesday	1842	Tuesday
1810	Thursday	1843	Wednesday
1811	Friday	1844	*Friday* †
1812	*Sunday* †	1845	Saturday
1813	Monday	1846	Sunday
1814	Tuesday	1847	Monday
1815	Wednesday	1848	*Wednesday* †
1816	*Friday* †	1849	Thursday
1817	Saturday	1850	Friday
1818	Sunday	1851	Saturday
1819	Monday	1852	*Monday* †
		1853	Tuesday

To find on what day of the week any given day in any month will fall, and the contrary.

EXAMPLE.

ON what day of the week will the 31st. day of January 1787 fall?

OBSERVE the day of the week annexed to the year in the outer column;—then, in the Table, under the given month, in the upper row of figures, you will find the day of the month on which that day falls.—According to this direction, I find that, in January 1787, Thursday is the 4th. 11th. 18th. and 25th. then reckoning on, Friday 26th. Saturday 27th, &c. I find the 31st. day falls on Wednesday;—or, that the last Wednesday in January is the 31st. day.

NOTE, In Leap-years, January and February must be taken in the columns marked thus.* Leap-years are marked in the outer columns, thus †

THE years 1800, 1900, and all other 100dth. years not to be Leap-years, except the years 2000, 2400, 2800, and every 400dth. year following, which must be Leap-years.

A Table for reducing Troy wt. to Avoirdupois.

Troy	Avoirdupois.			Tro.	Avoir.
lb.	lb.	oz.	dr.	pw.	drams.
4000	3291	6	13,68	19	16,67
3000	2528	9	2,26	18	15.79
2000	1645	11	6,84	17	14,92
1000	822	13	11,42	16	14,04
900	740	9	2,28	15	13,16
800	658	4	9,14	14	12,29
700	576	0	0	13	11,41
600	493	11	6,85	12	10,53
500	411	6	13,71	11	9,65
400	329	2	4,57	10	8,78
300	246	13	11,42	9	7,9
200	164	9	2,28	8	7,02
100	82	4	9,15	7	6,14
90	74	0	13,62	6	5,27
80	65	13	4,11	5	4,39
70	57	9	9,6	4	3,51
60	49	5	15,08	3	2,63
50	41	2	4,57	2	1,75
40	32	14	10,05	1	0,88
30	24	10	15,54	gr.	
20	16	7	5,03	23	,84
10	8	3	10,52	22	,8
9	7	6	7,86	21	,77
8	6	9	5,21	20	,73
7	5	12	2,56	19	,69
6	4	14	15,9	18	,66
5	4	1	13,25	17	,62
4	3	4	10,6	16	,58
3	2	7	7,95	15	,55
2	1	10	5,3	14	,51
1	0	13	2,65	13	,47
oz.				12	,44
11		12	1,09	11	,4
10		10	15,54	10	,36
9		9	13,99	9	,33
8		8	12,43	8	,29
7		7	10,88	7	,26
6		6	9,32	6	,22
5		5	7,77	5	,18
4		4	6,22	4	,15
3		3	4,66	3	,11
2		2	3,11	2	,07
1		1	1,55	1	,04

A Table for reducing Avoirdupois weight into Troy weight.

Avoir	Troy.				Av.	Troy.			
lb.	lb.	oz.	pw.	gr.	oz	lb	oz.	pw.	gr.
6000	7291	8	0	0	15	1	1	13	10,5
5000	6076	4	13	8	14	1	0	15	5
4000	4861	1	6	16	13		11	16	23,5
3000	3645	10	0	0	12		10	18	18
2000	2430	6	13	8	11		10	0	12,5
1000	1215	3	6	16	10		9	2	7
900	1093	9	0	0	9		8	4	1,5
800	972	2	13	8	8		7	5	20
700	850	8	6	16	7		6	7	14,5
600	729	2	0	0	6		5	9	9
500	607	7	13	8	5		4	11	3,5
400	486	1	6	16	4		3	12	22
300	364	7	0	0	3		2	14	16,5
200	243	0	13	8	2		1	16	11
100	121	6	6	16	1		0	18	5,5
90	109	4	10	0	dr				
80	97	2	13	8	15			17	2,1
70	85	0	16	16	14			15	22,76
60	72	11	0	0	13			14	19,42
50	60	9	3	8	12			13	15,08
40	48	7	6	16	11			12	12,74
30	36	5	10	0	10			11	9,4
20	24	3	13	8	9			10	6,06
10	12	1	16	16	8			9	2,72
9	10	11	5	0	7			8	23,38
8	9	8	13	8	6			7	20,04
7	8	6	1	16	5			6	16,7
6	7	3	10	0	4			5	13,36
5	6	0	18	8	3			3	10,02
4	4	10	6	16	2			2	6,68
3	3	7	15	0	1			1	3,34
2	2	5	3	8	¾			0	20,51
1	1	2	11	16	½				13,67

An

An account of the Gregorian *or* New Style, *together with some* Chronological Problems, *for finding the* Epact, Golden Number, Moon's Age, *&c.*

POPE GREGORY the XIIIth. made a reformation of the Calendar. The Julian Calendar, or Old Style, had, before that time, been in general use all over *Europe*. The year, according to the Julian Calendar, consists of three hundred and sixty-five days and six hours; which six hours being one fourth part of a day, the common years consisted of three hundred and sixty-five days, and every fourth year, one day was added to the month of *February*, which made each of those years three hundred and sixty-six days, which are usually called Leap-years.

THIS computation, though near the truth, is more than the Solar Year by 11 minutes, which, in one hundred and thirty-one years, amounts to a whole day. By which the vernal Æquinox was anticipated ten days from the time of the general council of *Nice*, held in the year 325 of the Christian Æra, to the time of Pope Gregory; who therefore caused ten days to be taken out of the month of October in 1582, to make the Æquinox fall on the 21st. of March, as it did at the time of that Council. And, to prevent the like variation for the future, he ordered that three days should be abated in every four hundred years, by reducing the Leap-year at the close of each century, for three successive centuries, to common years, and retaining the Leap-year at the close of each fourth Century only.

THIS was at that time esteemed as exactly conformable to the true solar year; but Dr. Halley makes the solar year to be three hundred and sixty-five days, five hours, forty-eight minutes, fifty-four seconds, forty-one thirds, twenty-seven fourths, and thirty-one fifths: according to which, in four hundred years, the Julian year of three hundred and sixty-five days and six hours will exceed the solar by three days, one hour and fifty-five minutes, which is near two hours, so that in fifty centuries it will amount to a day.

THOUGH the Gregorian Calendar, or New-Style had long been used throughout the greatest part of *Europe*, it did not take place in Great-Britain and America till the first of January 1752; and in September following, the eleven days were adjusted, by calling the third day of that month the fourteenth, and continuing the rest in their order.

CHRONOLOGICAL PROBLEMS.

PROBLEM I.

As there are three Leap-years to be abated in every four centuries: To shew how to find on which century the last year is to be a Leap-year, and in which it is not.

RULE.

CUT off the two cyphers, and divide the remaining figures by 4; if nothing remain, the year is a Leap-year.

EXAMPLES

EXAMP. 1. The year 18|00.

```
4)18(4
  16
  —
   2
```

EXAMP. 2. The year 19|00.

```
4)19(4
  16
  —
   3
```

EXAMP. 3. The year 20|00.

```
4)20(5
  20
  —
   0
```

EXAMP. 4. The year 40|00.

```
4)40(10
  4
  —
  0
```

THE first and second examples, having remainders, shew the years to be common years of three hundred and sixty-five days; but the third and fourth, having no remainders, are Leap-years of three hundred and sixty-six days.

PROBLEM II.

To find, with regard to any other years, whether any given year be Leap-year, and the contrary.

RULE.

DIVIDE the proposed year by 4, and if there be no remainder, after the division, it is Leap-year; but if 1, 2 or 3 remain, it is the first, second or third after Leap-year.

EXAMP. 1. For the year 1784.

```
4)1784(446
  16
  —
   18
   16
   —
    24
    24
    —
     0
```

EXAMP. 2. For the year 1786.

```
4)1786(446
  16
  —
   18
   16
   —
    26
    24
    —
     2 { second after
       { Leap-year.
```

PROBLEM III.

To find the Dominical Letter for any year, according to the Julian method of calculation.

RULE.

ADD to the year its fourth part and 4, and divide that sum by 7: if nothing remain, the Dominical Letter is G; but if there be any remainder, it shews the letter in a retrograde order from G, beginning the reckoning with F; or, if it be subtracted from 7, you will have the index of the letter from A, accounting as follows.

A	B	C	D	E	F	G
1	2	3	4	5	6	7

EXAMP. For the year 1786.

Add { *Given year* = 1786
Its fourth = 446
And 4 }

$$
\begin{array}{r}
7)2236(319 \\
21 \\
\hline
13 \\
7 \\
\hline
66 \\
63 \\
\hline
\end{array}
$$

And 7 — 3=4=D, *reckoning from* A.

PROBLEM IV.

To find the Dominical Letter for any year according to the Gregorian Computation.

RULE.

DIVIDE the year and its fourth part by 7; subtract the remainder, after the division, from 7, and this remainder will be the Index of the Dominical Letter, as before; if nothing remain it is G.

EXAMPLE 1. For the year 1786.

Add { *Given year* = 1786
Its fourth = 446 }

$$
\begin{array}{r}
7)2232(318 \\
21 \\
\hline
13 \\
7 \\
\hline
62 \\
56 \\
\hline
\end{array}
$$

And 7—6=1=A

EXAMP. 2. For the year 1788.*

1788
447

$$
\begin{array}{r}
7)2235(319 \\
21 \\
\hline
13 \\
7 \\
\hline
65 \\
63 \\
\hline
\end{array}
$$

And 7—2=5=E.

PROBLEM

* HERE it is to be observed, that every Leap-year has two Dominical Letters; that, found by this rule, is the Dominical Letter from the twenty-fifth day of *February* to the end of the year; and the next in the order of the Alphabet serves from the first of *January* to the twenty-fourth of *February*.

IN the 2d. Examp. E is the Dominical Letter for the year; but F, the next in the order of the alphabet, is the Dominical Letter for *January* and *February*. From this interruption of the Dominical Letter every fourth year, it is twenty-eight years before the Dominical Letter returns to the same order, which, were it not for the Leap-years, would return to the same, every seven years.

THIS Cycle of twenty-eight years is called the Cycle of the Sun.

PROBLEM V.

To find the Prime, or Golden Number.

RULE.

ADD 1 to the given year; divide the sum by 19, and the remainder, after the division, will be the Prime; if nothing remain, then 19 will be the Golden Number.

EXAMPLE. For the year 1786.

To the given year 1786
Add 1

19)1787(94
171

77
76

1 *Golden number.*

THE Golden number, or Lunar Cycle, is a period of nineteen years, invented by *Meton*, an *Athenian*, and from him called the *Metonic* Cycle. The use of this Cycle is to find the change of the moon; because, after nineteen years, the changes of the moon fall on the same days of the month as in the former 19 years; though not at the same time of the day, there being an anticipation of one hour, twenty-seven minutes, forty-one seconds, and thirty-two thirds; which, in three hundred and twelve years, amount to a whole day. Hence, the Golden number will not shew the true Change of the moon for more than three hundred and twelve years, without being varied. But the Golden number is not so well adapted to the *Gregorian*, as the *Julian* Calendar: the Epact being more certain in the New-Style, to find which, the Golden number is of use.

PROBLEM VI.

To find the Julian Epact.

RULE.

FIRST find the Golden number, which multiply by 11, and the product, if less than 30, will be the number required; If the Product exceed 30, then divide it by 30, and the remainder is the Epact.

EXAMPLE. 1. For the year 1786.

To the given year 1786
Add 1

19)1787(94
171

77
76

Golden number = 1 *and* 1×11=11 *the Julian Epact.*

EXAMPLE

EXAMPLE 2. For the year 1791.

```
     1791
        1
     ----
  19)1792(94
     171
     ---
       82
       76
       --
        6=Golden number, and 6×11=66, therefore 30)66(6
                                                   60
                                                   --
                                                    6 Epact.
```

PROBLEM VII.

To find the Gregorian Epact.

RULE.

SUBTRACT 11 from the Julian Epact:—if the Subtraction cannot be made, add 30 to the Julian Epact; then subtract, and the remainder will be the Gregorian Epact; if nothing remain, the Epact is 29.

OR, Take 1 from the Golden number, divide the remainder by 3; if 1 remain, add 10 to the dividend, which sum will be the Epact; if 2 remain, add 20 to the dividend; but if nothing remain, the dividend is the Epact.

EXAMP. 1. For the year 1786.

```
The Julian Epact being 11
              Subtract 11
                       --
                       00
```

Because nothing remains, the Epact is 29.

Or,

EXAMP. 2. For the year 1786.

```
The Golden number being 1
            Take from it 1
                         -
          Divide by 3)0(0
```

There being no remainder, the Epact is 29, as before.

EXAMP. 3. For the year 1791.

```
The Julian Epact being but 6
                 Add to it 30
                           --
                           36
                  Subtract 11
                           --
          Gregorian Epact = 25
```

Or,

EXAMP. 4. For the year 1791.

```
The Golden number being 6
            Take from it 1
                         -
                      3)5(1
                        3
                        -
                        2
```

Therefore, as 2 remains; add 20 to the dividend, and it gives the Epact 25, as before.

A

A general Rule for finding the Gregorian Epact forever.

DIVIDE the *Centuries* of any year of the Christian Æra by 4, (rejecting the subsequent numbers;) multiply the remainder by 17, and to this product add the quotient multiplied by 43; divide this sum plus 86 by 25, multiplying the Golden Number by 11, from which subtract the last quotient, and rejecting the *thirties*, the remainder will be the epact.

EXAMP. For the year 1786.

REJECTING the subsequent numbers 86, it will be 17.

4)17(4
16
—
1
Multiply by 17
—
17
Add 4×43 = 172
—
189
Add 86
—
25)275(11
25
—
25
25

Golden number = 1
Multiply by 11
—
11
Subtract the last quotient = 11
—
00

Therefore, as nothing remains, the Epact is 29, as before.

A TABLE of the nineteen Epacts for the *Julian* and *Gregorian* Accounts, by the Golden number.

G.N.	*Julian* Epact.	*Greg.* Epact	G.N.	*Julian* Epact.	*Greg.* Epact	G.N.	*Julian* Epact.	*Greg.* Epact
1	11	29	7	17	6	13	23	12
2	22	11	8	28	17	14	4	23
3	3	22	9	9	28	15	15	4
4	14	3	10	20	9	16	26	15
5	25	14	11	1	20	17	7	26
6	6	25	12	12	1	18	18	7
						19	29	18

PROBLEM VIII.

To calculate the Moon's age on any given day.

RULE.

To the given day of the month, add the Epact and number of the month: if the sum be less than 30, it is the Moon's age; but if it exceed 30, then take 30 from it, and the remainder will be the Moon's age.

Note,

Note, The numbers to be added to the following months, are as follow:

To	January	0	July	5
	February	2	Auguſt	6
	March	1	September	8
	April	2	October	8
	May	3	November	10
	June	4	December	10

EXAMPLE. 1. For January 25th. 1786

Add { *Given day* = 25
Epact = 29
N° *of the month* = 00

54
Subtract 30

24 = *Moon's age.*

PROBLEM IX.

To find the times of the new and full Moon, and the firſt and laſt Quarters.

RULE.

FIND the moon's age on the given day, then, if it be 15, the moon will be full on that day, and by counting $7\frac{1}{2}$ days backward and forward you will have the firſt and laſt quarters, and by counting backward and forward 15 days, you will have the times of the laſt and next change; but if the age of the moon be greater than 15, take 15 from it, and the remainder will ſhew how many days have paſt ſince the laſt full moon, and, counting theſe backward, you will have the day the laſt full moon happened on, and by knowing that, we can find the change, or either of the quarters, as before.—Again, if the age of the moon, on the aſſumed day, be leſs than 15, then take that from 15, and the remainder will ſhew how many days are to run till the next full moon, which you will have by adding the remainder to the aſſumed day; and, proceeding as before, you will have the days of the change, and either quarter as above.

EXAMPLE

EXAMPLE. For January 25th. 1786.

Add { *Assumed day* = 25
Epact = 29
Number of the month = 00 }

54
Subtract 30

Moon's age = 24
Subtract 15

Take the days since the last full moon = 9
From the assumed day = 25

To the day of the full moon = 16*th*.
Add 15

New Moon 31*st*.

From the full moon 16
Take $7\frac{1}{2}$

First quarter 9*th*.

To the full moon = 16
Add $7\frac{1}{2}$

Last quarter = 23*d*.

PROBLEM X.

THE Time of the Moon's coming to the South, after the Sun, being given, to find the age of the Moon.

RULE.

As 24 hours, the whole difference of time; are to 30, the number of days from change to change: So is the difference of time, to the moon's age.

EXAMP. I observed the Moon to be on the Meridian, or due south, at 6 o'clock in the afternoon: What is the Moon's age?

$24 : 30 :: 6 : 7\frac{1}{2}$ *days*, *Answer*.

PROBLEM XI.

To find the time of the Moon's Southing.

RULE.

MULTIPLY the Moon's age, on the given day, by 48 minutes, and divide the product by 60, the minutes in an hour, (or multiply by 4, and divide by 5,) and the quotient will shew how many hours and

and minutes the Moon is later, in coming on the meridian, than the Sun, and counting ſo many hours and minutes forward from 12 o' clock, we have the time of the Moon's ſouthing; if the hours and minutes, found as above, be leſs than 12, then, that will be the time of the Moon's ſouthing after noon; but, if greater than 12, then take 12 from them, and the remainder will be the time of the Moon's ſouthing in the morning.

EXAMP. 1. Required the time of the Moon's ſouthing on the 25th day of *January* 1786?

	Moon's age = 24	*Or* 24
h. m.	48	4
from 19 12	192	5)96
take 12 00	96 *h. m.*	*h. m.*
7 12	60)1152(19 12	$19\frac{1}{5}$ = 19 12 *as before.*
Hence, the Moon ſouths at 12 minutes paſt 7 in the morning.	60	
	552	
	540	
	12	

EXAMP. 2. For the 9th. of February 1786?

Moon's age = 10
48
h. m.
60)480(8 00 *Afternoon, is the time of the*
480 *Moon's ſouthing.*

Note, From the change to the full, the Moon comes to the South afternoon; but from the full to the change, before noon.

PROBLEM XII.

To find, on what day of the week, any given day in any month will fall.

As one of the firſt ſeven letters of the alphabet is prefixed to every day in the year, beginning with A, which is always prefixed to the firſt day of January: And as, in common years, the letter, annexed to the firſt Sunday in January, ſhews the Dominical Letter for that year; but every Leap-year having two Dominical letters, the firſt of which ſerving to the 24th. of February, and the other for the reſt of the year, conſequently, in any common year, the Dominical Letter being known, the firſt of January may be eaſily found, reckoning from A according to the natural order of the letters: And in any Leap-year, the firſt of its two Dominical Letters will ſhew as above, counting from A 1, B 2, C 3, &c. and by counting backward, you may have the day of the week, on which the firſt of January will happen.

RULE.

RULE.

FIND the day of the week answering to the first of January that year, then add together the days contained in each month from the beginning of the year to the proposed day of the month inclusively; divide this sum by 7, and if any thing remain, after the division, then, count so many forward, beginning with that day on which the first of January falls, and you will have the day of the week, on which the proposed day will fall: but if nothing remain, then, the day of the week, preceding that day on which the first of January falls, answers to the proposed day.

EXAMPLE. On what day of the week will the 5th. day of May 1786 fall?

BY the preceding observations, and by Prob. 4th. the first of January is found to fall on Sunday.

Jan.	31
Feb.	28
March	31
April	30
May	5th.

$$
\begin{array}{r}
7)125(17 \\
\underline{7} \\
55 \\
\underline{49} \\
6 \text{ \textit{from Jan.} 1.}
\end{array}
$$

Now, counting forward six days from Sunday, the first of January (inclusively) and the 5th. of May falls on Friday.

PROBLEM XIII.

To find the Cycle of the Sun.

RULE.

ADD 9 to the given year; divide the sum by 28, and the remainder, after division, is the Cycle required; but if nothing remain, the Cycle is 28.

EXAMPLE. For the year 1786?

$$
\begin{array}{lr}
\textit{To} & 1786 \\
\textit{Add} & \underline{9} \\
28)1795(64 & \\
 & \underline{168} \\
 & 115 \\
 & \underline{112} \\
 & 3 = \textit{Cycle required.}
\end{array}
$$

THE use of this Cycle is to find the Dominical Letter by the following Table.

A Table *of the Dominical Letters for the New-Style, according to the Cycle of the Sun.*

Cycle.	*Letter.*	*Cycle.*	*Letter.*	*Cycle.*	*Letter.*	*Cycle.*	*Letter*
1	D C	8	B	15	G	22	E
—	—	9	A G	16	F	23	D
2	B	—	—	17	E D	24	C
3	A	10	F	—	—	25	B A
4	G	11	E	18	C	—	—
5	F E	12	D	19	B	26	G
—	—	13	C B	20	A	27	F
6	D	—	—	21	G F	28	E
7	C	14	A				

This Table, by the preſent rule, will ſerve but to the end of this Century. The Leap-year being to be omitted in the year 1800, will make it neceſſary to add 25 to the date of the year, and then dividing by 28, it will give the Cycle right during the next Century. And this is a general rule to be obſerved, that, when a Leap year has been abated, add 16 to the number which was before added to the year, rejecting 28, when it exceeds it, and this number being added to the year, and the ſum divided by 28, the remainder, after diviſion, will be the Cycle for finding the Dominical Letter. Thus, in the nineteenth Century it will be $9+16=25$, and in the twentieth Century $25+16-28=13$, which number will ſerve two Centuries, for the year 2000 is a Leap-year.

Problem XIV.

To find the year of the Dionyſian Period.

Rule.

Add to the given year 457; divide the ſum by 532 and the remainder will be the number required.

Example. Required the year of the Dionyſian period for the year 1786?

To 1786
Add 457

532)2243(4
2128

115=*Dionyſian period.*

Problem XV.

To find the year of Indiction.

Rule.

Add 3 to the given year; divide the ſum by 15, and the remainder, after diviſion, will be the Indiction; if nothing remain, it will be 15.

Examp.

EXAMP. Required the year of indiction for 1786?

To 1786
Add 3

15)1789(119
15
28
15
139
135

4 = *Indiction.*

PROBLEM XVI.

To find the Julian Period.

RULE.

ADD 4713 to the given year, and the sum will be the Julian Period.

EXAMP. What year of the Julian Period will answer to the year 1786?

To 1786
Add 4713

6499 *Answer.*

PROBLEM XVII.

To find the Cycle of the Sun, Golden number, and Indiction for any current year.

RULE.

To the current year add 4713; divide the sum by 28, 19 and 15, respectively, and the several remainders will be the numbers required; when nothing remains, the divisor is the number required.

EXAMP. What are the Cycle of the Sun, Golden number, and Indiction for the year 1786?

1786
4713

28)6499(232
56
89
84
59
56

3 *Cycle of the Sun.*

19)6499(342
57
79
76
39
38

Golden number = 1

15)6499(433
60
49
45
49
45

Indiction = 4

PROBLEM XVIII.

HAVING the Cycle of the Sun, the Golden number and Indiction, to find the year of the Christian Æra.

RULE.

MULTIPLY 4845 by the Cycle of the Sun; 4200 by the Golden number and 6916 by the Indiction: Add the several products together, and divide the sum by 7980; the remainder, after division, will be the Julian period, from which subtract 4713, and the remainder will be the year required.

EXAMPLE. The Cycle of the Sun being 3, Golden number 1, and Indiction 4; What year of the Christian Æra is it?

4845	4200	6916	27664
3	1	4	4200
14535	4200	27664	14535

7980)46399(5
39900

6499
Subtract 4713

1786 *Answer*.

PROBLEM XIX.

To find the Time of High-Water.

RULE.

FIND the Moon's southing, to which add the point of the compass making full sea, on the full and change days, for the place proposed, and the sum will be the time required.

EXAMP. I demand the time of High Water at Boston, *January* 25th. 1786, admitting the tide to flow and ebb NW and SE on the days of change and full?

WE have before found the Moon's southing to be 7*h*. 12*m*. in the morning.

h m.
Therefore to 7 12
Add 4 00 = *the point of the compass, and it*

Gives 11 12 *in the morning, for the time of high water.*

PROBLEM XX.

To find on what day Easter will happen.

IT was ordered, by the *Nicene* Council, that Easter Sunday should be kept on the first *Sunday* after the first full moon, which happened upon or after the twenty-first day of *March*, the day on which they thought the Vernal Æquinox happened. Though this was a mistake, for the Vernal Æquinox, that year, fell on the twentieth of

of March. But yet, the full moon, which fell on, or next after the twenty-first of *March*, they called the *Paschal* full moon. And by the Introduction of the *Gregorian*, or New-Style, the Æquinox will now always happen on the *twentieth* or *twenty-first* of *March*. And the feast of *Easter* is now to be kept on the next *Sunday* after the *Paschal* full Moon, or the full moon which happens after the twenty-first of March; but, if the full moon happens on a Sunday, *Easter-day* is to be the next Sunday after.

RULE.

FIND the age of the moon on the 21st. of March, in the given year, and if it be 14, then find the day of the week answering to it, and the Sunday following is *Easter-Sunday*; but, if the Moon's age on the 21st of March be not 14, then reckon forward to the day on which the Moon's age is 14. and find the day of the week answering to that day; the Sunday following will be the day required.

N. B. On Leap-year take the 20th. of March.

EXAMP. When does Easter happen in the year 1786?

21 *of March*	*Jan.* 31
29 *Epact*	*Feb.* 28
1 *N°. of the month*	*March* 31
—	*April* 13*th*.
51	——
Subt. 30	7)103(14
—	7
21 *Moon's age*	——
Add 23 { *N°. of days to the Moon's being* 14 *days old.*	33
	28
—	—
44	5 *There-*
Take 31 = *days in March*	*fore, the first of January being Sunday, reckon forward* 5 *days, including Sunday, and you will find*
—	
13*th. of April, the day of the full moon, or Easter-limit,*	*the* 13*th. of April falls on Thursday, consequently the next Sunday is the* 16*th. which is Easter-Sunday.*

EASTER may be found, for any future time, by the following Table, which is calculated from 1753, the time of the commencement of the New-Style in America, and which shews, by the *Golden Number*, the days of the Paschal full Moons; by which and the *Dominical Letter*, the day, on which Easter will fall, may be found.

The use of the Table.

FIRST find the Golden number as before taught, which seek in the Column of Golden numbers under the time in which the given year is included; right against the Golden Number of the year, in the last column but one, you have the day of the month on which the Paschal full Moon happens, which is the Limit of Easter, from thence run your Eye down among the Dominical Letters, till you

come

come to the Letter of the given year, and against it you have the day of the month, on which Easter falls that year.

Examp. To know when Easter falls in 1786.

The Golden Number for the year being 1, and the Dominical Letter A: therefore seek in the first Column (the given year being included between the years 1753 and 1899) for the Golden Number: then cast your eye along to the last Column but one, under the title, Paschal full ●, and you will find the thirteenth of April to be the day of the full Moon; against which, in the last column, stands E, which shews it to be Thursday, therefore the next Sunday following is Easter-Sunday, which, by going down the Column of Letters to the next A, you will find to be the sixteenth of April.

Golden Numbers *from* 1753 *to* 1899, *and so on to* 4199, *inclusively.*

1753 to 1899	1900 to 2199	2200 to 2199	2300 to 2399	2400 to 2499	2500 to 2599	2600 to 2899	2900 to 3099	3100 to 3399	3400 to 3499	3500 to 3599	3600 to 3699	3700 to 3799	3800 to 4099	4100 to 4199	Paschal full ● Days of the Month	Dom. Letter.
14	—	6	17	6	17	—	9	—	1	12	1	12	—	4	March 21	C
3	14	—	6	—	6	17	—	9	—	9	—	1	12	--	22	D
—	3	14	—	14	—	6	17	—	9	—	9	—	1	12	23	E
11	—	3	14	3	14	—	6	17	—	9	—	9	—	1	24	F
—	11	—	3	—	3	14	—	6	17	—	17	—	9	--	25	G
19	—	11	—	11	—	13	14	—	6	17	6	17	—	9	26	A
8	19	—	11	—	11	—	3	14	—	6	—	6	17	--	27	B
—	8	19	—	19	—	11	—	3	14	—	14	—	6	17	28	C
16	—	8	19	8	19	—	11	—	3	14	3	14	—	6	29	D
5	16	—	8	—	8	19	—	11	—	3	—	3	14	--	30	E
—	5	16	—	16	--	8	19	—	11	—	11	—	3	14	31	F
13	—	5	16	5	15	—	8	19	—	11	—	11	—	3	April 1	G
2	13	—	5	—	5	16	—	8	19	—	19	—	11	--	2	A
—	2	13	—	13	—	5	16	—	8	19	8	19	—	11	3	B
10	—	2	13	2	13	—	5	16	—	8	—	8	19	--	4	C
—	10	—	2	—	2	13	—	5	16	—	16	—	8	19	5	D
18	—	10	—	10	—	2	13	—	5	10	5	16	--	8	6	E
7	18	—	10	—	10	—	2	13	—	5	—	5	16	--	7	F
—	7	18	—	18	—	10	—	2	13	—	13	—	5	16	8	G
15	—	7	18	7	18	—	10	—	2	13	2	13	--	5	9	A
4	15	—	7	—	7	18	—	10	—	2	—	2	13	--	10	B
—	4	15	—	15	—	7	18	—	10	—	10	—	2	13	11	C
12	—	4	15	4	15	—	7	18	—	10	—	10	--	2	12	D
1	12	—	4	—	4	15	—	7	18	—	18	—	10	--	13	E
—	1	12	—	12	—	4	15	—	7	18	7	18	--	10	14	F
9	—	1	12	1	12	—	4	15	—	7	—	7	18	--	15	G
—	9	—	1	—	1	12	—	4	15	—	15	—	7	18	16	A
17	17	9	—	9	—	1	12	12	4	15	4	15	15	7	17	B
6	6	17	9	17	9	9	1	1	12	4	12	4	4	15	18	C
															19	D
															20	E
															21	F
															22	G
															23	A
															24	B
															25	C

THE USE OF LOGARITHMS.

1. *In* MULTIPLICATION.

Given two numbers, viz. 275 *and* 12,6 *to find their product.*

RULE. *To the Logarithm of* 275, *viz.* - - - - - - 2,43933
Add the Logarithm of 12,6, *viz.* - - - - - - 1,10037

And their sum is the logarithm of their product, viz. 3465 = 3,53970

2. *In* DIVISION.

LET it be required to find the quotient, which arises by dividing one number by another; suppose 1425 by 57.

From the logarithm of the dividend, viz. 1425 = 3,15381
Take the logarithm of the divisor, viz. 57 = 1,75587

And the remainder is the logarithm of the quotient, viz. 25 = 1,39794

3. *In the* RULE OF THREE.

Three numbers given to find a fourth, in direct proportion.

RULE. From the Tables take the Logarithms of each of the proposed numbers, then, add the Logarithms of the second and third together, and from the sum take the Logarithm of the first, and the remainder will be the Logarithm of the fourth number.

LET the three proposed numbers be 18, 24, and 33, and the operation will stand thus;

1,38021 = *the Logarithm of* 24, *the* 2*d. term.*
1,51851 = *the Logarithm of* 33, *the* 3*d. term.*

2,89872 = *the Logarithm of their product.*
−1,25527 = *the Logarithm of the first term* 18.

1,64345 = *the Logarithm of the fourth term required, which, by the Table, answers to the natural number* 44, *the* 4*th, proportional to the three proposed numbers.*

4. *In* INVOLUTION *or* RAISING POWERS.

To find any power of any proposed number, or to involve any number to any proposed power, by Logarithms.

RULE. Multiply the Logarithm of the given Root by the power, *viz.* by 2 for the square, by 3 for the cube, &c. and the product is the Logarithm of the power sought.

REQUIRED to find the cube of 12?

1,07918 = *the Logarithm of* 12.
×3 = *the third power, or cube.*

3,23754 = 1728, *the Cube of* 12.

5. *In*

5. *In* EVOLUTION, *or* EXTRACTING ROOTS.

To Extract any Root of any proposed Number.

RULE. Divide the Logarithm of the proposed number by the Index of the required root, viz. by 2 for the Square, by 3 for the Cube, &c. and the quotient will be the Logarithm of the root required.

REQUIRED to find the Cube Root of 1728?

3,23754 = *the Logarithm of* 1728, *and* 3,23754 ÷ 3 = 1,07918 *is the Logarithm of the Cube Root of* 1728, *viz.* 12.

6. *In* COMPOUND INTEREST.

To find the Amount of any Sum for any Time, and at any Rate, at Compound Interest.

RULE. Multiply the Logarithm of the Ratio (*i. e.* the amount of £1 for 1 year) by the number of years, and to the product add the Logarithm of the principal; the sum will be the Logarithm of the amount.

WHAT will £45 amount to, forborn 12 years, at 6 *per Cent. per Annum*, Compound Interest?

Log. of 1,06, *the Ratio, is*	,02533
Multiply by the time	12
	,30396
Add Log. of 45, *the principal*	1,65321
The Sum is	1,95717 *which is the Logarithm of* 90,7 = £90 14*s*. *Ans.*

7. *In* DISCOUNT, *at* COMPOUND INTEREST.

To find the present worth of any sum of Money, due any time hence, at any rate, at Compound Interest.

RULE. From the Logarithm of the sum to be discounted, subtract the Logarithm of the rate multiplied by the time; and the remainder is the Logarithm of the present worth.

WHAT present money will pay a debt of £90 14*s*. due 12 years hence, discounting at the rate of 6 *per Cent. per Annum*?

From the Logarithm of £90 14 =	1,95717
Subt. Product of the Log. of the Ratio × *by the time* =	,30396
The Remainder	1,65321 *is the Logarithm of* £45 *Ans.*

PLAIN GEOMETRY.

Definitions.

1. A *Point* in the Mathematics is considered only as a Mark, without any regard to dimensions.

2. A *Line* is considered as length, without regard to breadth or thickness.

3. A

3. A *Plain* or *Surface* has two dimenſions, Length and Breadth, but is not conſidered as having thickneſs.

4. A *Solid* has three dimenſions, Length, Breadth, and thickneſs, and is uſually called a *Body*.

5. A Line is either *ſtraight*, which is the neareſt diſtance between two Points; or *crooked*, called a Curve line, whoſe ends may be drawn further aſunder.

6. If two Lines are at equal diſtance from one another in every part, they are called *parallel* Lines, which, if continued infinitely, will never meet.

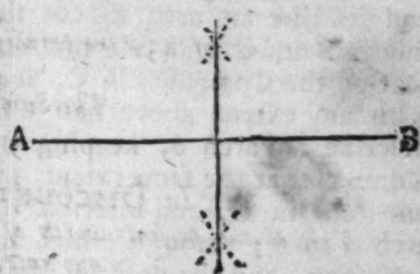
7. If two Lines incline one towards another, they will, if continued, meet in a point: by which meeting is formed an *Angle*.

8. If one Line fall directly upon another, ſo that the Angles on both ſides are equal, the Line, ſo falling, is called a *perpendicular*, and the Angles, ſo made, are called *right* Angles, and are equal to 90 degrees, each.

9. All Angles, except right Angles, are called oblique Angles, whether they are *accute*, that is, leſs than a right Angle; or *obtuſe*, that is, greater than a right Angle.

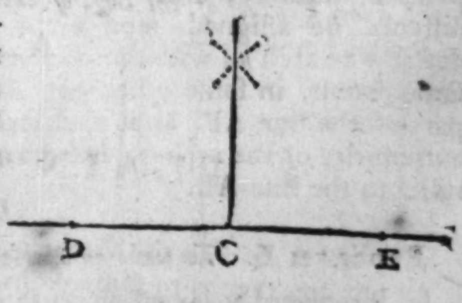
GEOMETRICAL PROBLEMS.

Problem I. *To divide a Line* AB *into two equal parts.*



Set one foot of the Compaſſes in the point A, and, opening them beyond the middle of the line, deſcribe arches above and below the line; with the ſame extent of the Compaſſes, ſet one foot in the point B, and deſcribe two arches croſſing the former: draw a line from the interſection of the arches above the line, to the interſection below the line, and it will divide the line AB into two equal parts.

Problem 2. *To erect a Perpendicular on the point* C *in a given Line.*

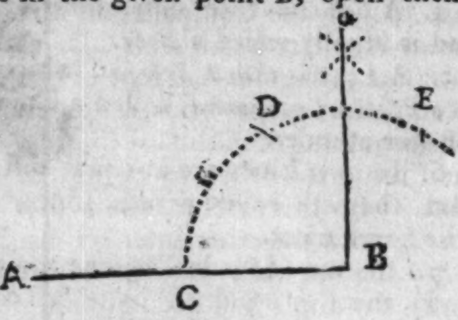


Set one foot of the Compaſſes in the given point C, extend the other foot to any diſtance at pleaſure, as to D, and with that extent make the marks D, and E. With the Compaſſes, one foot in D, at any extent above half the diſtance of D and E, deſcribe an arch above the line, and with the ſame extent, and one foot in E deſcribe an arch, croſſing the former; draw a line from the interſection of the arches to the given point C, which will be perpendicular to the given line in the point C.

Problem

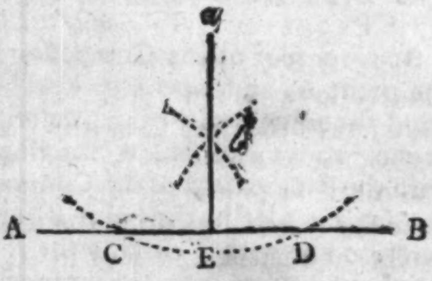
PROBLEM 3. *To erect a perpendicular upon the end of a line.*

SET one foot of the Compaſſes in the given point B, open them to any convenient diſtance, and deſcribe the arch CDE; ſet one foot in C, and with the ſame extent, croſs the arch at D: with the ſame extent croſs the arch again, from D to E; then with one foot of the Compaſſes in D, and with any extent above the half of DE, deſcribe an arch *a*: take the Compaſſes from D, and, keeping them at the ſame extent with one foot in E, interſect the former arch *a* in *a*; from thence draw a line to the point B, which will be a perpendicular to A B.



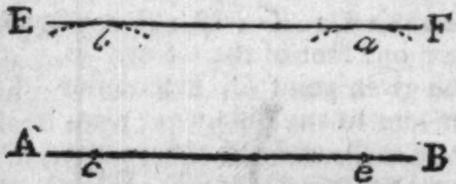
PROBLEM 4. *From a given point, a, to let fall a perpendicular to a given line* A B.

SET one foot of the Compaſſes in the point *a*, extend the other ſo as to reach beyond the line AB, and deſcribe an arch to cut the line A B in C and D; put one foot of the Compaſſes in C, and, with any extent above half CD, deſcribe an arch *b*, keeping the Compaſſes at the ſame extent, put one foot in D, and interſect the arch *b* in *b*; through which interſection, and the point *a*, draw *a* E, the perpendicular required.



PROBLEM 5. *To draw a Line parallel to a given Line* AB.

SET one foot of the Compaſſes in any part of the line, as at *c*; extend the Compaſſes at pleaſure, unleſs a diſtance be aſſigned, and deſcribe an arch *b*; with the ſame extent, in ſome other part of the line AB, as at *e*, deſcribe the arch *a*; lay a ruler to the extremities of the arches, and draw the line E F, which will be parallel to the line AB.

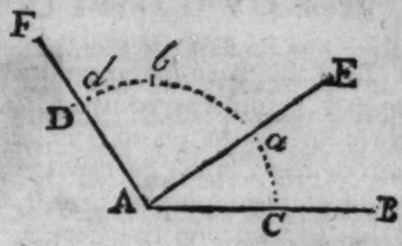


PROBLEM 6. *To make an Angle equal to any number of Degrees.*

IT is required to lay off an acute Angle of 35° on a given Line AB.

TAKE

Take 60 degrees from the line of Chords in the Compaſſes, ſet one foot of the Compaſſes in the point A, deſcribe an arch CD, at pleaſure; then ſet one foot of the Compaſſes in the braſs centre in the beginning of the line of Chords, and bring the other to 35 on the line; with this extent, ſet one foot



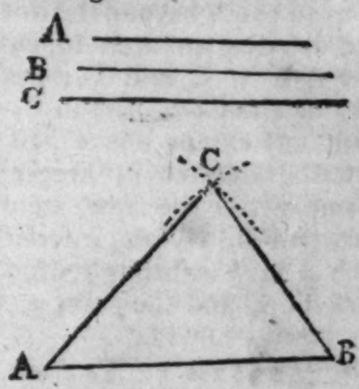
in C, with the other interſect the arch CD, in *a*, and through *a* draw the line AE, ſo will EAB be an angle of 35 degrees.

If the angle had been obtuſe, ſuppoſe 125°, then take 90° from the line of chords; ſet one foot in C, and interſect the arch in *b*; then take 35° from the ſame line of chords, and ſet them from *b* to *d*; a line drawn from A through *d* to F will make an angle, FAB of 125°.

To meaſure an Angle by the Line of Chords is only to take the diſtance on the arch between the lines A B and A E, or A B and A F, and laying it on the line of Chords.

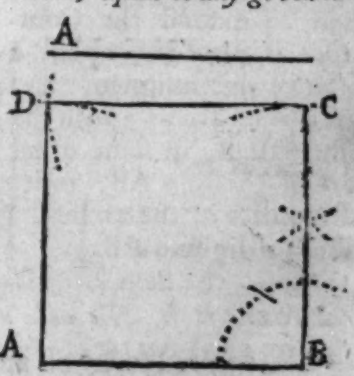
Prob. 7. *To make a Triangle, whoſe ſides ſhall be equal to three given Lines, provided any two of them be longer than the third.*

Let A,B,C, be the three given lines; Draw a line, A B, at pleaſure; take the line C in the Compaſſes, ſet one foot in A, and with the other make a mark at B; then take the given line B in the Compaſſes, and ſetting one foot in A, interſect the Arch C in C; laſtly, draw the lines AC and B C, and the Triangle will be completed.



Prob. 8. *To make a Square, having equal Sides, equal to any given line.*

Let A be the given line; draw a line AB equal to the given line; from B raiſe a Perpendicular to C equal to AB, with the ſame extent, ſet one foot in C, and deſcribe the Arch D; alſo with the ſame extent, ſet one foot in A and interſect the Arch D; laſtly, draw the line A D and CD, and the ſquare will be completed.

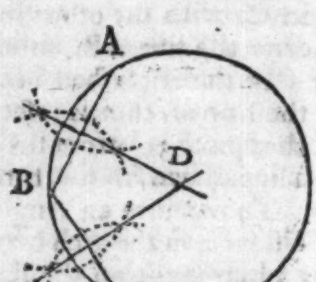


In like manner may a Parallelogram be conſtructed, only attending to the difference between the length and breadth.

Prob. 9. *To deſcribe a Circle, which ſhall paſs through any three given points, which are not in a ſtraight line.*

Let the three given points be ABC, through which the Circle is to paſs. Join the points AB and BC with right lines, and biſect theſe lines; the point D, where the biſecting lines croſs each other, will be the Centre of the Circle required. Therefore, place one point of the Compaſſes in D, extending the other to either of the given points, and the Circle, deſcribed by that Radius, will paſs through all the points.

Hence, it will be eaſy to find the Centre of any given Circle; for, if any three points are taken in the Circumference of the given Circle, the Centre will be readily found as above. The ſame may alſo be obſerved, when only a part of the Circumference is given.



Prob. 10. *To deſcribe an Ellipſis or Oval mechanically.*

Draw two parallel lines, as L and M, at a moderate diſtance, by Prob. 5; then draw two others at the ſame diſtance, acroſs the former, as N and O: by the croſſing of theſe lines will be made a figure, ABCD, of four ſides; extend the Compaſſes at pleaſure, and ſetting one foot in D, deſcribe the Arch *cde*; with the ſame extent, ſet one foot in B, and deſcribe the Arch *fgh*, then, ſet one foot in C, and contract them ſo as to reach the point *e*, and deſcribe the Arch *lm*; with the ſame extent, and one foot in A, deſcribe the Arch *i k*, and the Oval will be completed: In the ſame manner, with a greater or leſs extent of the Compaſſes, may a greater or leſs Oval be made by the ſame four-ſided figure ABCD.

Of Plain TRIGONOMETRY, *right and oblique Angled.*

1. Plain Trigonometry is that Science, by which we meaſure the Sides and Angles of plain Triangles.

Sect. 1. *Of Rectangular Trigonometry.*

In a right Triangle, the longeſt ſide is uſually called the Hypothenuſe, the next longeſt, the Baſe, and the ſhorteſt, the Perpendicular.

Logarithmic Sines, Tangents, and Secants are called the *Tabular* ſides of a Triangle, and are the ſines, &c. of the oppoſite Angles; The length of the ſides are called the *natural* ſides.

All

All the three Angles of a Triangle are equal to two right Angles, or 180°.

The Proportion ought to be made between Sides and Sides; and between Angles and Angles.

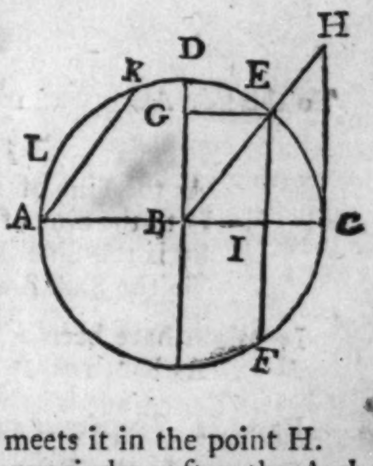
When a ſide is required, any ſide (whether known, or not) may be made Radius; but when an Angle is required, then a known ſide, only, muſt be made Radius.

Note, a ſide is ſaid to be made Radius, when one foot of the Dividers is ſet in one end of the ſide, and ſuch a Circle deſcribed, of which the ſide is the Semi-diameter;—Alſo, that when the Hypothenuſe is Radius, it is the ſign of the right Angle, or 90°, and the Baſe and Perpendicular, uſually called the Legs, become ſines of their oppoſite Angles: But when one of the Legs is made Radius, the other becomes the Tangent of the oppoſite Angle, and the Hypothenuſe, the Secant of the ſame Angle.*—Tangent's Radius is 45°.

When a ſide is to be found, the two firſt Terms of the Proportion muſt be Tabular ſides, and the laſt a real one; but when an Angle is to be found, the two firſt Terms muſt be real ſides, and the third, a Tabular one

The given parts, whether ſides or angles, are marked with —, and the part required, with O.

Angles are meaſured by the Arch of a Circle. The Periphery of every Circle, whether great or ſmall, is divided into 360 degrees, each degree into 60 minutes, every minute into 60 ſeconds, and ſo on, to thirds, fourths, &c.



Any Portion of the Periphery of a Circle, as E C F, is called an *Arch*, and a line drawn from the ends of an Arch, as, E I F, is called the *Chord* of the Arch. Half the Chord of any Arch, as E I, is called the *Sine* of the Arch E C, and IC, is called the *verſed ſine* of the ſame Arch E C: So alſo, E G is the *Sine* of the Arch E D. A Line drawn perpendicular to the diameter of a Circle, ſo as to touch the Circle, and not cut it, is called a Tangent, as C H, which is the Tangent of the Arch E C, becauſe the line B H, drawn from the Centre B through E, called the *Secant*, meets it in the point H.

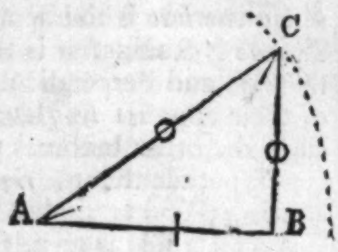
The Complement of an Arch is the remainder, after the Arch is taken from 90°, thus, KD is the Complement of the Arch ALK, taken from the Arch AD. The Co-Sine or Sine Complement of an Arch is the Sine of the Complement of that Arch, as ED, is the Complement of EC. The

* To work on the Scale with a Secant, you muſt take the ſines backwards, that is, 80 ſines for 10 ſecants, &c.

PROBLEM I. *The Angles and one of the Legs given, to find the Hypothenuse and other Leg.*

EXAMPLE. In the Triangle ABC, right angled at B, suppose the Leg AB, 86 equal parts (as feet, yards, miles, &c.) the angle A = 33°, 40′, and the Angle C = 56°, 20′; required the length of the Hypothenuse AC, and the other Leg BC?

Geometrically. Draw AB equal to 86, from any line of equal parts, then upon the point B, erect the perpendicular BC; lastly, from the point A, draw the line AC, making with AB an angle = 33°, 40′, and that line produced will meet BC in C, and so constitute the Triangle. The length of AC and BC may be found by taking them in your Compasses, and applying them to the same line of equal parts that AB was taken from.



By Calculation. By making the Hypothenuse Radius, the Legs will become the Sines of the opposite angles;—And as natural sides are required, the proportions must begin with tabular sides: therefore, for the Hypothenuse,

As the Sine of C	56°,20′	9,92027
Is to Radius	90 ,00	10,
So is the Side AB	86	1,93450
		11,93450
		9,92027
To the side AC	103,3	2,01423

HERE, I add the Logarithms of the 2d. and 3d. Terms, and from their sum subtract the first, and the remainder is the Log. of the side sought, which gives 103,3 the same must be done in all the following cases.

For the Leg B C.

As the Sine of the Angle C	56°,20′	9,92027
Is to the Sine of the Angle A	33 ,40	9,74380
So is the Side A B	86	1,93450
To the Side B C	57, 28	1,75803

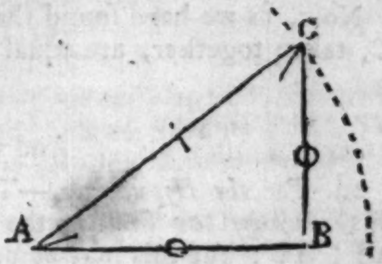
IT might have been as easily found by the following Proportion:
As R : S,A :: AC : BC.

PROB. 2. *The Angles and Hypothenuse given, to find the Legs.*

EXAMP. In the Triangle A B C, suppose the Hypothenuse A C = 146, the Angle A = 36°,25′, and the Angle C = 53°,35′: Required the Legs AB and BC?

Geometrically. Draw the Line AB at pleasure, and make the Angle A = 36°,25′,; then take AC = 146 from any line of equal Parts; lastly,

laſtly, from the point C let fall the Perpendicular CB on the line AB: So the Triangle is conſtructed, and AB and BC may be meaſured from the line of equal parts.



By Calculation. Making AC Radius, the Legs become Sines, as before, and as the Angles are given to find the Sides, we muſt begin the Proportions with Angles, or Tabular ſides.

For the Leg AB.

As Radius	90°,00′	10,
Is to the Sine of C	53 ,35	9,90565
So is Side AC	146	2,16435
To Side AB	117,5	2,07000

For the Leg BC.

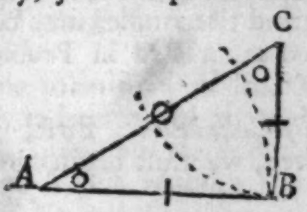
As Radius	90°,00	10,
Is to the Sine of A	36 ,25	9,77353
So is Side AC	146	2,16435
To Side BC	86,67	1,93788

As we had before found AB, the Proportion might have been, As S,C : S,A :: AB : BC.

Prob. 3 and 4. *The two Legs given, to find the Angles and Hypothenuſe.*

Examp. In the Triangle ABC, ſuppoſe the Leg AB = 94, and BC = 56, Required the Angles and Hypothenuſe?

Geometrically. Draw AB = 94 from any line of equal parts, then, from the point B raiſe BC perpendicular to AB, and take BC from the former line of equal parts = 56; laſtly, join the points A and C with the ſtraight line AC, ſo the Triangle is conſtructed;—AC may be found by taking it in your Dividers and applying it to the line of equal parts; and the Angles may be meaſured by the 6th Geometrical Problem.



By Calculation, 1ſt. For the Angle A; ſuppoſing the Baſe AB the Radius, then the Hypothenuſe becomes Secant of the Angle A, and the perpendicular BC, the Tangent of the Angle A:—And as an Angle is required, we muſt begin the Analogy with a natural ſide.

As AB	94	1,97313
Is to BC	56	1,74819
So is Tangent's Radius	45°,00	10,
To the Tangent of A	30°,47′	9,77506

The Perpendicular might have been made Radius, and then the Proportion would have been As BC ; AB :: Tan. Rad. : Tan. of C.

Now,

Now, as we have found the Angle A, and as the Angles A and C, taken together, are equal to 90°, therefore from 90°,00′

Take the Angle A = 30°,47

And we have the Angle C = 59°,13′

2d. *For the Hypothenuse.*—The Base still being Radius, we have this analogy for finding the Hypothenuse: As T. R : Sec. A :: AB : AC : but this may be done without the help of Secants: For, having found the Angles, we may now make the Hypothenuse Radius; and as a natural side is required, we must begin the proportion with a tabular side; therefore,

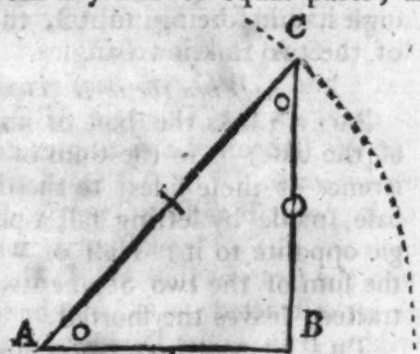
As the Sine of C	59°,13′	9,93405
Is to Radius	90 ,00	10,
So is AB	94	1,97313
To AC	109,4	2,03908

Or the Analogy might have been, As, S. C : R :: BC : AC.

PROBLEMS 5 *and* 6. *The Hypothenuse and one of the Legs given, to find the Angles, and other Leg.*

EXAMP. In the Triangle ABC, suppose the Leg AB=83, and the Hypothenuse AC=126; Required the Angles A & C, & the Leg BC?

Geometrically. Draw AB = 83 from any line of equal parts; and from the point B, raise the perpendicular BC of any length, then, take the length of AC 126 from the same line of equal parts, and setting one foot of the dividers in A, with the other, cross the perpendicular BC in C; lastly, join AC, so the Triangle will be constructed, and the Angles may be measured as directed in Problem 3d and 4th.



By Calculation. First, for the Angle C; and as an Angle is required, we must begin with a Side, making the Hypothenuse Radius.

As AC	126	2,10037
Is to AB	83	1,91908
So is Radius	90°,00	10,
To Sine of C	41 ,12	9,81871

From 90°,00′

Take the Angle at C = 41 ,12

And we have the Angle A = 48°,48′

For the Side BC.—As a Side is now required, we must begin with an Angle; therefore,

As Radius	90°,00′	10,
Is to the Sine of A	48°,48′	9,87646
So is AC	126	2,10037
To BC	94,8	1,97683

SECT.

SECT. 2. *Of Oblique-angular Trigonometry.*

IN any Triangle, the Sides are proportional to the Sines of the opposite angles.

WHEN two angles of any Triangle are given, their sum, being subtracted from 180°, leaves the third angle: and when one angle is given, that being subtracted from 180°, leaves the sum of the two unknown angles.

WHEN an angle exceeds 90°, subtract it from 180° and work with the remainder.

When the given and required parts, viz. Sides and Angles, are opposite.

RULE 1. As in right-Angled Triangles.

AS the Sine of any angle is to the Sine of any other angle: So is the Side opposite to the first angle, to the Side opposite to the other angle.

OR, As any one side is to any other side: So is the Sine of the angle opposite to the first Side, to the Angle opposite to the other Side.

When any two Sides with the Angle included between them are given.

RULE 2. As the Sum of any two sides is to their difference; So is the Tangent of the Half-Sum of the two opposite angles, to the Tangent of half the difference of those two angles; which Half-difference, being added to the half-sum, gives the greater of the two angles, and, being subtracted from the Half-sum, leaves the less of the two unknown angles.

When the three sides are given to find the Angles.

RULE 3. As the base of any Triangle (or sum of the Segments of the base) is to the Sum of the other two Sides; So is the difference of those sides, to the difference of the two Segments of the base, made by letting fall a perpendicular to the base from the angle opposite to it: Half of which difference, being added to half the sum of the two Segments, gives the longest, and, being subtracted, leaves the shortest.

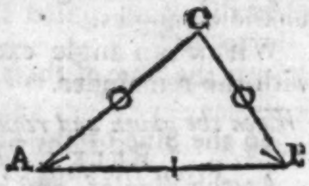
THE Learner being now somewhat acquainted with the common method of working by Logarithms, it will be proper to shew how to perform those Proportions without subtracting the first number from the Sum of the second and third; which is done by setting down the Arithmetical Complement of the first Term instead of the Logarithm. This may be readily done thus; Subtract the first figure of the Logarithm from 10, and set down the remainder: then subtract each of the other figures, Index and all, from 9, setting down the remainders, and place a dot before the Index, as in the Case of the Logarithm. Thus the Arithmetical Complement (usually marked Co. Ar.) of the Logarithm 9,66004 is 0,33996 and so of any other.

WHEN the Arithmetical Complement of the first Term is used instead of the Logarithm, add all the three numbers together, and reject 10 out of the Index of their sum, as in those cases where the Radius is the first Term.

PROB. 1. *In the oblique-angled Triangle ABC, given two Angles and a Side opposite to one of them, to find the two other Sides.*

SUPPOSE the angle at A 36°, 40′, the Angle at B, 60°, 51′, and the base AB 85,6; Required AC and BC?

Geometrically. Draw the base A B, and from any Scale of equal Parts, lay thereon 85,6 from A to B; then, from the line of chords, lay off an angle of 36°, 40′ at A and an angle of 60°, 51′ at B, and the meeting of these two lines in C completes the Triangle, and A B and B C may be measured by the same line of equal parts.



From the sum of all the angles	180°,00′
Take the Sum of the angles A and B, viz.	97°,31′
And we have the angle C equal to	82°,29′

HERE we have the angle at C opposite to the given base, and the angles at A and B opposite the two required sides, which may be found by the first Rule, as follows:

By Calculation. For the Side BC. Having to find a Side, we begin with an angle.

As the Sine of the angle at C,	82°,29′	Co. Ar. 0,00375
Is to the Sine of the angle A,	36°,40′	9,77609
So is the base AB	85,6	1,93247
To Side BC	51,55	1,71231

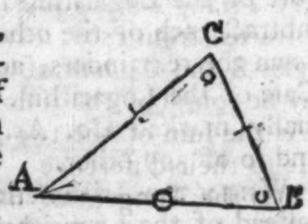
For the Side AB.

As the Sine of C	82°,29′	Co. Ar. 0,00375
Is to the Sine of B	60°,51′	9,94118
So is AB	85,6	1,93247
To AC	75,4	1,87740

PROB. 2*d*. and 3*d*. *Two Sides, and an Angle opposite to one of them given, to find the two other Angles and remaining Side.*

IN the oblique-angled Triangle ABC, given the Side AC 75,4, the Side BC 51,56, and the angle at A 36°, 40′, to find the base AB, and the angles at B and C.

Geometrically. Draw the base A B at pleasure, and on any point assumed, as A, make an angle of 36°,40′; take 75,4 from the Scale of equal parts and set it from A to C; then take 51,56 from the same Scale; set one foot of the dividers in C, and with the other, intersect the base in B; lastly, draw BC, and the Triangle is completed, and the base may be measured by the same scale of equal parts.



By

By Calculation. Here we have the ſide BC oppoſite the known angle at A, and the ſide AC oppoſite the unknown angle at B, which may be found by Rule 1ſt.

To find the angle at B; Having to find an angle, we begin with a Side.

As BC	51,55	Co.Ar.	8,28778
Is to AC	75,4		1,87737
So is the Sine of the angle A	36°,40′		9,77609
To the Sine of the angle B	60°,51′		9,94124

From the ſum of all the angles 180°,00′
Take the ſum of the angles A and B 97°,01′

And we have the angle C equal to 82°,59′

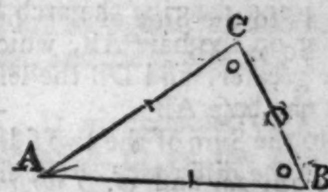
For the Baſe AB. Having to find a Side, we begin with an angle.

As the Sine of A	36°,40′	Co. Ar.	0,22392
Is to the Sine of C	82°,29′		9,99625
So is BC - - - - -	51,55		1,71223
To AB - - - - -	85,6		1,93240

PROB. 4th and 5th. *Two Sides, and the Angle included between them at* A, *given, to find the two other Angles and the other Side.*

IN the oblique-angled Triangle ABC, given the ſide AC 75,4, the baſe AB 85,6, and the included angle at A 36°,40′, to find the angles B and C, and the ſide BC.

Geometrically. Draw the baſe AB, and, from any ſcale of equal parts, ſet off 85,6 from A to B; make an angle at A of 36°, 40′, and draw AC, and from the ſame ſcale of equal parts, ſet 75,4 from A to C; laſtly, draw the line BC, and the Triangle is completed: BC may be meaſured by the ſame ſcale of equal parts, and the angles B and C, on the line of Chords.



By Calculation. Here we have given the two ſides AB and AC, with the angle included between them; and therefore theſe caſes muſt be ſolved by Rules 2d and 1ſt. Now, as the three angles of every Triangle are equal to 180°, the angle at A 36°,40′, being ſubtracted from 180°, leaves 143°, 20′, the ſum of the two unknown angles B and C, half of which is 71°, 40′; and half their difference may be found by the following Proportion, according to Rule 2.

As the ſum of the two ſides AB and AC	161	Co.Ar.	7,79318
Is to their difference	10,2		1,00860
So is the Tangent of half the ſum of the unknown angles B and C	71°,40′		10,47969
To the Tangent of half their difference	10°,49′		9,28147

To

To the half-sum	71°,40′	From the half-sum	71°,40′
Add the half-difference	10 ,49	Take the half-diff.	10°,49′
The sum is the greater angle C	82 ,29	The remaind. is the less angle B	60°,51′

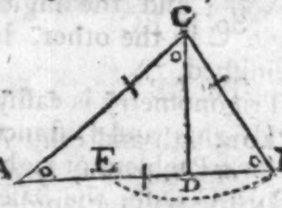
Having found the angles B and C, the side BC may be found by Rule 1.

As the sine of C	82°,29′	Co. Ar.	0,00375
Is to the sine of A	36°,40′	- - -	9,94118
So is AB - - -	85,6		1,03247
To BC - - - -	51,56	- -	1,87740

Prob. 6. *The three sides given to find the Angles*

In the oblique-angled Triangle ABC, given the base AB 85,6, the Side AC 75,4, and the Side BC 51,56; Required the angles.

Geometrically. Draw the base AB, and set off 85,6 from any Scale of equal parts from A to B; take 75,4 from the same Scale, and setting one foot in A, describe an arch; then from the scale take 51,56, and, setting one foot in B, intersect the former arch in C; from C draw lines to A and B, and the Triangle is completed. The angles may all be measured upon the line of Chords.



By Calculation. Here being no angle given, these cases must be solved by Rule 3d, in the following manner: Place one foot of the dividers in C, and extend the other so as to take in the shortest side BC, and describe the arch BE; then, from C let fall a perpendicular on the base AB, which will divide it into two Segments AD the greater, and DB the less, whose difference is AE: Then,

As the base AB	85,6	—Co. Ar.	8,06753
Is to the Sum of the two sides AC and BC	126,96	——	2,10366
So is the difference of the sides AC & BC	23,84	——	1,37730
To the difference of the Segments of the base, or AE — —	35,36	——	1,54849
Half the difference of the Segments is	17,68		

To half the base	42,8	From half the Base	42,8
Add half the difference	17,68	Take the half-difference	17,68
And the Sum is the greater Segment AD	60,48	And the remainder is the less Segment DB	25,12

Thus is the oblique-angled Triangle ABC divided into two right-angled Triangles ADC and BDC, both right-angled at D, in each of which are given the base and Hypothenuse, to find the other Parts.

First, for the angle at C in the right-angled Triangle ADC, making the Hypothenuse Radius.

As

As AC	75,4	Co. Ar.	8,12263
Is to AD	60,48		1,78161
So is Radius	90°,00′		10,
To the Sine of C	53°,20′		9,90424

The angle A, being the Complement of the angle C, is 36°,40′, Then for the angle C in the right-angled Triangle BDC

As BC	51,56	Co. Ar.	8,28778
Is to BD	25,12		1,40002
So is Radius	90°,00′		10,
To the Sine of C	29°,09′		9,68780

Whence the angle A is 60°,51′, being the Complement of 29°,09′; and the angle at C, in one Triangle, being added to the angle C in the other, is 82°,29′: thus the ſolution of the problem is finiſhed.

Trigonometry is eaſily applied to navigation, and the Menſuration of Heights and Diſtances. With reſpect to the former; ſuppoſe in the firſt Problem of right-angled Trigonometry, the angle at A is the Ship's Courſe, the Baſe to be the true, (or meridional,) difference of Latitude, the perpendicular to be the departure, or difference of Longitude, and the Hypothenuſe to be the diſtance the ſhip is to run; then, we have the courſe and true (or meridional) difference of Latitude given, to find the diſtance, and departure from the meridian, (or difference of Longitude.)

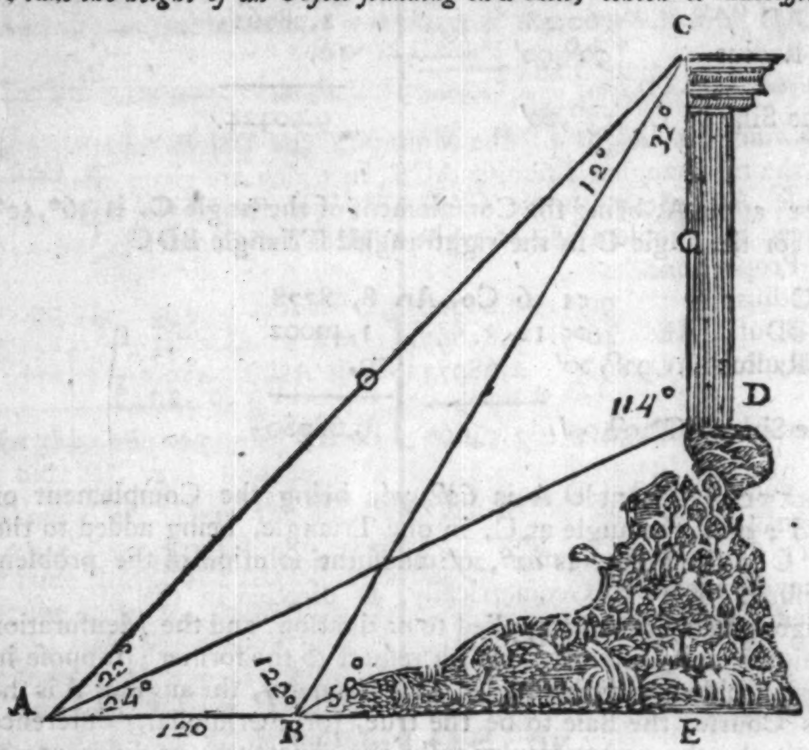
In Problem 2d, we have the Courſe and diſtance given, to find the true (or meridional) difference of Latitude, and the departure, (or difference of Longitude.)

With reſpect to heights and diſtances; If we ſuppoſe, in the firſt Problem before-mentioned, the angle at A to be the angle which the Top of any diſtant object makes with the Surface of the Earth; where we ſtand,—the baſe to be the diſtance of the object, (on level ground,) and the Perpendicular, the object's height; then, we have the angle A, and the diſtance AB, to find the height BC; but this will ſerve only on level ground, and where the Object is acceſſible.

The diſtance of any *inacceſſible* Object may be found by Prob. 1, of oblique Trigonometry: For, if we ſuppoſe the object at C, then, at *two* ſtations, as at A and B, take the bearing of the place; alſo meaſure the ſtationary diſtance AB, and you will then have *two* angles and a ſide oppoſite to one of them, to find either of the other Sides.

To

To take the height of an Object standing on a Hill, which is inaccessible.



At two Stations, as at A and B, take the angles, *viz.* CAE and CBE, which the Top of the Object makes with an horizontal line, and that, which the bottom of the object makes with the first Station, at A, viz. DAE, then take DAE from CAE, and the remainder is CAD.

Note, when an angle is expressed by three letters, the middle one shews the angle. Now, suppose the Stationary distance AB 120, the angle ACB 12°, and Angle CBA 122°, then by Problem 1st, of oblique Trigonometry, we have two angles and a Side opposite to one of them given, to find the side AC. Therefore,

			Co. Ar.
As S. of ACB	12°,00′	—	0,68213
Is to S. of CBA	122 ,00′	—	9,92842
So is Stationary distance	120	—	2,07918
To Side AC	489,5	—	2,68973

Note, I subtracted 122° from 180°, and worked with the remainder, and in the following, 114° from 180°. Now, having found AC 489,5, suppose the angle CDA 114°, and the angle CAD 22, and we have two angles and a Side opposite to one of them, as before, to find the perpendicular height of the object CD, Therefore,

As

	deg. min.				deg.	Co. Ar.
As S. of CDA	114,00 Co. Ar.	0,03927	As S. of CDA,	114 —	0,03927
Is to S. of CAD	22,00	9,57358	Is to S. of ACD	44 —	9,84177
So is Side AC	489,5	2,68973	So is Side AC	489,5 —	2,68973
To perpendi. Hht. CD	200,7	2,30258	To Side AD	372,2 —	2,57077

To find the height of the Mountain and Object together; we have the right angled Triangle ACE, in which are given the Hypothenuse AC 489,5, angle CAE 46°, and the angle ACE 44°, whence, by Problem 2d, of right-angled Trigonometry, we have these Proportions.

As Radius	90°	10,00000	As Radius	90°—	10,00000
Is to S. of CAE	46°	9,85693	Is to S. of ACE	44°—	9,84177
So is Hypoth. AC	489,5	2,68973	So is AC	489,5—	2,68973
To Perp. ht. CE	252,1	2,54666	To AE	340 —	2,53150

If you subtract CD from CE, you will have the height of the hill 151,4.

Any figure in Navigation, or mensuration of heights and distances may be measured Geometrically, as directed in the aforegoing Problems of Trigonometry.

MENSURATION OF SUPERFICIES AND SOLIDS.

Section I. Of SUPERFICIES.

Superficies, or Surfaces, are measured by the Superficial Inch, Foot, Yard, &c. according to the measures peculiar to different artists.

The Superficial Inch, Foot, &c. is one inch, foot, &c. in length and breadth;—and, because 12 inches make 1 foot of long measure, therefore, $12 \times 12 = 144$ inches make one superficial foot, $3 \times 3 = 9$ feet, a yard, &c.

The Superficial Content of every Surface is found by the proper Rule of its figure, whether Square, Triangle, Polygon, or Circle.

Article I. *To measure a Square, having equal Sides.*

Rule. Multiply the Side of the Square into itself, and the Product will be the area or superficial Content, of the same name with the denomination taken, either in inches, feet, or yards, respectively.

Let ABCD represent a square, whose Side is 12 feet. Multiply the Side 12 by itself, thus,

12 *Inches*	12 *Feet*
12 *Inches*	12 *Feet*
Area=144 *Inches.*	144 *Feet.*

By

By the Sliding Rule.

SET 1 to the length on B, then, find the breadth on A, and opposite to this on B, you will have the Content.

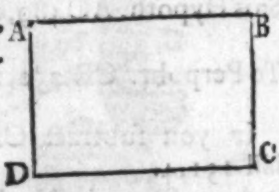
By Gunter's Scale.

EXTEND the Dividers from 1, on the line of numbers, to the length; that distance, laid the same way from the breadth, will point out the Answer.

ART. 2. *To measure a Parallelogram, or long Square.*

RULE. Multiply the length by the breadth, and the Product will be the area, or Superficial Content.

LET ABCD represent a Parallelogram, whose length is 16 feet, and breadth, 12 feet. Multiply 16 by 12.



Length	16
Breadth	12
	192 *Area.*

THE Content of this figure is found on the sliding Rule and Scale, as the former.

ART. 3. *When the breadth of a Superficies is given, to find how much in length will make a square foot, yard, &c.*

RULE. As the breadth is to a foot, yard, &c. So is a foot, yard, &c. to the length required to make a foot, yard, &c.—Or, divide 144 by the breadth, and the Quotient will be the length required.

How much, in length, of a board $2\frac{1}{2}$ feet wide will make a square foot?

In. br. *In. leng.* *In. br.* *In. leng.*
As 30 : 12 :: 12 : 4,8
12

30)144(4,8 *Inches, length required.*
120

240
240

In.
Breadth=30)144(4,8 *In. Ans.*

ART. 4. *To measure a Rhombus.*

Definition. A Rhombus is a figure with four equal Sides, in the form of a Diamond on Cards, having two angles greater, and two less, than the angles of a Square: The former are called *obtuse* angles, and the latter, *acute*, or sharp, angles.

RULE.

RULE. Multiply the Side by the length of a Perpendicular let fall from one of the obtuse angles to the Side opposite such angle.

LET ABCD represent a Rhombus, each of whose sides is 15 feet: A perpendicular let fall from the obtuse angle, at B, on the Side DC, will intersect it in the point E, So will BE be 12 feet; and this being multiplied into the given Side, the Product will be the area of the Rhombus.

Side = 16
Perp. = 12

192 *Area.*

By the Sliding Rule.

Set 1 on A to the length on B; find the Perpendicular height on A, against which on B is the Content.

By Gunter.

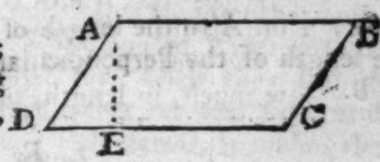
THE extent from 1 to the perpendicular-height will reach from the length to the content.

ART. 5. *To find the Area of a Rhomboides.*

Definition. A Rhomboides is a figure whose opposite sides and opposite angles are equal.

RULE. Multiply one of the longest sides by the perpendicular let fall from one of the obtuse angles on one of the longest sides.

LET ABCD represent a Rhomboides; The longest Sides AB and CD being 16,5 feet, and the Perpendicular AE, 9,7 feet.



Side = 16,5
Perp. 9,7

1155
1485

Ans. 160,05 *feet.*

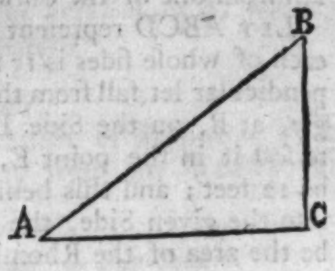
The content is found on the Sliding Rule, and Scale, as in the last figure.

ART. 6. *To measure a Triangle.*

RULE. If it be a right-angled Triangle, multiply the base by half the Perpendicular, or half the base by the Perpendicular, and the Product will be the area: But, if it be an oblique-angled Triangle (whether obtuse, or acute) multiply half the base by the length of the Perpendicular let fall on the base from the angle opposite to it, and the Product will be the area. The longest Side of a Triangle is usually called the base, except in a right-angled Triangle, where

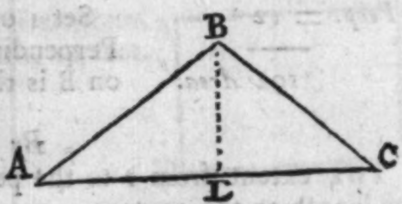
the longest of the two Legs, which include the right angle, is called the base.

In the right-angled Triangle ABC right-angled at C; the base AC; is 18,8 feet, and the Perpendicular BC=12,6



Base=18,8	*Or Perp.*=12,6
Perp. = 6,3	½ *Base*= 9,4
564	504
1128	1134
118,44 *Area.*	118,44 *Area.*

The oblique-angled Triangle ABC being given, let fall a Perpendicular from the angle at B on the base AC, and that Perpendicular is the height of the Triangle. The base AC being 15,6, and the Perpendicular BD=9, to find the area.



7,8=*Half the Base*
9=*Height of the Triangle.*

70,2=*Area.*

By the Sliding Rule.

Set 1 on A to the length of the base on B, and opposite to half the length of the Perpendicular, on A, you will have the Content on B.

By Gunter.

The Extent from 1 to half the length of the Perpendicular will reach from the length of the base to the Content.

In this place it may be proper to instruct the Learner in one of the Properties of a Right-angled Triangle: viz. That the square of the longest side of a Right-angled Triangle, usually called the Hypothenuse, is equal to the sum of the squares of the two other sides, usually called the Legs, which is of great use, for, by this mean, any two sides of a right-angled Triangle being given, the other may be found by common Arithmetic. Thus, in the right-angled Triangle ABC, the base AC and perpendicular BC being given, the Hypothenuse AB may be found by extracting the square root of the sum of the squares of the base and Perpendicular.

18,8

18,8 *Base*	12,6 *Perp.*	353,44=*Square of the Base.*
18,8	12,6	158,76=*Square of the Perp.*
1504	756	512,20(22,63 *Hypothenuse.*
1504	252	4
188	126	42)112
353,44	158,76	84
		446)2820
		2676
		4523)14400
		13569
		831

And, if the Hypothenuse and one of the Legs be given, the other may be found by subtracting the square of the given Leg from the Square of the Hypothenuse.

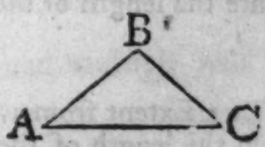
There are some numbers, the sum of whose squares make a perfect square, of which sort are 3 and 4, whose squares, being added together, make 25, which is the square of 5; Therefore, if the base of a Triangle be 4, and the Perpendicular 3, the Hypothenuse will be 5; and if any of these numbers be multiplied by any other number, those Products will be the Sides of right-angled Triangles, as 6, 8, 10 and 15, 20, 25, &c. Thus, artificers, when they set off the Corner of a building, usually measure 6 feet on one side, and 8 feet on the other, then laying a ten-feet Pole across, it makes the corner a true right-angle.

Art. 7. *There is another method of finding the Area of Triangles, the three sides being given.*

Rule. Add the three sides together, then take the half of that sum, and out of it subtract each side severally; multiply the half of the sum and these remainders continually, and the square root of this Product will be the area of the Triangle.

In the oblique Triangle ABC, the base AC is given 15,6, the side AB is 10,4, and the side BC is 9,2, to find the area.

15,6	17,6	17,6	17,6
10,4	—15,6	—10,4	— 9,2
9,2	2	7,2	8,4
35,2 *Sum*			
67,1=*Half the Sum.*			



17,6

```
17,6
   2
————
35,2
 7,2
————
 704
2464
————
253,44
   8,4
————
101376
202752
————
2128,896
```

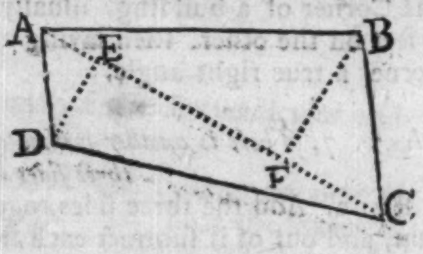
```
      . . . .
      2128,8960(46,139 = Area.
      16
      ——
   86)528
      516
      ———
  921)1289
       921
      ————
 9223)36860
      27669
      —————
92269)919100
      820421
      ——————
       98679
```

ART. 8. *To measure a Trapezium.*

Definition. A Trapezium is an irregular figure of four unequal sides, and unequal angles.

RULE. Draw a diagonal line from one of the angles to the opposite angle, as AC, and then will the Trapezium be divided into two Triangles, of which the diagonal is the common base: Then letting fall perpendiculars from the other opposite angles on the diagonal, add those perpendiculars together, and multiply half that sum into the diagonal, or half of the diagonal into the sum of the perpendiculars, and that product will be the area of the Trapezium.

IN the Trapezium ABCD, the diagonal AC is 24, the perpendicular DE 6, and the perpendicular BF 10. The sum of the perpendiculars is 16, whose half is 8, which being multiplied into 24, will give the area.




```
 24
  8
———
192 = Area.
```

By the Sliding Rule.

SET 1 on A to ½ the Sum of the perpendiculars on B, and opposite the length of the diagonal on A, you will have the area on B.

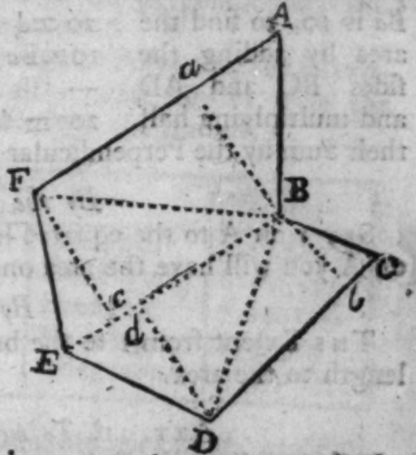
By Gunter.

THE Extent from 1 to ½ the sum of the perpendiculars will reach from the length of the diagonal to the area.

ART.

ART. 9. *To measure any irregular figure.*

RULE. Divide the figure into Triangles, by drawing diagonals from one angle to another; then measure all the Triangles by either of the rules, already taught, at Art. 6 or 7, and the sum of the several areas of all the Triangles will be the area of the given figure.



THE irregular figure ABCDEF being given, divide it into Triangles by the diagonals FB, EB, and DB; Then may the Triangles be measured by letting fall Perpendiculars on their respective bases, as B*a*, B*b*, D*c*, F*d*, and multiplying those Perpendiculars by half their respective bases.

IN the Triangle AFB the base FA is 100, and the Perpendicular B*a* 49; In the Triangle FBE the base BE is 92, and the Perpendicular F*d* 52; In the Triangle EBD, the base BE is the same as before, and the Perpendicular D*c* 44; and in the Triangle DCB, the base DC is 80, and the Perpendicular B*b* 38; by which the area of each may be found by Art. 6, as follows.

50 = *Half AF*	46 = *Half BE*	2450
49 = *Perp. aB*	52 = *Perp. Fd*	2024
——	——	2392
2450 = *Area of AFB.*	92	1520
46 = *Half BE*	230	——
44 = *Perp. Dc*	——	8386 = *Area of the Figure ABCDEF.*
——	2392 = *Area of FBE*	
184	38 = *Perp. Bb.*	
184	40 = *Half DC.*	
——	——	
2024 = *Area of EBD.*	1520 = *Area of DCB.*	

IN dividing any irregular figure into Triangles, the Triangles will be less, by two, and the diagonals less by three, than the number of the sides of the figure.

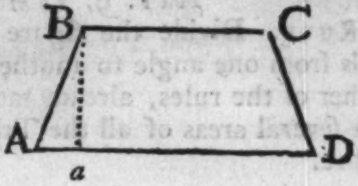
ART. 10 *To measure a Trapezoid.*

Definition. A Trapezoid is the Segment of a Triangle, cut by a line parallel to the base.

RULE. Add the parallel Sides together, and multiply half that Sum by the Perpendicular breadth.

In the Trapezoid ABCD, the Side AD is 24, the Side BC is 16 and the perpendicu. breadth B*a* is 10, to find the area by adding the sides BC and AD and multiplying half their Sum by the Perpendicular breadth B*a*.

24=AD
16=BC
—
40=*Sum*
—
20 =$\frac{1}{2}$ *Sum*
10=B*a*
—
200=*Area.*



By the Sliding Rule.

Set 1 on A to the equated length on B, and against the breadth on A you will have the area on B.

By Gunter.

The Extent from 1 to the breadth will reach from the equated length to the area.

Art. 11. *To measure any regular Polygon.*

Definition. A regular Polygon is a figure whose sides and angles, are all equal; they are usually denominated from the number of their sides:

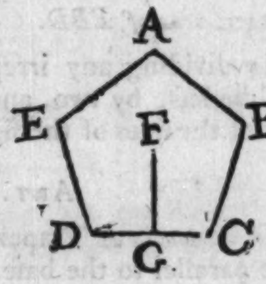
Thus, A figure having { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 } *equal sides and angles is a* { Trigon. Tetragon. Pentagon. Hexagon. Heptagon. Octagon. Enneagon. Decagon. Endecagon. Dodecagon. }

3	Trigon.
4	Tetragon.
5	Pentagon.
6	Hexagon.
7	Heptagon.
8	Octagon.
9	Enneagon.
10	Decagon.
11	Endecagon.
12	Dodecagon.

Rule. Multiply the length of one of the sides by the number of sides: then this product by the half of a perpendicular let fall from the centre of the figure to the middle of one of the sides, and the product will be the area of the Polygon.

In the Pentagon ABCDE, each side is 95, & the perpendicular FG 65,36, to find the area.

95=*Length of a side.*
5=*Number of sides.*
—
475=*Sum of the sides.*
32,68
—
3800
2850
950
1425
—
15523,00 = *Area of the Pentagon.*



By

By the Sliding Rule.

SET 1 on A to ½ the perpendicular on B, and againſt the ſum of the ſides on A you will have the area on B.

By Gunter.

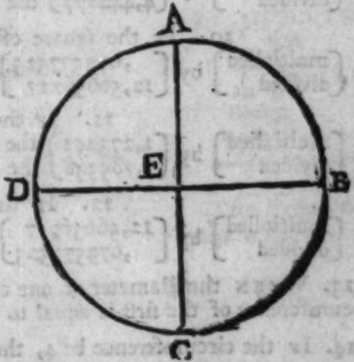
THE extent, from 1 to ½ the length of the perpendicular, will reach from the ſum of the ſides to the content.

BUT, for the more ready meaſuring regular Polygons, the following Table, containing multipliers for all regular figures from the Triangle to the Dodecagon, will be of uſe to the learner.

Number of ſides.	Names.	Multipliers.	Number of ſides.	Names.	Multipliers.
3	Trigon.	,433013	8	Octagon.	4,828427
4	Tetragon.	1,	9	Enneagon.	6,181827
5	Pentagon.	1,720477	10	Decagon.	7,694209
6	Hexagon.	2,589076	11	Endecagon.	8,51425
7	Heptagon.	3,633959	12	Dodecagon.	9,330125

If the ſquare of the ſide of a Polygon be multiplied by the multiplier of the like Figure, the product will be the Area of the Figure ſought.

To meaſure a Circle and its Parts.



IN the annexed Circle ABCD, the Arch-line ABCD is called the *Periphery*, the length of which is called the *Circumference*: Any Line, as DB or AC, paſſing through the Centre E, cuts the Circle into two equal Parts, called *Semicircles*, or half-circles; and ſuch Lines are called Diameters of the Circle: If two Diameters be drawn thro' a Circle, at right-angles to each other, then, the four equal diviſions of the Circle are called *Quadrants*; Half the diameter, as EB, is called the *Radius*, or Semidiameter.

ART. 12. *The Diameter of a Circle being given, to find the Circumference.* *

RULE. This may be done by either of the following Proportions; in

* *Note* 1. IF the diameter of any Circle

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}3{,}14159\\ {,}31831\end{array}\right\}\begin{array}{l}\text{the product}\\ \text{the quotient}\end{array}\Big\}\text{ is the Circumference.}$$

2. If the diameter of any Circle

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}{,}886227\\ 1{,}128379\end{array}\right\}\begin{array}{l}\text{the product}\\ \text{the quotient}\end{array}\Big\}\text{ is the ſide of an equal ſquare.}$$

3. IF the diameter of any Circle

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}{,}866024\\ {,}1547\end{array}\right\}\begin{array}{l}\text{the product}\\ \text{the quotient}\end{array}\left\{\begin{array}{l}\text{is the ſide of the equilateral}\\ \text{Triangle inſcribed.}\end{array}\right.$$

4. IF

in whole numbers, as 7 is to 22, or more exactly, as 113 is to 355: or in Decimals, as 1 is to 3,14159; so is the Diameter of a Circle to the Circumference.—EXAMP. A Circle whose Diameter is 12, to find the Circumference.

As

4. IF the diameter of any Circle be {multiplied / divided} by {,707016 / 1,414213} {the product / the quotient} is the side of the square inscribed.

5. IF the square of the diameter of any Circle be {multiplied / divided} by {,785398 / 1,273241} {the product / the quotient} is the Area.

6. IF the circumference of any Circle be {multiplied / divided} by {,31831 / 3,14159} {the product / the quotient} is the diameter.

7. IF the Circumference of any Circle be {multiplied / divided} by {,282094 / 3,544907} {the product / the quotient} is the side of the square equal.

8. IF the Circumference of any Circle be {multiplied / divided} by {,2756646 / 3,6275939} {the product / the quotient} is the side of the equilateral Triangle inscribed.

9. IF the Circumference of any Circle be {multiplied / divided} by {,225079 / 4,442877} {the product / the quotient} is the side of the square inscribed.

10. IF the square of the Circumference of any Circle be {multiplied / divided} by {,079577525 / 12,56636217} {the product / the quotient} is the Area.

11. IF the Area of any Circle be {multiplied / divided} by {1,273241 / ,785398} {the product / the quotient} is the Square of the Diameter.

12. IF the Area of any Circle be {multiplied / divided} by {12,56636217 / ,079577525} {the product / the quotient} is the Square of the Circumference.

13. WHEN the diameter of one circle is 1, and the diameter of another is 2, the circumference of the first is equal to the area of the second, =3,141592.

14. IF the circumference be 4, the diameter and area are equal. =1,273241.

15. IF the diameter be 4, the circumference and area are equal. =12,566368.

HENCE, because circles are the most capacious of all figures, if the *fourth* part of a circle be *squared*, it will not be equal to the *area* of that circle (as is commonly supposed) although the *four* sides added together are equal to the *Circumference* of that circle.

IN a circle, whose diameter is 24, circumference 75,4, and area 452,4, the *fourth* part of the circumference is 18,85, the *square* of which is only 355,3225, that is 97,0775 less than the truth; and the larger the circle is, the greater will the error be.

FOR further proof of this matter; If a cylindrical pint, beer-measure, whose content is 35,25 cubic inches, be beaten into a perfectly *square form*, it will contain only 28,902 cubic inches, which is less than the truth by 6,3484+; the area of the circle is 8,7615859288, and the area of the square only 6,8813320653076624.

HENCE appears the reason, why taking the *fourth part* of the Girth in measuring a Cylinder (or a round stick of timber) is false.

16. IF the Diameter of one circle be double to *that* of another, the *area* of the first circle will be *four times* the area of the second.

As 7 : 22 :: 12
12

7)264(37,71=*Circumference*
21

54
49

50
49

10
7

3

As 113 : 355 :: 12
12

113)4260(37,699 *Cir.*
339

870
791

790
678

1120
1017

103

As 1 : 3,14159 :: 12
12

37,69908 *Cir.*

Note, 3,14159 may be contracted to 3,1416 without any sensible difference.

ART. 13. *The Circumference of a Circle being given, to find the Diameter.*

RULE. As 22 is to 7; or 355 to 113; or as 1 to ,31831, So is the Circumference of a Circle to the Diameter.

EXAMP. The circumference of a circle being 326, to find the diameter.

As 22 : 7 : : 326
7

22)2282(103,72 *Diameter.*
22

82
66

160
154

60
44

16

355 : 113 : : 326
326

678
226
339

355)36838(103,76 *Diameter.*
355

1338
1065

2730
2485

245

1 : ,31831 : : 326
326

190986
63662
95493

103,76906=*Diameter.*
This Proportion is the most accurate.

ART. 14. *To find the Area of a Circle.*

RULE. Multiply half the Diameter by half the Circumference, and the Product is the Area.

IF the Diameter be given, find the Circumference by Art. 12.

IF the Circumference be given, find the Diameter by Art. 13.

EXAMP. A Circle, whose Diameter is 12, and Circumference is 37,7, given, to find the Area?

18,85=*Half the Circumference.*
6=*Half the Diameter.*

113,1=*Area of the given Circle.*

ART. 15. *The Diameter being given, to find the Area of a Circle without finding the Circumference.*

RULE. Multiply the ſquare of the Diameter by ,7854, and the Product will be the Area of the Circle, whoſe Diameter was given.

EXAMP. The Diameter of a Circle being 12, to find the area?

```
           ,7854
12×12=  144
        ------
          31416
         31416
        7854
        --------
        113,0976=Area.
```

By the Sliding Rule.

SET 1 on A to the Diameter on B. then find ,7854 (which expreſſes the Area of a Circle whoſe Diameter is 1) on A, againſt which on B is a 4th number, then find this 4th number on A, againſt which on B is the Area.

By Gunter.

THE Extent from 1 to the length of the Diameter reaches from ,7854 to a 4th number, and from that 4th number to the area.

ART. 16. *The Circumference of a Circle being given, to find the Area, without finding the Diameter.*

RULE. Multiply the Square of the Circumference by ,07958, and the Product will be the area of the Circle.

EXAM. The Circumference of a Circle being 37,7, to find the area?

```
   37,7                 1421,29
   37,7                 ,07953
  ------               --------
   2639                  426387
  2639                 710645
 1131                 1279161
 --------             994903
 1421,29=Square.     -----------
                     113,0351937=Area of the Circle.
```

ART. 17. *The Dimenſions of any of the Parts of a Circle being given, to find the Side of a Square equal to the Circle.*

RULE. If the area of the Circle be given, extract the ſquare root of the area, which will be the Side of a ſquare equal to the Circle: If the Diameter or Circumference be given, find the area by Art. 15 or 16, and then extract the ſquare Root, as before. And this is a *general Rule* to find the Side of a Square equal to any ſuperficial Figure, regular or irregular: For the Square Root of the

the area of any figure whatever, is the Side of a Square equal to the given Figure.—But, with regard to Circles, if the Diameter be given; multiply it by ,886 and the Product will be the Side of an equal Square:——Or, As 13,545 is to 12, or 1354 to 1200: So is the Diameter of a Circle to the Side of a Square equal to the given Circle. And, if the Circumference be given; multiply it by ,282 for the Side of an equal Square.—Or, divide it by 3,545, and the Quotient will be the Side of an equal Square.

EXAM. 1.

LET the Diameter of a Circle be 12, to find the Side of a Square equal to the Circle?

$,886 \times 12 = 10,632 =$ *Side of the Square.* *Or,*

As $13,545 : 12 :: 12 : 10,631 =$ *the side.*

EXAM. 2.

THE Circumference being 37,7 to find the Side of an equal square?

$37,7 \times ,282 = 10,631 =$ *Side of the Square.*

Or $37,7 \div 3,545 = 10,634$

ART. 18. *The Area of a Circle being given, to find the Diameter.*

RULE. Multiply the given area by 1,2732, and the product will be the square of the diameter; then, extracting the square root of the product, you will have the diameter.

EXAMP. The area of a circle being 113,09, to find the diameter.

```
 1,2732
 113,09
 ------
 114588
381960
12732
12732
-------------
143,986188
```

```
        12
143,986188(11,999=12=Diameter.
1
-
21)43
   21
   ---
229)2298
    2061
    ----
2289)23761
     20601
     -----
22989)316088
      206901
      ------
      109187 Remainder.
```

ART. 19. *The Area of a Circle being given, to find the Circumference.*

RULE. Multiply the given area by 12,566, and extract the square root of the product, which root will be the circumference required.

EXAMP. The area of a circle being 113,03 to find the circumference. 12,566

12,566	1420,3349(37,68 = *Circumference.*
113,03	9
37698	67)520
376980	469
12566	746)5133
12566	4476
1420,33498	7528)65749
	60224
	5525 *Remainder.*

ART. 20. *The side of a Square being given, to find the diameter of a Circle equal to the Square, whose Side is given.*

RULE. Multiply the given side by 1,128, and the product will be the diameter of a circle, whose area is equal to the area of the given square.—Or, If the side of the square be divided by ,886, the quotient will be the diameter.—Or, As 12 to 13,54, So is the side of any square to the diameter of an equal circle.

EXAM. The Side of a Square being 10,635, to find the Diameter of a Circle equal to that Square?

$10,635 \times 1,128 = 12$ *nearly,* Or $10,635 \div ,886 = 12 =$ *Diameter.*

Or, As $12 : 13,54 :: 10,635 : 12$ *nearly.*

ART. 21. *The Side of a Square being given, to find the Circumference of a Circle equal to the given Square.*

RULE. Multiply the given Side by 3,545 and the Product will be the Circumference required.—Or divide it by ,282, and the quotient will be the circumference.

EXAMP. The side of a square being 10,631, to find the circumference of a circle equal to that square. Or

$10,631 \times 3,545 = 37,686 =$ *Circumf.* ,282)10,631(37,698 *Circumf.*

ART. 22. *To find the Area of a Semicircle, the Diameter being given.*

RULE. Find the area of the circle by Art. 15, and take the half of it.

IN the same manner may the area of a quadrant, or a quarter of a circle, be found, by taking a fourth part of the area of the whole circle.

BUT with regard to measuring a Sector, or a Segment of a Circle, it will be necessary first to shew how to find the length of the arch-line of a sector, and the diameter of the circle to a given segment.

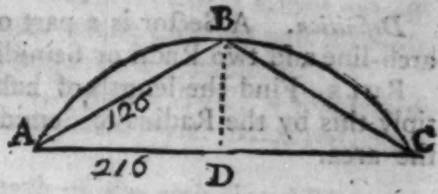
ART. 23. *A Segment of a Circle being given, to find the length of the Arch-line.*

RULE. Divide the segment into two equal parts; then measure the chord of the half-arch, from the double of which subtract the chord of the whole segment; and one third of that difference, be-

in

ing added to the double of the chord of the half-arch, will give the length of the arch-line.

EXAMP. In the segment ABCD, the whole chord ADC is 216, and the chord AB or BC 126, to find the arch-line ABC?



126 = *Chord* AB *or* BC.
2

252 = *Double.*
216 = ADC, *to be subtracted.*

3)36 = *difference.*

12 = $\frac{1}{3}$ *difference.*

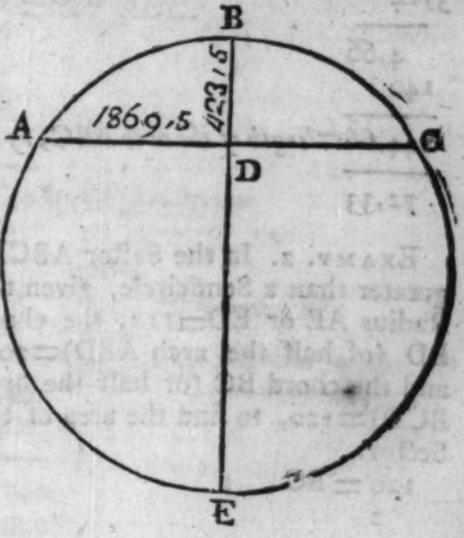
252 = *Double of* AB.
12 = $\frac{1}{3}$ *difference, added.*

264 = *Length of the arch* ABC.

ART. 24. *The Chord and versed sine of a Segment being given, to fina the diameter of a Circle.*

RULE. Multiply half the chord by itself, and divide the product by the versed sine; then add the quotient to the versed sine, and the sum will be the diameter of the circle.

EXAMPLE. In the Segment ABCD, the chord AC is 1869,5, and the versed sine BD 423,5, to find the diameter.



934,75 { *Half the chord* AC
934,75

467375
654325
373900
280425
841075

423,5)873557,5625(2062,7 = DE.
8470

26557
25410

11475
8470

30056
29645

411

423,5 = BD, *add.*

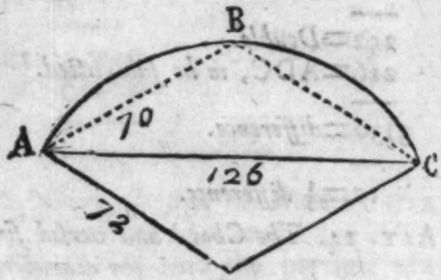
2486,2 = *Diameter* BDE.

ART. 25. *To measure a Sector.*

Definition. A Sector is a part of a Circle, contained between an arch-line and two Radii or Semidiameters of the Circle.

RULE. Find the length of half the Arch by Art. 23: then multiply this by the Radius or Semidiameter, and the product will be the area.

EXAMP. 1. In the sector ABCD, given the Radius AD or DC 72 feet, the chord AC = 126 feet, and the chord AB or BC = 70, to find the area of the Sector.



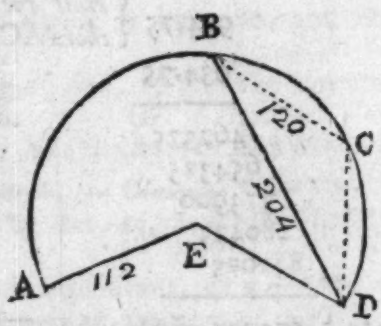
First.

70 = *Chord* AB *or* BC.
2
———
140
126 = AC, *subtract.*
———
3)14
———
4,66
140
———
144,66 = *length of the arch* ABC, *by Art.* 23.
———
72,33

Secondly.

72,33 = *Half the Arch.*
72 = *Radius.*
———
14466
50631
———
5207,76 = *Area.*

EXAMP. 2. In the Sector ABCD, greater than a Semicircle, given the Radius AE or ED = 112, the chord BD (of half the arch ABD) = 204, and the chord BC (of half the arch BCD) = 120, to find the area of the Sector.



120 = BC.
2
———
240
204 *Subtract.*
———
3)36
———
12
240 *Add.*
———
252 = { *Length of the arch* BCD, *by Art.* 23.

126 = *Half the arch* ABD.
112 = *Radius.*
———
252
126
126.
———
14112 = *Area of the Sector.*

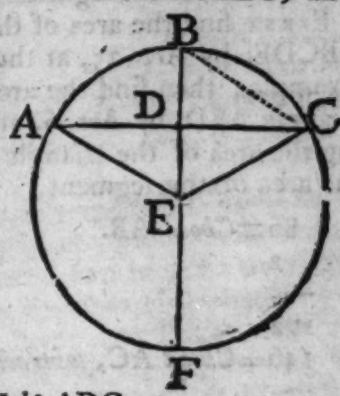
ART.

ART. 26. *To find the Area of a Segment of a Circle.*

Definition. A Segment of a Circle is any part of a Circle cut off by a right line drawn acroſs the circle, which does not paſs through the centre, and is always greater or leſs than a Semicircle.

EXAMP. I. To find the area of the ſegment ABC, whoſe chord AC is 172, the chord of half the arch ABC, viz. BC=104, and the verſed ſine BD=58,48.

RULE. By Art. 23, find the length of the arch-line ABC, and by Art. 24, the diameter BF; then multiply half the chord of the arch ABC by half the diameter, and the product will be the area of the Sector ABCE: Then find the area of the triangle AEC, whoſe baſe AC is 172, and perpendicular-height 34, found by ſubtracting the verſed ſine BD from half the diameter; and the area of the Triangle AEC, being ſubtracted from the area of the Sector ABCE, will leave the area of the Segment ABC.



104=BC.
2
208
172=AC, *Subtract.*
3)36
12
208 *Add.*
220= *Arch-line* ABC.
110=*Half-arch*

92,475=*Radius.*
110
924750
92475
10172,25=*Area of the Sector.*

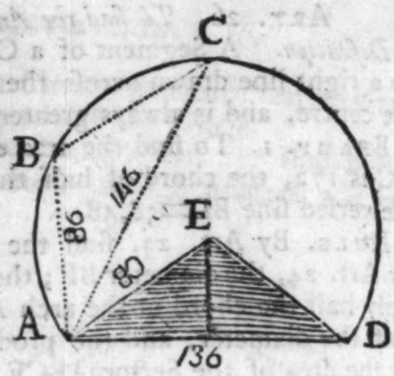
86=*Half the Baſe*=AD.
34=*Perpendicular* DE.
344
258
2924=*Area of the Triangle.*

86=*Half* ADC.
86
516
688
58,48)7396,00(126,47=DEF.
5848
15480
11696
37840
35088
27520
23392
41280
40936
344

58,48=BD *Add.*
184,95=*Diameter* BF.
92,475 = { *Radius or Semidiameter.*

10172,25=*Area of the Sector.*
2924 =*Area of the Triangle.*
7248,25=*Area of the Segment.*

EXAMP.



EXAMP. 2. In the ſegment, ABCD greater than a ſemicircle, given the chord of the whole ſegment AD=136, the chord AC of half the arch ACD=146, the chord AB or BC of one fourth of the arch ACD=86, and the radius AE or ED=80, to find the area of the ſegment ABCD.

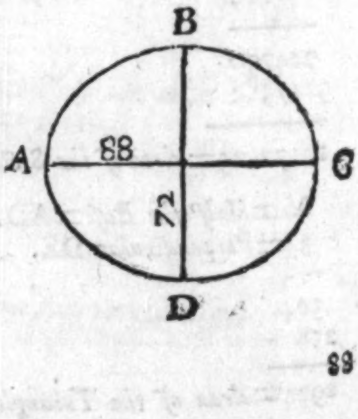
FIRST find the area of the ſector ABCDE, by Art. 25, at the ſecond Example; then find the area of the triangle AED, by Art. 6, and, adding the area of the triangle to the area of the ſector, you will have the area of the ſegment.

86=*Chord* AB.
2
———
172
146=*Chord* AC, *ſubtract.*
———
3)26
———
8,666
172 =*Double of* AB, *add.*
———
180,666=*Arch-line* ABC.
80=*Radius.*
———
14453,280=*Area of the ſector.*

68=*Half the Baſe* AD.
42=*Perpendicular* E 136.
———
136
272
———
2856=*Area of the triangle* AED
14453,28=*Area of the ſector, add.*
———
17309,28=*Area of the ſegment.*

ART. 27. *To find the Area of an Ellipſis.*

Definition. An Ellipſis, or Oval, is a Curve which returns into itſelf like a circle, but has two diameters, one longer than the other, the longeſt of which is called the tranſverſe, and the ſhorteſt, the conjugate Diameter.



RULE. Multiply the two diameters of the Ellipſis together; then, multiplying the product by ,7854, this laſt product will be the area of the Ellipſis.

EXAMP. In the Ellipſis ABCD, the tranſverſe diameter AC is 88, and the conjugate diameter BD is 72, to find the area.

$$
\begin{array}{r}
88 \\
72 \\
\hline
176 \\
616 \\
\hline
6336 \\
,7854 \\
\hline
25344 \\
31680 \\
50688 \\
44352 \\
\hline
4976,2944
\end{array}
= \textit{Area.}
$$

The Content is found by the Sliding Rule and Gunter, in the same way as the Circle, only using the product of the two diameters as the diameter of a circle.

Mensuration of Superficies is easily applied to *Surveying*: thus, take the angles of the plot with a good compass, then measure the sides with Gunter's chain, which note down in links (or chains and links, which is done by separating the two right-hand figures of your links by a comma, your chain being 100 links) then cast up the contents, according to the rule of the figure, cutting off the five right-hand figures of the product, and those at the left-hand, if any, are Acres; then, multiply the five figures, cut off, by 4, by 40, and by $272\frac{1}{4}$, cutting off as before, and those at the left-hand will be Roods, Poles and Feet, respectively.

Section 2. *Of* SOLIDS.

Solids are measured by the solid inch, foot, or yard, &c.—— 1728 of these inches, that is $12\times 12\times 12$, make 1 cubic or solid foot.

The solid content of every body is found by rules adapted to their particular figures.

Art. 28. *To measure a Cube.* ‡

Definition. A Cube is a solid of six equal sides, each of which is an exact square.

The

‡ Here follows a Table of the Proportions, which the following *Solids* have to the *Cube* and *Cylinder*, having the same *Base* and *Altitude*.

	Solid Inches.
1. A *Cube*, whose side is 12 inches, contains - - - - - -	1728
2. A *Prism*, having an equilateral Triangle, whose side is 12 inches for its *Base*, and its *Altitude* 12 inches, contains - - -	784,24
3. A *Square Pyramid*, whose height, and the side of its Base, are each 12 inches, is $\frac{1}{3}$ of the above Cube, and therefore contains - - -	576
4. A *Triangular Pyramid*, whose height and side of its triangular base are each 12 inches, is near $\frac{1}{7}$ of the Cube, and contains - -	249,413
5. A *Cylinder*, whose diameter and height are each 12 inches, is $\frac{11}{14}$ of the above Cube, and contains - - - - - -	1357,17
6. A *Sphere* or *Globe*, whose axis or diameter is 12 inches, equal to the side of the Cube, is $\frac{11}{21}$ of it, and contains - - - -	904,78
7. A *Cone*, whose base and altitude are each 12 inches, equal to the side of the Cube, is $\frac{5}{19}$ of it, and contains - - - - -	452,38829

8. A

THE solid foot is composed of 1728 inches: for a solid, that is 1 foot, or 12 inches every way, that is $12 \times 12 \times 12$, contains 1728 inches.

	Solid Inches.
8. A *Parabolic Conoid*, whose diameter at the base, and height are each 12 inches, being $\frac{1}{2}$ its circumscribing Cylinder, contains - -	678,583
9. A *Hyperbolic Conoid*, whose height, and diameter at the base, are each 12 inches, is $\frac{5}{12}$ of its circumscribing Cylinder, and contains	565,49
10. A *Parabolic Spindle*, whose height and middle diameter are each 12 inches, is $\frac{8}{15}$ of its circumscribing Cylinder, and contains - -	723,824

HENCE arises a different method of finding their contents.

General Rule. If the base of the solid, whose content you would find, be rectilinear, consider it as a *Parallelopipedon*:—if curved, as a *Cylinder*, and find the content accordingly: then take such a part of the content, thus found, as is specified in the preceding Table, which, if the parts be taken in inches, will be the solid content of the given figure, in inches, which, divided by 1728, will give the cubic feet.

Example 1. There is a triangular Prism, the side of whose base is 48 inches, and whose perpendicular height is 108 inches; what is its solid content?

THE base being right-lined, I consider it as a *parallelopipedon*, the side of whose base is 48 inches, and whose length is 108 inches, and as 784,24 is contained 2,20340712 times in a cubic foot; 2,20340712 is a divisor, to divide the content of the parallelopipedon by; therefore $48 \times 48 \times 108 \div 2{,}20340712 = 112930{,}56$ solid inches $= 65{,}353$ solid feet.

HAD the dimensions been given in feet, it would have been $4 \times 4 \times 9 \div 2{,}20340712 = 65{,}353$ feet.

Example 2. There is a square Pyramid, whose height is 12 feet, and the side of whose base is 3,5 feet; what is its content?

$3{,}5 \times 3{,}5 \times 12 \div 3 = 29$ *feet, Answer.*

Example 3. There is a Triangular Pyramid, whose height is 15 feet, and the side of whose base is 5 feet; what is its content?

$5 \times 5 \times 15 \div 7 = 53{,}57$ *feet, Answer.*

Example 4. There is a Cylinder, whose diameter is 2,5 feet, and whose length is 24 feet; what is its content?

HERE, the *diameter* is to be considered as the side of the base of a Parallelopipedon. Therefore, $2{,}5 \times 2{,}5 \times 24 \times 11 \div 14 = 117{,}857$ *feet.*

Example 5. There is a spherical Balloon, whose diameter is 50 feet; how many cubic feet of air does it contain?

HERE, the diameter is to be considered as the side of a cube. Therefore,

$50 \times 50 \times 50 \times 11 \div 21 = 65476{,}19$ *feet, Answer.*

Example 6. There is a Cone whose height is 15 feet, and the diameter of whose base is 5 feet; what is its content?

HERE, the diameter of the base is to be considered as the side of the base of a Parallelopipedon, and its height, as the length. Therefore,

$5 \times 5 \times 15 \times 5 \div 19 = 98{,}684$ *feet.*

Example 7. There is a Parabolic Conoid, whose diameter, at the base, is 2,9 feet, and whose height is 6 feet; what is the content?

THIS Solid, being $\frac{1}{2}$ of a Cylinder; we must first find the content as of that of a Cylinder, and then halve it. Therefore,

$2{,}9 \times 2{,}9 \times 6 \times 11 \div 14 = 39{,}647$, *and* $39{,}647 \div 2 = 19{,}823$, *Answer.*

Example 8. There is a *Hyperbolic Conoid*, whose diameter at the base is 2,9 feet and whose height is 6 feet; what is the content?

FIRST, find the content of a Cylinder.

$2{,}9 \times 2{,}9 \times 6 \times 11 \div 14 = 39{,}647$, *and* $39{,}647 \times \frac{5}{12} = 16{,}519$ *feet.*

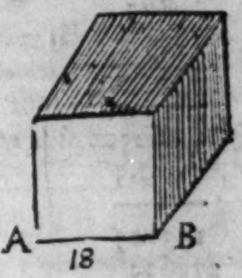
Example 9. There is a *Parabolic Spindle*, whose middle diameter is 2,9 feet, and whose length is 6 feet: Required the content?

FIRST, find the content of a Cylinder.

$2{,}9 \times 2{,}9 \times 6 \times 11 \div 14 = 39{,}647$, *and* $39{,}647 \times \frac{8}{15} = 21{,}145$ *feet.*

RULE. Multiply the side by itself, and that product by the same side, and this last product will be the solid content of the cube.

EXAMP. The side of a cube AB, being 18 inches, or 1 foot and 6 inches, to find the content?



1 *Foot*, 6 *Inches*= 1,5 *Foot*.	18 *Inches*.
1,5	18
75	144
15	18
2,25	324
1,5	18
1125	2592
225	324
3,375	1728)5832(3,375
In this operation the inches are changed into the decimal parts of a foot.	5184
	6480
	5184
	12960
	12096
	8640
	8640

I have done this two different ways, that the learner may see they come out the same. The content in inches is 5832, which being divided by 1728, the inches in a solid foot, and the division continued by annexing cyphers, it comes out the same as the decimal operation.

Note, The area of the surface, or superficial content of the cube and parallelopipedon is found by adding the areas of the several quadrilateral figures which compose them.

ART. 29. *To measure a Parallelopipedon.*

Definition. A Parallelopipedon is a solid of three dimensions, length, breadth and thickness; as a piece of timber exactly squared, whose length is more than the breadth and thickness. The ends are called Bases, which are equal.

RULE. Find the area of the base, then multiply that by the length, and it will give the solid content.

EXAMP. 1. The side AB is 1,75 foot, and the length AD 9,5 feet, to find the solid content?

1,75=

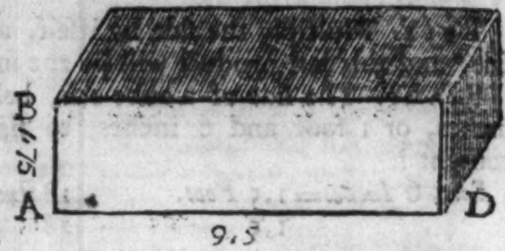
1,75 = 1 *foot, 9 inches.*
1,75

875
1225
175

3,0625 = *Area of the base.*
9,5

153125
275625

29,09375 = *Solid Content.*



EXAMP. 2. A Vessel 3,5 feet each side within, and 5 feet deep, to find the content?

3,5
3,5

175
105

12,25
5

61,25 = *the Content.*

If a piece of Timber, or any other thing, be of an equal bigness through its whole length, though there be a difference between the breadth and thickness, if the breadth and thickness are multiplied together, and that product multiplied by the length, this last product will be the solid content.

EXAMP. 3. A piece of Timber being 1 foot and 6 inches, or 18 inches broad, 9 inches thick, and 9 feet 6 inches, or 114 inches long, to find the content?

1 *Foot,* 6 *Inches* = 1,5 *Foot.*
9 *Inches* = ,75 *Foot.*

75
105

1,125
9 *Ft.* 6 *Inch.* = 9,5

5625
10125

10,6875 = *Content.*

In this operation the Inches are changed into the decimal fractions of a foot.

Breadth = 18 *Inches.*
Depth = 9 *Inches.*

162
Length = 114 *Inches.*

648
162
162

1728) 18468 (10,6875 = *Content as before.*
1728

11880
10368

15120
13824

12960
12096

8640
8640

Note,

Note, When the end is given in Inches and the length in feet, find the area at the end in Inches, multiply that by the length in feet, and divide this Product by 144 (the square inches in a foot) and the Quotient will be feet.

Take the last Example.

Foot.

1,5 = 18 *Inches*
,75 = 9 *Inches*

162 *Area in inches*,
9,5 *feet* = *length*.

810
1458

144)1539(10,6875 = *Content*.
144

990
864

1260
1152

1080
1008

720
720

By the Sliding Rule.

SET 12 inches on the girt-line D to the side of the square end on C, then, against the length on D, you will have the answer on C.

By Gunter.

EXTEND the Compasses from 12 inches to the length of the Side of the square end; that distance, twice turned over from the length, will reach to the Content.

WHEN the Side of a square solid is given, in inches, to find how much in length will make a foot solid.

RULE. As the given Side is 12, so is 12 to a fourth number, and so is that fourth number to the required length.—Or divide 1728 by the Area at the end, and the Quotient will be the length making a solid foot.

IF the given Side is in foot-measure, then,

RULE. As the given Side is to 1; so is 1 to a fourth number, and so is that fourth number to the required length.

WHEN two Sides of an unequal square solid (that is, of unequal breadth) are given, to find what length will make any number of solid feet:

RULE. Multiply the proposed number of feet by 144; Divide that Product by the Product of the breadth and depth, and the Quotient will be the length required.

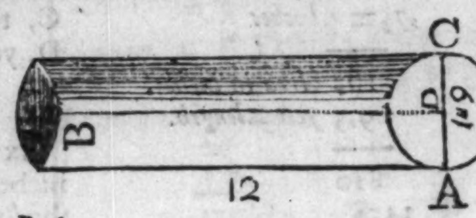
ART.

Art. 30. *To measure a Cylinder.*

Definition. A Cylinder is a round body, whose Bases are Circles, like a round Column, or a Rolling-Stone of a garden.

Rule. The Diameter of the Base being given, find the Area of the end by Art. 15, then, multiplying the Area of the Base by the length, that Product will be the Content of the Cylinder.

Exam. The Diameter of the Base AC being 1 foot, and 9 inches, and the length BD 12 feet and 6 inches, to find the content.



1,75 = *Diam. of the Base.*
1,75

875
1225
175

3,0625
,7854

122500
153125
145000
214375

2,40528750 = *Area of the Base.*

2,405 = *Area of the Base.*
12,5 = *Length.*

12025
4810
2405

30,0625 = *Content.*

If the square of the Diameter of a Cylinder be multiplied by ,7854, and the Solidity divided by that Product, the Quotient will be the length.

The learner may, for his practice, reduce all the dimensions to inches, and find the solid content in inches, which being divided by 1728, the quotient will be the solid content in feet: Or if he finds the area at the end in inches, and multiplies that by the length in feet, and divides by 144, the quotient will be feet.

This is a general rule for finding the content of any straight solid body, of equal bigness from end to end, of whatever form the bases are: For, if the area of the base be multiplied by the length, the product will be the solid content.

By the Sliding Rule.

Set 13,5, the square root of 183,34 (which is a gauge point arising from the division of 144 by ,7854) found on D, to the diameter

ameter found on C, and opposite to the length, on D, you will find the content on C.

Or, As 42,54 is to the circumference; So is the length in feet to a fourth number, and so is that fourth number, to the answer.

Note, The superficial content of a cylinder is found by multiplying the circumference of one of the bases into the length, and to the product adding the areas of the two bases, or ends.

When the diameter is given in inches, to find what length will make a solid foot.

Rule. As the given diameter is to 13,531; So is 12 to a 4th number, and so is that 4th number to the required length.——If the diameter be given in foot-measure; *Rule*, As the given diameter is to 1,128; So is 1 to a 4th number, and so is that 4th number to the required length ——Or, divide 1728 by the area at the end in inches, and the quotient will be the required length.

To find how much a Cylindric or round Tree, that is equally thick from end to end, will hew to, when made square.

Rule. Multiply twice the square of its semi-diameter by the length, then divide the product by 144, and the quotient will be the answer.

If the diameter of a round stick of Timber be 24 inches from end to end, and its length 20 feet; How many solid feet will it contain, when hewn square; and what will be the content of the slabs, which reduce it to a square?

$$\frac{12\times12\times2\times20}{144}=40 \text{ feet, the solidity when hewn square.}$$

$$\frac{24\times24\times,7854\times20}{144}=62,8 \text{ feet, or } 2\times2\times,7854\times20=62,8 \text{ the}$$

total solidity, whence 62,8—40=22,8 *feet, the solidity of the slabs.*

Art. 31. *To measure a Prism.*

Definition. A Prism is a body with two equal or parallel ends, either square, triangular, or polygonal, and three or more sides, which meet in parallel lines, running from the several angles at one end, to those of the other.

Rule. Prisms of all kinds, whether square, triangular or polygonal, are measured by one general rule, viz. Find the superficial content, or area at the base (or end) by the proper rule of Sect. 1. and this multiplied by the length, or height of the Prism, will give the solid content.

Examp.

EXAMP. The side of a stick of Timber, AB, hewn three square, is 10 inches, and the length, AC, is 12 feet, to find the content?

Side = 10 *Inches.*
$\frac{1}{2}$ *Perpend.* = 4,2 *Inches.*

42 = *Area at the end.*
12 *feet* = *length.*

144)504(3,5 *feet, content.*
432

720
720

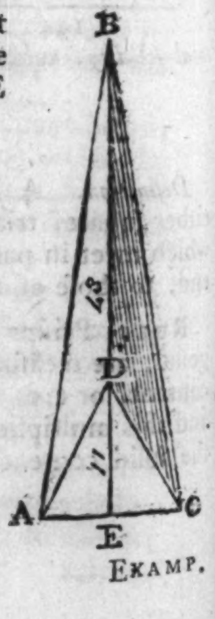
Note, The superficial content is found by adding the areas of the several quadrilateral and triangular figures, which compose it.

ART. 32. *To measure a Pyramid.*

Definition. Solids, which decrease gradually from the base till they come to a point, are generally called Pyramids, and are of different kinds, according to the figure of their bases; thus, if it has a square base, it is called a *square Pyramid*: if a triangular base, a triangular Pyramid; if the base be a circle, a *circular Pyramid*, or simply a *Cone.* The point, in which the top of the Pyramid ends, is called a *Vertex*, and a line drawn from the Vertex, perpendicular to the base, is called the height of the Pyramid.

RULE. Find the area of the base, whether triangular, square, polygonal or circular, by the Rules in superficial measure; then, multiply this area by one third of the height, and the product will be the solid content of the pyramid.

EXAMP. 1. In a triangular Pyramid, the height BE, being 48, and each side of the base 13: the base being a triangle, let the perpendicular height DE be 11; to find the content?



5,5 = *Half* ED
13 = *Base* AC

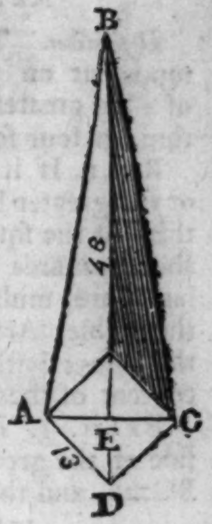
165
55
71,5 = *Area of the base,*
16 = $\frac{1}{3}$ *of the height* EB

4290
715

1144,0 = *Content.*

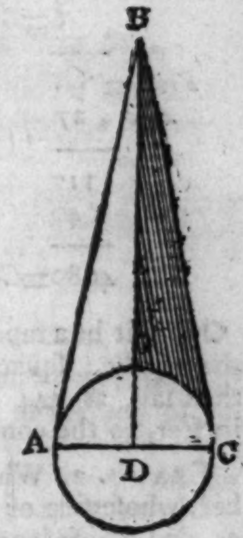
EXAMP.

EXAMP. 2. In a quadrangular Pyramid, the height BE being 48, and each side of the base 13, to find the content.



13
13
—
39
13
—
169 = *Area of the Base.*
16 = $\frac{1}{3}$ *of the height* EB
—
1014
169
—
2704 = *Content.*

EXAMP. 3. *To measure a Cone.*—The diameter AC being 13, and the height BD 48, to find the content.



13
13
—
39
13
—
169
,7854
—
676
845
1352
1183
—
132,7326 = *Area of the Base.*
16 = $\frac{1}{3}$ *of the height.*
—
7963956
1327326
—
2123,7216 = *Content.*

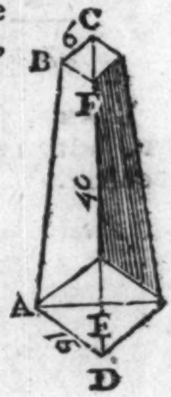
Note. The superficial content of all Pyramids is found by taking the sum of the several areas, which compose them. That of a cone, by multiplying the circumference of the base into half the line joining the vertex and any point in that circumference, and adding the area of the base to the product.

ART. 33. *To measure the frustum of a Pyramid.*

Definition. The frustum of a Pyramid is what remains after the top is cut off by a plane parallel to the base, and is in the form of a log greater at one end than the other, whether round, or hewn three or four square, &c.

RULE. If it be the frustum of a square Pyramid, multiply the side of the greater base by the side of the less; to this product add one third of the square of the difference of the sides, and the *sum* will be the mean area between the bases; but, if the base be any other regular figure, multiply this *sum* by the proper multiplier of its figure in the Table. Art. 11. and the product will be the mean area between the bases: lastly, multiply this by the height, and it will give the content of the frustum.

EXAM. 1. In the frustum of a square Pyramid, the side of the greater base AD=15, the side of the less, BC=6, and the height EF=40, to find the content.



	15=AD	15
	6=BC	6
Prod.	= 90	9=*Difference*
Add	27	9
	117	3)81 = *square of the difference.*
	×40	
		27=$\frac{1}{3}$ *of the square.*
	4680=*Content.*	

OR, if it be a tapering square stick of timber, take the girth of it in the middle; square $\frac{1}{4}$ of the girth (or multiply it by itself in inches) then say, as 144 (inches) to that product; so is the length, taken in feet, to the content in feet.

EXAMP. 2. What is the content of a tapering square stick of timber, whose side of the largest end is 12 inches, of the least end, 8 inches, and whose length is thirty feet?

ONE fourth of the girth in the middle = 10, and 10×10=100 the area in the middle, then, As 144 : 100 :: 30 feet : 20,83 feet, the content.

By the Sliding Rule.

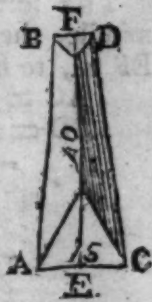
Set 12 on D to $\frac{1}{4}$ of the circumference on C; and against the length on D is the answer on C.

By Gunter.

The extent from 12 to $\frac{1}{4}$ of the circumference doubled, or twice turned over, will reach from the length to the content.

EXAM.

Exam. 3. In the fruſtum of a triangular pyramid, the ſide of the greater baſe $AC=15$, as before, the ſide of the leſs, $BD=6$, and the height $EF=40$, to find the content.



$15=AC$
$6=BD$

$9=$ *Difference of the ſides.*
9

$3)81=$ *Square of the difference.*

$27=\frac{1}{3}$ *of the ſquare.*

15
6

90
Add 27

117
$,433$ *Multiplier.*

351
351
468

$50,661=$ *Mean area.*
$40=$ *Height.*

$2026,440=$ *Content.*

Or, If it be a tapering three-ſquare ſtick of Timber, you may find the area midway from end to end, then, As 144 is to that area; ſo is the length, taken in feet, to the content in feet.

Examp. 4. *To meaſure the Fruſtum of a Cone.*

Rule. Multiply the diameters of the two baſes together, and to the product add one third of the ſquare of the difference of the diameters: then multiplying this ſum by ,7854, it will be the mean area between the two baſes, which being multiplied by the length of the fruſtum, will give the ſolid content.

Or, To the areas of the top and bottom add the ſquare root of the product of thoſe areas, and the ſum, multiplied by *one third* of the height of the fruſtum, will give the ſolidity.

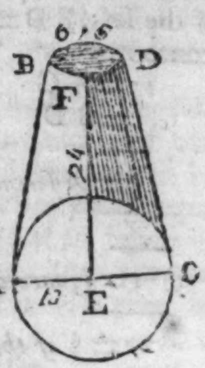
When figures run uniformly taper; but not to a point (they being conſidered as portions of the Cone or Pyramid) we may find the ſolidity by ſupplying what is wanting to complete the figure, and then deducting the part cut off.

A General Rule for completing every ſtrait ſided ſolid, whoſe ends are parallel and ſimilar.

As the difference of the top and bottom-diameters is to the perpendicular height, (or depth, which is the ſame:) So is the longeſt diameter to the altitude of the whole cone or pyramid.

The

The former cone in Art. 32. Examp. 3. being cut off in the middle, the greater diameter AC is 13, the less, BD $6\frac{1}{2}$, and height EF 24, to find the content of the frustum.



AC = 13 *Inches.*
BD = 6,5 *Inches.*

65
78
84,5
Add 14,083
98,583
,7854
394332
492915
788664
690081
77,427|0882 = *Mean area.*
24 *Feet* = *Length.*
309708
154854
1858,248 = *Content.*

13
6,5
6,5 = *Difference.*
6,5
325
390
3)42,25 = { *Square of the differ.*
14,083 = $\frac{1}{3}$ *of the squa.*

144)1858,248(12,973 *feet, content.*
144
418
288
1302
1296
1064
1008
568
432
136

Art. 34. *To measure a Sphere, or Globe.*

Definition. A Sphere or Globe is a round solid body, in the middle of which is a point, from which all lines drawn from the surface are equal.

Rule. Multiply the cube of the diameter by ,5236, and the product will be the solid Content.

Or multiply the Circumference by the diameter, which will give the superficial content; then multiply the surface by *one sixth* of the diameter, and it will give the solidity.

Or, Multiply the cube of the diameter by 11, and the product divided by 21, will give the solidity.

Examp. The diameter, AB, of a Globe is 4,5 feet; to find the solid content.

4,5

4,5
4,5
———
225
180
———
20,25
4,5
———
10125
8100
———
91,125
,5236
———
546750
273375
182250
455625
———
47,7130500

Note. If the circumference, or greateſt circle of the Sphere, be given, multiply the cube of it by ,016887 for the content.

THE ſurface of the Globe may be found by multiplying the ſquare of the diameter by 3,1416—or by multiplying the area of its greateſt circle by 4, or the ſquare of the circumference by ,3183.

WHEN the ſolidity of a Globe is given, the diameter may be found by dividing the ſolidity by ,5236, and extracting the cube root of the quotient.

OR, If the circumference be required, divide the ſolidity by ,016887 and the cube root of the quotient will give it.

ART. 35. *To meaſure the ſolidity of a fruſtum or Segment of a Globe.*

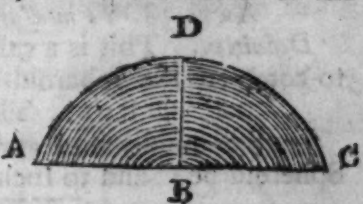
Definition. The fruſtum of a Globe is any part cut off by a plane.

RULE. To three times the ſquare of the Semidiameter of the Baſe, add the ſquare of the height; then multiplying that ſum by the height, and the product by ,5236, you will have the ſolid content.

EXAM. The height BD being 9 inches, and the diameter of the Baſe AC 24 inches: to find the content.

12 = *Semi-diam.*
12
———
144 = *Square.*
×3
———
432
Add 9×9 = 81 = { *Square of the height.*
———
513
×9 = *Height.*
———
4617

4617
,5236
———
27702
13851
9234
23085
———
2417,4612 = *Solid content.*



To

To measure the Surface of a Frustum or Segment of a Globe.

RULE. Find the diameter of the Globe by Art 24, and the surface of the whole Globe, by Art. 34; then, As the diameter of the Globe, is to the height of the frustum; So is the Surface of the Globe, to the Surface of the Frustum; then, by Art. 15, find the area of the Base; Add these two together, and the sum will be the whole Surface of the frustum.

ART. 36. *To measure the middle Zone of a Globe.*

Definition. This part of a Globe is somewhat like a Cask, two equal Segments being wanting, one on each side of the Axis.

RULE. To twice the square of the middle diameter, add the square of the end diameter; multiply that sum by ,7854, and that Product multiplied by *one third* of the length, will give the Solidity.

OR, To four times the square of the middle diameter add twice the square of the end diameter, that sum multiplied by ,7854, and that product by *one sixth* of the length, will give the solidity.

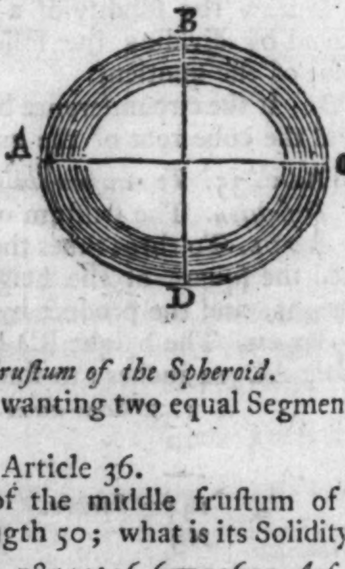
NOTE, *This Rule is applicable to the frustum of a Cone or Pyramid.*

IF the middle diameter of a Zone be 20 inches, the end-diameters each 16 inches, and length 12 inches; required its solidity?

$\overline{20\times20\times2+16\times16}\times,7854\times4=3317,5296$ *Answer.*

ART. 37. *To measure a Spheroid.*

Definition. A Spheroid is a Solid body like an Egg, only both its ends are the same.



RULE. Multiply the square of the diameter of the greatest Circle, *viz.* the diameter of the middle (DB in the figure) by the length AC, and that product by ,5236, and you will have the Solidity.

EXAMP. The diameter BD being 20, and the length AC 30, to find the content.

$20\times20\times30\times,5236=6283,2,$ *Answer.*

ART. 38. *To measure the middle Frustum of the Spheroid.*

Definition. This is a cask-like Solid, wanting two equal Segments to complete the Spheroid.

RULE. The same as in Article 36.

IF the middle and end diameters of the middle frustum of a Spheroid be 40 and 30 Inches, and its length 50; what is its Solidity?

$50\div3=16,6,$ *then* $\overline{40\times4\times2+30\times30}\times,7854\times16,6=53699$ *Ans.*

ART. 39. *To measure a Segment, or Frustum of a Spheroid.*

Definition. This is a part of a Spheroid made by a plane, parallel to its greatest circular diameter.

RULE.

Rule. To four times the ſquare of the middle diameter add the ſquare of the baſe diameter, then multiply that ſum by ,7854, and the product by *one ſixth* of the altitude, and it will give the ſolidity.

If the Baſe-diameter of the end-fruſtum of a Spheroid be 36, diameter at the middle of the height 30, and the height 20 inches; Required its ſolidity?

$\overline{30\times30\times4}+\overline{36\times36}\times,7854\times3,3=12817,728$ *Anſ. N.B.* $20\div6=3,3$.

Art. 40. *To meaſure a Parabolic Conoid.*

Definition. This Solid may be generated by turning a Semi-parabola about its abſciſſa or altitude.

Rule. As a Parobolic Conoid is half of its circumſcribing Cylinder, of the ſame baſe and altitude; multiply the area of the baſe by half the height, for the Solidity.

If the diameter of the baſe of a Parabolic Conoid be 40 inches, and its height 42; what is the Solidity?

$40\times40\times,7854\times21=2639,44$ *Anſwer.*

Art. 41. *To meaſure the lower Fruſtum of a Parabolic Conoid.*

Definition. This Solid is made by a plane paſſing through the Conoid, parallel to its baſe.

Rule. Multiply the ſum of the ſquares of the diameters of the baſes by ,7854, and that product by half the height, for the Solidity.

If the diameters of a Fruſtum of a parabolic Conoid be 40 and 30 inches, and its height 20 inches; Required its Solidity?

$\overline{40\times40}+\overline{30\times30}\times,7854\times10=19635$ *Anſ.*

Art. 42. *To meaſure a parabolic Spindle.*

Definition. This Solid is formed by an obtuſe parabola, turned about its greateſt ordinate.

Rule. This Solid being eight fifteenths of its leaſt circumſcribing Cylinder, multiply the Area of its middle or greateſt diameter by eight fifteenths of its perpendicular length, and it will give its Solidity.

If the Diameter at the middle of a parabolic Spindle be 20 Inches, and its length 60; Required its Solidity?

$20\times20\times,7854\times32(=60\times8\div15)=10053,12$ *Anſ.*

Art. 43. *To meaſure the middle Zone, or middle Fruſtum of a Parabolic Spindle.*

Definition. This is a caſk-like Solid, wanting two equal ends of ſaid Spindle.

Rule. To the Sum and half ſum of the ſquares of the two diameters add three tenths of the difference of their ſquares, which multiply by a third of the length, and the Product will be the Solidity.

If

If the middle and end Diameters of the middle frustum of a Parabolic Spindle be 40 and 30 Inches, and its length 60; what is its Solidity?

$40\times 40=1600$ $\qquad$ $1600-900=700$ *the diff. of the Squares.*

$30\times 30=\;\;900$ $\qquad$ $700\times,3=210=$ *three tenths of ditto, then,*

Sum $=2500$ $\qquad$ $2500+1250+210\times 20(=\frac{1}{3}$ *of* $60)=79200$ *Anf.*

Half Sum $=1250$

Art. 44. *To measure a Cylinderoid, or Prismoid.*

Definition. A Cylinderoid is a solid somewhat like the frustum of a cone, one base may be an ellipsis, and the other a disproportional ellipsis or circle.

A Prismoid is a solid somewhat like the frustum of a Pyramid, but its bases are disproportional.

Rule. The same as for the frustum of a cone or Pyramid— Or, to the areas of both bases add a mean area, that is, the square root of the product of the two bases, then multiply that sum by a third of the height or length, and it will give the solidity.

If the diameters of the greater base of a Cylinderoid be 30 and 20 inches, the diameter of the less base 12, and length 60 inches; what is the soldity?

$$\left.\begin{array}{r} 30\times 20=600 \\ 12\times 12=144 \\ \sqrt{144\times 600}=293,9 \\ \hline 1037,9 \end{array}\right\} 1037,9\times,7854\times 20(=60\div 3)=16033,3,\ \textit{Anf.}$$

If the diameters of the greater base of a Prismoid be 30 and 20 inches, the less base 20 by 10 inches, and length 30 inches; what is its solidity?

$$\left.\begin{array}{r} 30\times 20=600 \\ 20\times 10=200 \\ \sqrt{600\times 200}=346,4 \\ \hline 1146,4 \end{array}\right\} 1146,4\times 10(=30\div 3)=11464\ \textit{Solidity in inches.}$$

Art. 45. *To measure a Solid Ring.*

Rule. Measure the internal diameter of the Ring, and its girth, or circumference, then multiply the girth by ,31831 and the Product will be the Diameter of the wire, which add to the internal Diameter; multiply this sum by 3,1416 and the Product will be the length of a Cylinder equal to the Ring of the same Base. Then the area of a Section of the Ring multiplied by the length of the said Cylinder will give the Solidity of the Ring.

If

If an Iron Ring be 12 Inches in Girth, and its internal diameter be 20 Inches; what is its Solidity?

$,31831 \times 12 = 3,8 =$ Ring's Diameter. $\overline{20+3,8} \times 3,1416 = 74,76$ the length of a Cylinder equal to the Ring, And

$3,8 \times 3,8 \times ,7854 \times 74,76 = 847,86 =$ Solidity.

Art. 46. *To measure the Solidity of any irregular body, whose dimensions cannot be taken.*

Take any regular vessel, either square or round, and put the irregular body into it; Pour so much water into the vessel as will exactly cover the body, and measure the dry part from the Top of the vessel to the water;—then take out the body, and measure again from the Top of the vessel to the water, and subtract the first measure from the second, and the difference is the fall of the water; Then, if the vessel be square, multiply the Side by itself, and that product by the fall of the water, and you will have the Content of the Body; but if it be a long square, multiply the length by the breadth, and that product by the fall of the water;—Or, lastly, if it be a round vessel, multiply the square of the Diameter by ,7854, and that Product by the fall of the water, and you will have the Content.

Exam. 1. A Body being put into a vessel 18 inches square, on taking out the body, the water sunk 9 Inches, Required the Content of the Body?
18 *Inch.* $=1,5$ *foot.*
9 *Inch.* $=,75$ *foot.*
$1,5 \times 1,5 \times ,75 = 1,6875$ *foot, Content.*

Exam. 2. A Body put into a Cistern 4 feet by 3, on taking it out, the water fell 6 Inches; Required the Content of the Body?
$4 \times 3 \times ,5 = 6$ *feet, Content.*

Exam. 3. A body being put into a round Tub, whose Diameter was 1,5 foot, on taking out the Body, the water fell 1,5 foot; what was the Content of the Body?
$1,5 \times 1,5 \times ,7854 \times 1,5 = 2,65$ *feet, Cont.*

Of the five Regular Bodies.

There are five Solids contained under equal regular Sides, which, by way of distinction, are called *the five regular Bodies.*

These are the *Tetraedron*, the *Hexaedron* or *Cube*, the *Octaedron*, the *Dodecaedron*, and the *Eicosiedron*. The measuring of the Cube was shewn at Art. 28. I shall now shew how to measure the other four, by the following Table, which is the shortest method.

A Table of the solid and superficial Content of each of the five Bodies, the Sides being unity, or 1.

Names of the Bodies.	*Solidity.*	*Superficies.*
Tetraedron.	0.11785	1.73205
Hexaedron.	1.	6.
Octaedron.	0.4714	3.464
Eicosiedron.	2.181695	8.66025
Dodecaedron.	7.663119	20.6457

All like ſolid bodies being in proportion to one another as the cubes of their like ſides; the ſolid content of any of theſe bodies may be found by multiplying the cubes of their ſides by the numbers in the ſecond column under *Solidity*; and their Superficies, by multiplying the ſquares of their ſides into the numbers in the third column, under *Superficies*.

Of the Tetraedron.

This ſolid is contained under four equal and equilateral triangles, that is, it is a triangular Pyramid of four equal faces, the ſide of whoſe baſe is equal to the ſlant height of the Pyramid, from the angles to the vertex

Art. 47. The ſide of a Tetraedron being 3, to find the ſolid and ſuperficial content.

Cube $= 3 \times 3 \times 3 = 27$, *and* $27 \times ,11785 = 31,8195 =$ *Solidity*.
Square $= 3 \times 3 = 9$, *and* $9 \times 1,73205 = 15,58845 =$ *Superficies*.

Of the Octaedron.

This Solid is contained under eight equal and equilateral triangles, which may be conceived to conſiſt of two quadrangular pyramids of equal baſes joined together, the ſides of whoſe baſes are equal to the given ſides of the triangles, under which it is contained.

Art. 48. The ſide of an Octaedron being 3, to find the ſolid and ſuperficial content.

Cube $= 3 \times 3 \times 3 = 27$, *and* $27 \times ,4714 = 12,7278 =$ *Solidity*.
Square $= 3 \times 3 = 9$, *and* $9 \times 3,464 = 31,176 =$ *Superficies*.

Of the Dodecaedron.

This Solid is contained under twelve equilateral Pentagons, and may be conceived to conſiſt of twelve pentagonal pyramids, of equal baſes and altitude, whoſe vertices meet in the centre of the Dodecaedron.

Art. 49. The ſide of a Dodecaedron being 3, to find the ſolid and ſuperficial content.

Cube $= 3 \times 3 \times 3 = 27$, *and* $27 \times 7,663119 = 206,904$.
Square $= 3 \times 3 = 9$, *and* $9 \times 20,6457 = 185,8113$.

Of the Eicosiedron.

This Solid is contained under twenty equal and equilateral triangles, and may be conceived to conſiſt of twenty equal triangular pyramids, whoſe vertices all meet in the centre.

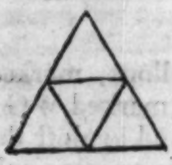
Art. 50. The ſide of an Eicoſiedron being 3, to find the ſolid and ſuperficial content.

Cube $= 3 \times 3 \times 3 = 27$, *and* $27 \times 2,18169 = 58,90563 =$ *Solidity*.
Square $= 3 \times 3 = 9$, *and* $9 \times 8,66025 = 77,94225 =$ *Superficies*.

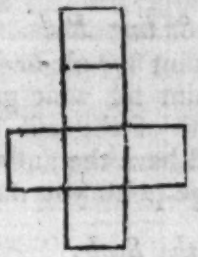
As the figures of ſome of theſe bodies would give but a confuſed idea of them, I have omitted them; but the following figures, cut out

out in pasteboard, and the lines cut half through, will fold up into the several bodies.

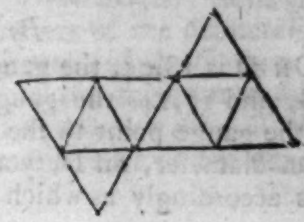
Tetraedron.



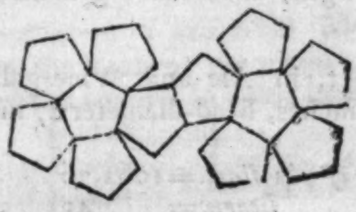
Hexaedron.



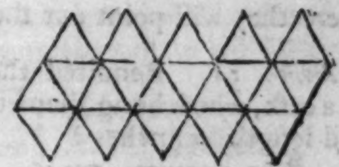
Octaedron.



Dodecaedron.



Eicosiedron.



Of CASK GAUGING.

AMONG the many different Canons, drawn from Stereometry, for gauging *Casks*, the following is as exact as any.

TAKE the dimensions of the cask in inches, viz. the diameter at the bung and head, and length of the cask: Subtract the head-diameter from the bung-diameter, and note the difference.

IF the staves of the cask be much curved or bulging *between the bung and the head*, multiply the difference by ,7; if not quite so curve, by ,65; if they bulge yet less, by ,6; and if they are almost or quite strait, by ,55, and add the product to the head-diameter; the sum will be a mean diameter, by which the cask is reduced to a cylinder.

SQUARE the mean diameter, thus found, then multiply it by the length; divide the product by 359 for ale or beer-gallons, and by 294 for wine-gallons.

Note 1. The length is most conveniently taken by Callipers, allowing, for the thickness of both heads, 1 inch, $1\frac{1}{2}$ inch, or 2 inches, according to the size of the cask: But if you have no callipers, do thus; measure the length of the stave; then take the depth of the chimes, which, with the thickness of the head, being subtracted from the length of the stave, leaves the length within.

Note 2. You must take the head-diameter, close to its outside, and, for small casks, add 3 tenths of an inch; for casks of 30, 40, or 50 gallons, 4 tenths, and for larger casks, 5 or 6 tenths, and the sum

ſum will be very nearly the head-diameter within. In taking the bung-diameter, obſerve, by moving the rod backward and forward, whether the ſtave, oppoſite the bung, be thicker or thinner than the reſt, and if it be, make allowance accordingly.

By the Sliding Rule.

On D is 18,94, the gauge-point for ale or beer-gallons, marked AG, and 17,14, the gauge-point for wine-gallons, marked WG: ſet the gauge-point to the length of the caſk on C, and againſt the mean-diameter, on D, you will have the anſwer in ale or wine gallons accordingly as which gauge-point you make uſe of.

By the Scale.

Take the extent from the gauge-point to the mean diameter, ſet one foot of the dividers in the length, and, turning them twice over, they will point out the content.

Art. 51. Required the content, in Ale and Wine-gallons, of a caſk, whoſe bung-diameter is 35 inches, head-diameter 27 inches, and length 45 inches?

$$
\begin{array}{rr}
\textit{Bung-diameter} = & 35 \\
\textit{Head-diameter} = & 27 \\
\hline
\textit{Difference} = & 8 \\
 & ,7 \\
\hline
 & 5,6 \\
\textit{Add the head-dia.} = & 27 \\
\hline
\textit{Mean-diameter} = & 32,6 \\
 & 32,6 \\
\hline
 & 1956 \\
 & 652 \\
 & 978 \\
\hline
\textit{Squared} & 1062,76
\end{array}
$$

$$
\begin{array}{rr}
\textit{Square of the diam.} = & 1062,76 \\
\textit{Length} = & 45 \\
\hline
 & 531380 \\
 & 425104 \\
\hline
 & 359)47824,20(133,21 \\
 & [\textit{Ale gall.} \\
 & 294)47824,2(159,26\ \textit{wine gall.}
\end{array}
$$

Art. 52. A round Maſh-Tub is 42 inches diameter at the top, within, and 36 inches at the bottom, and the perpendicular height 48 inches; Required the content in beer and wine gallons?

This being the lower fruſtum of a cone, to the product of the diameters add $\frac{1}{3}$ of the ſquare of their difference; multiply this ſum by the length, and it will give the ſolidity in ſuch parts as the dimenſions are taken in. If they be taken in inches, divide by 359 for beer, and 294 for wine gallons.

$$42 \times 36 + \frac{42-36 \times 42-36}{3} \times 48 \div \left\{ \begin{array}{l} 359 = 203\frac{3}{4}\ \textit{Ale Gallons.} \\ 294 = 248\frac{3}{4}\ \textit{Wine Gallons.} \end{array} \right.$$

Art.

ART. 53. Let the difference of diameters of this Tub be 6 inches, the height 48 inches, and the content 203¾ gallons, to find the diameters?

MULTIPLY the content, if beer-meaſure, by 359; if wine-meaſure, by 294, and divide the product by the length: from the quotient ſubtract $\frac{1}{3}$ of the ſquare of the difference of the diameters; to this remainder add the ſquare of $\frac{1}{2}$ the difference of the diameters, and extract the ſquare root of the ſum; from the ſquare root ſubtract $\frac{1}{2}$ the difference of the diameters, and it will give the leaſt diameter to great exactneſs, to which add the difference of the diameters, and the ſum is the greateſt diameter.

$\sqrt{\frac{203{,}75\times 359}{48} - \frac{6\times 6}{3} + 3\times 3} - 3 = 36$, *and* $36+6=42$. *The diameters are* 36 *and* 42.

THE content of any veſſel, in gallons, &c. may be thus found: Meaſure the inſide of the veſſel, according to the rule of the figure, and find the content in cubic inches, then,

Divide by	1728	*and the Quotient will be the Content in*	*Cubic Feet.*
	282		*Ale or Beer Gallons.*
	231		*Wine Gallons.*
	250,425		*Buſhels.*

ART. 54. *To ullage a Caſk, lying on one ſide, by the Gauging-Rod, when the Bung diameter, and the content, one, or both, are greater or leſs than the Table on the Rod is made for.*

RULE. As the Bung-diameter of the Caſk to be meaſured : is to the Bung-diameter that the table is made for :: ſo are the dry inches of the Caſk : to a *fourth* number, which find in the Table on the Rod, and note the number of gallons anſwering to it. Then, as the content of the Caſk that the table is made for : is to the content of the Caſk to be meaſured :: ſo is the number of gallons anſwering to the aforeſaid *fourth* number : to the number of gallons your Caſk wants of being full.

Art. 55. *To find a Ship's Burthen, Or to Gauge a ſhip.*

THERE is ſuch diverſity in the forms of Ships that no general Rule can be applied to anſwer all varieties; however, the following Rules are practiced.

RULE. 1. Multiply the breadth at the main beam, half the breadth, and length together: divide the product by 94, and the quotient is the Tuns.

RULE 2. Divide the continued product of the length, breadth and depth, in feet, by 100, for Ships of war, and 95 for Merchant-Ships, in which nothing is allowed for guns, &c: and the quotient is the Tons.

RULE

RULE 3. Take the length, from the Stern-post to the upper part of the Stem; subtract two thirds of her breadth from that length: multiply the remainder by the whole breadth, and that product by half the breadth, in feet, and divide by 100 for war, and 94 for Merchant-Tonnage.

RULE 4. The weight of a Ship's burthen is half the weight of Water she can hold.

What is the tonnage of a Ship, whose length is 97 feet, breadth 31 feet, and depth $15\frac{1}{2}$ feet?

By Rule 1st.

½ Breadth 15,5
Breadth 31
155
465
480,5
Length, 97
33635
43245
94)46608,5(495,83 *Tons.*
376
900
846
548
470
785
752
330
282
48

100)46608,5(466 *Tons.*
400
660
600
608
600
85

By Rule 2d.

Length 97
Breadth 31
97
291
3007
½ Breadth = 15,5
15035
15035
3007
95)46608,5(490,61 *Tons.*
380
860
855
585
570
150
95
55

By

By Rule 3.

Length$=97$

Subt. $\frac{2}{3}$ *of Breadth*$=20,66$

$$\begin{array}{r} 76,33 \\ \textit{Mult. by the breadth}\quad 31 \\ \hline 7633 \\ 22899 \\ \hline 3366,23 \\ \textit{Mult. by } \tfrac{1}{2} \textit{ breadth}\quad 15,5 \\ \hline 1683115 \\ 1683115 \\ 336623 \\ \hline \end{array}$$

94)52176,565(555,069 *Tons.*

The Proportions of Noah's Ark were as follow, viz.

	Feet.	
Length of the Keel,	300	Its Burthen as a Man of War
Breadth by the Mid-ship-beam	50	4500 Tons.
Depth in the Hold	30	As a Merchant-ship $4736\frac{80}{95}$ [Tons.

QUESTIONS IN MENSURATION.

1. THE largest of the Egyptian Pyramids is square at the Base, and measures 693 feet on a Side: How much ground does it cover?

$\frac{693 \times 693}{272,25} = 1764$ Poles, and $\frac{1764}{160} = 11$ Acres and 4 Poles, Ans.

2. WHAT difference is there between a floor 20 feet square, and two others, each 10 feet square?

$\overline{20 \times 20} - \overline{\overline{10 \times 10} + \overline{10 \times 10}} = 200$ *feet, Ans.*

3. THERE is a square of 2500 yards in area; what is each Side of the square, and the breadth of a walk along one Side and one end, which may take up just one half of the square?

$\sqrt{2500} = 50$ yards, each side. $\sqrt{\frac{2500}{2}} = 35,35$, and $50 - 35,35 = 14,65$ yards, breadth of the walk, Answer.

4. A Pine plank is 16 feet and 5 inches long, and I would have just a square yard slit off: At what distance from the edge must the line be drawn?

A square yard $= 1296$ Inches, and 16 feet 5 inches$=197$ inches.

Therefore, $\frac{1296}{197} = 6\frac{114}{197}$ Inches, Answer.

5. IF

5. If the Area of a Triangle be 900 yards, and the Perpendicular 40 yards; Required the length of the Base?

$$\frac{900\times2}{40}=45 \text{ yards, Anf.}$$

6. If the three Sides of a plain Triangle be 24, 16 and 12 perches; Required its Area?

$\frac{24+16+12}{2}=26$; $26-24=2$; $26-16=10$; $26-12=14$, and $\sqrt{26\times14\times10\times2}=85,32$ Perches, =Area. Again, As 24 : 16 + 12 :: 16—12 : 4,6 +, the difference of the Segments of the base; then, $12-\frac{4,6+}{2}=9,6$, and $\sqrt{12\times12-9,6\times9,6}=7,11$ the perpendicular on the longest Side; whence $24\div2\times7,11=85,32$, Area, as above.

7. Required the Area of a circular Garden, whose diameter is 12 Rods? $12\times12\times,7854=113,142$ *Poles, Anf.*

8. The wheel of a Perambulator turns just once and an half in a Rod; what is its diameter?

$16,5\times\frac{2}{3}=11$, *Circumf. and* $11\times,31831=3\frac{1}{2}$ *feet, Anf.*

9. Agreed for a Platform to the Curb of a round well, at $7\frac{1}{2}$*d.* per square foot; the inward part, round the mouth of the well, is 36 Inches diameter, and the breadth of the Platform was to be $15\frac{1}{2}$ Inches; what will it come to?

$\overline{36+15,5\times2}=67$ *the greatest diam.* $\overline{67\times67}\times,7854-\overline{36\times36}\times,7854$ $=\frac{2507,8722}{144}=17,4157$ *Square feet, at* $7\frac{1}{2}$*d per foot,* = 10*ſ*10 $\frac{6}{10}$ *Anf.*

10. Required the difference between the Area of a Circle, whose Radius (or semi-diameter) is 50 yards, and its greatest inscribed Square?

$50\times2=100$ *the Diameter, and* $100\times100\times,7254=7854$ *the Area of the Circle, then,* $50\times50\times2=5000$ *the Area of the greatest inscribed square, and* $7854-5000=2854$ *Anf.*

11. There is a Section of a Tree 25 Inches over; I demand the difference of the Areas of the inscribed and circumscribed squares, and how far they differ from the area of the section?

$\overline{25\times25}-\overline{12,5\times12,5\times2}=312,5$ *the difference of the squares.*

$\overline{25\times25}-\overline{25\times25\times,7854}=234,125$ *the circumscribed square, more than the section, and* $25\times25\times,7854-12,5\times12,5\times2=178,375$ *inscribed square, less than the Area of the Section.*

12. Four men bought a Grindstone of 60 inches diameter: How much of its diameter must each grind off, to have an equal share of the Stone, if one first grind his share, and then another, till the Stone is ground away, making no allowance for the Eye?

Rule.

Rule. Divide the ſquare of the diameter by the number of men, ſubtract the Quotient from the ſquare, and extract the ſquare root of the remainder, which is the length of the diameter after the firſt man has ground his ſhare; this work being repeated by ſubtracting the ſame quotient from the Remainder, for every man, to the laſt; extract the ſquare root of the remainders, and ſubtract thoſe roots from the Diameters, one after another; the ſeveral remainders will be the Anſwers.

60
60
4)3600
Quot. = 900
From 3600
Take 900
$\sqrt{2700} = 51{,}9615$, *to be taken from* 60.
Subt. 900
$\sqrt{1800}$
Subt. 900 = 42,4264, *from* 51,9615
$\sqrt{900} = 30$; *from* 42,4264.

From 60
Take 51,9615
Remains 8,0385 = *1ſt ſhare.*
From 51,9615
Take 42,4264
Rem. 9,5351 = *2d. ſhare.*
From 42,4264
Take 30
Rem. 12,4264 = *3d. ſhare.*
And 30 *Inches* = *4th. ſhare.*

13. If a cubic foot of Iron were hammered, or drawn, into a ſquare Bar, an Inch about, that is $\frac{1}{4}$ of an inch ſquare; Required its length, ſuppoſing there is no waſte of Metal?

$$\frac{12 \times 12 \times 12}{,25 \times ,25} = 27648 \text{ inches}, = 2304 \text{ feet, Anſ.}$$

14. Required the Axis of a Globe, whoſe Solidity may be juſt equal to the area of its Surface?

$$\frac{,7854 \times 4}{,5236} = 6 \text{ Inches; Anſ.}$$

15. A Joiſt is $7\frac{1}{2}$ inches wide, and $2\frac{1}{4}$ thick; but I want one juſt twice as large, which ſhall be $3\frac{3}{4}$ inches thick; what will be the breadth?

$$\frac{7{,}5 \times 2{,}25 \times 2}{3{,}75} = 9 \text{ Inches, Anſ.}$$

16. I have a ſquare ſtick of timber 18 inches by 14; but one of a third part of the Timber in it, provided it be 8 inches deep, will ſerve; How wide will it be?

$$\frac{18 \times 14}{3} \div 8 = 10\tfrac{1}{2} \text{ Inches, Anſ.}$$

17. A had a Beam of Oak-timber 18 inches ſquare throughout, and 25 feet long, which he bartered with B, for an equilateral tri-

angular Beam of the ſame length, each ſide 24 Inches; Required the balance, at 1/3 per foot?

$$\frac{18\times 18\times 25}{144} = 56,25 \text{ Solidity of the Square Beam.}$$

The Perp. let fall on one of the Sides of the triangular Beam is 21 *inches, and the half-perp.* = 10,5, *then,* $\frac{10,5\times 24}{144} = 1,75$ *foot, Area at the end, and* $1,75\times 25=43,75$ *feet, Solidity of the triangular Beam; therefore* $56,25-43,75=12,4$ *feet, at* 1/4 *per foot*$=16/6\frac{4}{10}$ *balance due to A, Anſ.*

18. What is the difference between a ſolid half-foot, and half a foot ſolid?

$$\frac{12\times 12\times 6}{6\times 6\times 6} = 4, \text{ therefore, one is but } \tfrac{1}{4} \text{ of the other.}$$

19. A lent B a ſolid Stack of Hay, meaſuring 20 feet every way; ſometime afterward, B returned a quantity, meaſuring every way 10 feet; what proportion of the Hay remains due?

$$20\times 20\times 20-10\times 10\times 10 = 7000 \text{ feet } = \tfrac{7}{8} \text{ Anſ.}$$

20. A ſhip's hold is $75\frac{1}{2}$ feet long, $18\frac{1}{2}$ wide, and $7\frac{1}{4}$ deep; How many Bales of Goods $3\frac{1}{2}$ feet long, $2\frac{1}{4}$ deep, and $2\frac{3}{4}$ wide may be ſtowed therein, leaving a gang-way the whole length, of $3\frac{1}{4}$ feet wide?

$$\frac{75,5\times 18,5\times 7,25-75,5\times 7,25\times 3,25}{3,5\times 2,25\times 2,75}=385,44 \text{ Bales, Anſ.}$$

21. If a Stick of Timber be $20\frac{1}{2}$ feet long, 16 inches broad, and 8 inches thick, and $3\frac{1}{2}$ ſolid feet be ſawed off one end; How long will the Stick then be?

$$20\tfrac{1}{2}-\frac{1728\times 3,5}{16\times 8}=16 \text{ feet, } 6\tfrac{3}{4} \text{ Inches, Anſ.}$$

22. The ſolid content of a ſquare Stone is found to be $136\frac{1}{2}$ feet; its length is $9\frac{1}{2}$ feet; what is the Area of one end; and if the breadth be 3 feet 11 inches; what is the depth?

$$\frac{136,5\times 1728}{9,5\times 12}=\text{Area } 2069,0526 \text{ inches, and } \frac{2069,0526}{47} = 44,022 \text{ inches, Anſwer.}$$

23. I would have a cubic Box made capable of receiving juſt 50 Buſhels, the Buſhel containing 2150,425 ſolid Inches; what will be the length of the Side?

$$\sqrt[3]{2150,4\times 50}=47,55 \text{ Inches.}$$

24. A Statute Buſhel is to be made 8 Inches high, and $18\frac{1}{2}$ Inches diameter, to contain 2176 cubic Inches; (though the content of the Dimenſions is but 2150,425 inches) I demand what the diameter of the Buſhel muſt be, the height being 8 inches; and what the

the height, the Diameter being $18\frac{1}{2}$ Inches, to contain 2176 cubic Inches?

$$\text{Height}=8)\overset{\textit{Solidity}}{2176} \quad \textit{and} \quad \sqrt{272\times1,273}=18,6 \textit{ Diameter.} \quad 18,5\times18,5\times$$

$$\textit{Area}=272 \quad ,7854=268,80315=\textit{Area, and the Solidity } 2176 \div 268,8=8,0956 \textit{ Inches, height.}$$

25. There is a Garden-rolling-ſtone 66 Inches in Circumference, and $3\frac{1}{2}$ cubic feet are to be cut off from one end, perpendicular to the axis: where muſt the Section be made?

$$\frac{1728\times3,5}{\textit{Area}=412,5}=14,65 \textit{ Inches from one end, Anſ.}$$

26. I would have a Syringe of $1\frac{1}{2}$ Inch Diameter in the bore, to hold a quart, wine meaſure; what muſt be the length of the Piſton, ſufficient to make an Injection with?

$\overline{1,5}|^3=2,650725$, *and* $231\div4$ *cubic Inches*$=57,75$ *the cubic Inches in a quart, then,* $\frac{57,75}{2,507\ 5}=23,033$ *Inches, Anſ.*

27. If a round Pillar, 9 Inches diameter, contain 5 feet; of what diameter is that column, of equal length, which meaſures 10 times as much?

As $5 : 9\times9 :: 5\times10 : 810$, *and* $\sqrt{810}=28,46$ *inches, Anſ.*

28. There is a ſquare Pyramid, each ſide of whoſe Baſe is 30 inches, and whoſe perpendicular height is 120 inches, to be divided by Sections parallel to its Baſe into 3 equal parts; Required the Perpendicular height of each part?

$30\times30\times40=36000$ *the Solidity in Inches, now* $\frac{2}{3}$ *thereof is* 24000, *and* $\frac{1}{3}$ *is* 12000.

Therefore, As $36000 : 120\times120\times120 :: \begin{cases}24000\\12000\end{cases} : \begin{cases}1152000\\576000\end{cases}$ *Then,*

$\sqrt[3]{1152000}=104,8$. *Alſo,* $\sqrt[3]{576000}=83,2$. *Then* $120-104,8=15,2$ *length of the thickeſt part, and* $104,8-83,2=21,6$ *length of the middle part, conſequently* 83,2 *is the length of the top-part.*

29. Suppose the diameter of the Baſe of a conical ingot of gold to be 3 inches, and its height 9 inches; what length of wire may be expected from it, without loſs of metal, the Diameter of the wire being one hundredth part of an inch?

$3\times3\times,7854\times3=21,2058$ *the Solidity of the Cone.*

$$\frac{21,2058}{,01\times,01\times,7854}=270000 \textit{ inches}=4 \textit{ miles and } 460 \textit{ yards, Anſ.}$$

30. Suppose a Pole to ſtand on an horizontal plane 75 feet in height above the Surface: At what height from the Ground muſt it be cut off, ſo as that the top of it may fall on a point 55 feet from the bottom of the Pole, the end, where it was cut off, reſting on the ſtump or upright part?

As

As the whole length of the Pole is equal to the Sum of the Hypothenuse and Perpendicular of a Triangle, (the 55 feet on the Ground being the Base) this, as well as the following Question, may be solved by this

RULE. From the Square of the length of the Pole (that is, of the Sum of the Hypothenuse and Perpendicular) take the Square of the Base; divide the Base by twice the length of the Pole, and the Quotient will be the Perpendicular, or height at which it must be cut off.

$$\frac{\overline{75\times75}-\overline{55\times55}}{75\times2}=17\tfrac{1}{3}\ \textit{Feet, Ans.}$$

31. SUPPOSE a Ship sails from Lat. 43°, North, between North and East, till her departure from the Meridian be 45 Leagues, and the Sum of her distance and difference of Latitude to be 135 Leagues; I demand her distance sailed, and Latitude come to?

$$\frac{\overline{135\times135}-\overline{45\times45}}{135\times2}=60\ \textit{Leagues, and}\ 60\times3=180\ \textit{miles}=3\ \textit{degrees}$$

the difference of Latitude, 135—60=75 *Leagues the Distance.—Now, As the Vessel is sailing from the Equator, and consequently the Latitude is increasing. Therefore,*

To the Latitude sailed from	43°,00′N
Add the Difference of Latitude	3 ,00
And the Sum is the Lat. come to	=46 ,00N.

A N

AN *INTRODUCTION*

TO

A L G E B R A,

DESIGNED FOR THE USE OF

A C A D E M I E S.

DEFINITIONS.

ALGEBRA is the art of computing by Symbols.

1. *Like quantities* are thoſe which conſiſt of the ſame Letters.
2. *Unlike quantities* are thoſe which conſiſt of different Letters.
3. *Given Quantities* are thoſe whoſe values are known.
4. *Unknown quantities* are thoſe whoſe values are unknown.
5. *Simple quantities* are thoſe which conſiſt of one term only.
6. *Compound quantities* are thoſe which conſiſt of ſeveral terms.
7. *Poſitive* or *affirmative quantities* are thoſe to be added.
8. *Negative quantities* are thoſe to be ſubtracted.
9. *Like ſigns* are all + or all —.
10. *Unlike ſigns* are + and —.
11. *The co-efficient* of any quantity is the number prefixed to it.
12. *A binomial quantity* is one conſiſting of two terms; a *trinomial*, of three terms; and a *quadrinomial*, of four terms, &c.
13. *A reſidual quantity* is a binomial, where one of the terms is a negative.

IN the computation of Problems, the firſt letters of the Alphabet are put for known quantities, and the laſt letters for thoſe which are unknown.

AXIOMS.

1. IF equal quantities be added to, ſubtracted from, multiplied or divided by, equal quantities, the wholes, remainders, products and quotients will be reſpectively equal.
2. THE equal powers or roots of equal quantities are equal.
3. Two quantities reſpectively equal to a third, are equal to each other.
4. THE whole is equal to all its parts taken together.

ADDITION.

CASE I. *To add quantities which are alike, and have like ſigns.**

RULE. Add all the co-efficients together, and to their ſum adjoin the Letters common to each term, prefixing the common ſign.

* WHEN a leading quantity has no ſign before it, + is always underſtood; and a quantity, without any co-efficient prefixed to it, is ſuppoſed to have unity, or 1.

$$\begin{array}{ccccc}
5a & -6bx & 8bxy & 5x^2+xy & 7ax-y \\
7a & -3bx & 7bxy & 3x^2+2xy & 8ax-3y \\
8a & -2bx & 3bxy & x^2+3xy & 6ax-2y \\
10a & -7bx & 4bxy & 7x^2+8xy & 4ax-3y \\
2a & -bx & 5bxy & x^2+xy & ax-y \\
a & -5bx & bxy & 2x^2+3xy & 3ax-2y \\
\hline
33a & -24bx & 28bxy & 19x^2+18xy & 29ax-12y \\
\hline
\end{array}$$

CASE 2. *To add quantities which are like, but have unlike signs.*

RULE. 1. Add all the affirmative co-efficients into one sum, and all the negative ones into another.

2. SUBTRACT the least sum from the greatest, and to the difference prefix the sign of the greatest, with the common quantity.

$$\begin{array}{ccccc}
-3a & +8ax^2 & +6x^{\frac{1}{3}}+8y & -2xy+8 & +8x^2-y+3\sqrt{x} \\
+7a & +7ax^2 & -3x^{\frac{1}{3}}+7y & -3xy+7 & -10x^2-3y+2\sqrt{x} \\
+8a & -3ax^2 & -13x^{\frac{1}{3}}+8y & +xy-10 & -4x^2-2y+\sqrt{x} \\
-a & -4ax^2 & +2x^{\frac{1}{3}}-3y & +5xy-7 & +9x^2+6y-10\sqrt{x} \\
-2a & +4ax^2 & +x^{\frac{1}{3}}-y & -xy+2 & +x^2 * -\sqrt{x} \\
\hline
+9a & +12ax^2 & -7x^{\frac{1}{3}}+19y & * & +4x^2 * -5\sqrt{x} \\
\hline
\end{array}$$

CASE 3. *To add quantities which are unlike, and have unlike signs.*

RULE. Collect the like quantities together by the last rule, and set down those which are unlike, one after another, with their proper signs.

$$\begin{array}{ccc}
2x & 2x-x^2 & 12ax-{}^2-6+\sqrt{ax-x^2} \\
3y & 3a-x & -6ax+x^2-x+10 \\
-a & 2ax+6x^2 & 3y-ax-4-2\sqrt{ax-x^2} \\
x^2 & 3\sqrt{x}-2ax & \\
\hline
2x+3y-a+x^2 & x+5x^2+3a+3\sqrt{x} & 5ax-\sqrt{ax-x^2}-x+3y \\
\hline
\end{array}$$

SUBTRACTION.

RULE. Change the signs of all the quantities to be subtracted and then add them together, as in Addition.

$$\begin{array}{cccc}
3a^2-2b & 6x^2-8y+2 & 35xy-2+8x-y^{\frac{1}{2}} & 8ax-2\sqrt{xy}-10 \\
2a^2-3b & x^2+9y-20 & 24xy-8-8x-3y & 10x-6\sqrt{xy}-ax \\
\hline
a^2+b & 5x^2-17y+22 & 11xy+6+16x-y^{\frac{1}{2}}+3y & 9ax+4\sqrt{xy}-10-10x \\
\hline
\end{array}$$

MULTIPLI-

MULTIPLICATION.

CASE 1. *To multiply simple quantities.*

RULE. Multiply the co-efficients of the two terms together, and to the product annex all the letters in those terms.

Note. Like signs produce $+$, and unlike signs $-$.

$$\begin{array}{ccccccc}
2a & -2a & 5a & -9x & 6a^2x & -x\ y & -7xy \\
3b & +4b & -6x & -5b & 5x & xy^2 & -xy \\
\hline
6ab & -8ab & -30ax & +45bx & 30a^2x^2 & -x^3y^3 & +7x^2y^2 \\
\hline
\end{array}$$

CASE 2. *When one of the factors is a compound quantity.*

RULE. Find the products of the multiplier and every particular term of the multiplicand separately, and place them one after another with their proper signs.

$$\begin{array}{cccc}
4a-2b & 6xy-8 & 8a^2-2x+6 & 3y-8+2xy \\
3a & 2x & 3xy & xy \\
\hline
12a^2-6ab & 12x\ y-16x & 24a^2xy-6x^2y+18xy & 3xy^2-8xy+2x^2y^2 \\
\hline
\end{array}$$

CASE 3. *When both the factors are compound quantities.*

RULE. Multiply every particular term of the multiplier into every term of the multiplicand respectively, and set down the products one after another with their proper signs, and their sum will be the whole product.

$$\begin{array}{lll}
x+y & x-y & 3x^2-2xy+5 \\
x+y & x-y & \quad x^2+2xy-3 \\
\hline
x^2+xy & x^2-xy & 3x^4-2x^3y+5x^2 \\
\quad +xy+y^2 & \quad -xy+y^2 & \quad +6x^3y-4x^2y^2+10xy \\
 & & \qquad -9x^2+6xy-15 \\
\hline
x^2+2xy+y^2 & x^2-2xy+y^2 & 3x^4+4x^3y-4x^2-4x^2y^2+16xy-15 \\
\hline
\end{array}$$

WHEN two surd numbers are to be multiplied together, multiply them without any regard to the radical sign, and prefix the radical sign to the product. Thus, $\sqrt{3}\times\sqrt{2}=\sqrt{6}$; $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$, &c.

DIVISION.

CASE 1. *When the divisor is a simple quantity.*

RULE. 1. Place the dividend above a line, and the divisor under it, like a vulgar fraction.

2. EXPUNGE

2. Expunge those letters which are common to both the factors, and divide the co-efficients of all the terms by any number which will divide them without a remainder.

Note. Like signs make $+$, and unlike signs $-$, as in multiplication.

$$\frac{a}{a}=1;\ \frac{8bc}{2b}=4c;\ \frac{abc}{bcd}=\frac{a}{d};\ \frac{10ab+15ac}{20ad}=\frac{2b+3c}{4d};\ \frac{ab+b^2}{2b}=\frac{a+b}{2};\ \frac{12xy}{6x^2}=\frac{2y}{x};\ \frac{30ax-54ay}{12ab}=\frac{5x-9y}{2b};\ \frac{10x^2y-15y^2-5y}{5y}=2x^2-3y-1;\ \frac{3a^2-15+6a+3b}{3a}=a-\frac{5}{a}+2+\frac{b}{a}.$$

Case 2. *When the divisor and dividend are both compound quantities.*

Rule. 1. Range the terms of both the quantities according to the dimensions of some letter in them, so that the first term may have the highest power of that letter, and the second term the next highest power; and so on.

2. Divide the first term of the dividend by the first term of the divisor, and place the result in the quotient.

3. Multiply the whole divisor by the quotient-term last found, and subtract the result from the dividend.

4. To this remainder bring down the next term of the dividend, and divide as before, and so on, as in common Arithmetic.

$$\begin{array}{l}
a+x)\,a^3+5a^2x+5ax^2+x^3\,(a^2+4ax+x^2 \\
\quad a^3+a^2x \\
\hline
\qquad 4a^2x+5ax^2 \\
\qquad 4a^2x+4ax^2 \\
\hline
\qquad\qquad ax^2+x^3 \\
\qquad\qquad ax^2+x^3 \\
\hline
\qquad\qquad\quad *
\end{array}$$

$$\begin{array}{l}
x-3)\,x^3-9x^2+27x-27\,(x^2-6x+9 \\
\quad x^3-3x^2 \\
\hline
\quad -6x^2+27x \\
\quad -6x^2+18x \\
\hline
\qquad\quad 9x-27 \\
\qquad\quad 9x-27 \\
\hline
\qquad\qquad *
\end{array}
\qquad
\begin{array}{l}
a-x)\,a^3-x^3\,(a^2+ax+x^2 \\
\quad a^3-a^2x \\
\hline
\qquad a^2x-x^3 \\
\qquad a^2x-ax^2 \\
\hline
\qquad\quad ax^2-x^3 \\
\qquad\quad ax^2-x^3 \\
\hline
\qquad\qquad *
\end{array}$$

ALGEBRAIC FRACTIONS.

PROBLEM I. *To reduce a mixed quantity to an improper fraction.*

RULE. Multiply the integer by the denominator of the fraction, and to the product add the numerator, and the denominator being placed under this sum will give the improper fraction required.

$$x+\frac{x^2}{a}=\frac{ax+x^2}{a};\quad a-\frac{b}{c}=\frac{ac-b}{c};\quad 1-\frac{2x}{a}=\frac{a-2x}{a};$$

$$a-x+\frac{a^2-ax}{x}=\frac{a^2-x^2}{x}.$$

PROB. 2. *To reduce an improper fraction to a whole, or mixed quantity.*

RULE. Divide the numerator by the denominator, for the integral part, and place the remainder over the denominator, for the fractional part.

$$\frac{a^2+a^2}{x}=a+\frac{a^2}{x};\quad \frac{ay+2y^2}{a+y}=y+\frac{y^2}{a+y};\quad \frac{ab-a^2}{b}=a-\frac{a^2}{b};$$

$$\frac{a^2+x^2}{a-x}=a+x+\frac{x^2}{a-x}.$$

PROB. 3. *To reduce fractions of different denominators, to those of the same value, which shall have a common denominator.*

RULE. Multiply every numerator separately into all the denominators but its own, for new numerators, and all the denominators together, for a common denominator.

1. REDUCE $\frac{a}{b}$ and $\frac{b}{c}$ to fractions of equal values, having a common denominator.

$a\times c=ac$ *New Numer.*
$b\times b=b^2$ *New Numer.* $\frac{ac}{bc}$ *and* $\frac{b^2}{bc}$ = *fractions required.*

$b\times c=bc$ *Common Denominator.*

2. REDUCE $\frac{a}{b}$, $\frac{b}{c}$ & $\frac{c}{d}$ to equivalent fractions, having a common denominator.

$a\times c\times d=acd$
$b\times b\times d=b^2d$ $\frac{acd}{bcd}$, $\frac{b^2d}{bcd}$ and $\frac{c^2d}{bcd}$, *Ans.*
$c\times b\times c=c^2b$

$b\times c\times d=bcd$

PROB. 4. *To find the greatest common measure of a fraction.*

RULE. 1. Range the quantities according to the dimensions of some letter, as was shewn in division.

2. DIVIDE the greater term by the leſs, and the laſt diviſor by the laſt remainder, and ſo on, 'till nothing remain, and the diviſor laſt uſed, will be the common meaſure required.

Note. All the letters or figures, which are common to the diviſor, and dividend, muſt be cancelled in the diviſor before they be uſed in the operation.

1. To find the greateſt common meaſure of $\frac{cx+x^2}{ca^2+a^2x}$

$$\ddagger\ cx+x^2)ca^2+a^2x$$
$$\text{or } c+x)ca^2+a^2x(a^2$$
$$ca^2+a^2x$$
$$*$$

Therefore the greateſt common meaſure is $c+x$.

2. To find the greateſt common meaſure of $\frac{b^3-b^2x}{x^2+2bx+b}$

$$x+2bx+b^2)x^3-b^2x(x$$
$$x^3+2bx^2+b^2x$$
$$*\ -2bx^2-2b^2x)x^2+2bx+b^2$$
$$\text{or } x+b)x^2+2bx+b^2(x+b$$
$$x^2+bx$$
$$bx+b^2$$
$$bx+b^2$$
$$*$$

Therefore $x+b$ *is the greateſt common Diviſor.*

PROB. 5. *To reduce a fraction to its loweſt Terms.*

RULE. 1. Find the greateſt common meaſure, as in the laſt Problem.

2. Divide both the terms of the fraction by the common meaſure thus found.

1. REDUCE

‡ HERE I find that x is common to both diviſor and dividend, I therefore cancel x in the diviſor, that is, I divide $cx+x^2$ by x, and $c+x$ is the quotient: Thus,

$$x)cx+x^2(c+x, \textit{ for the diviſor.}$$
$$cx$$
$$+x^2$$
$$+x^2$$
$$*$$

* HERE $-2bx$ is common to the diviſor and dividend; I therefore firſt divide $-2bx^2-2b^2x$ by x, and the quotient is $-2bx-2b^2$, thus,

$$x)-2bx^2-2b^2x(-2bx-2b^2$$
$$-2bx^2$$
$$-2b^2x$$
$$-2b^2x$$
$$*$$

I then divide $-2bx-2b^2$ by $-2b$, and the quotient is $x+b$, thus,

$$x)-2bx-2b^2(x+b, \textit{ for the diviſor.}$$
$$-2bx$$
$$-2b^2$$
$$-2b^2$$
$$*$$

1. REDUCE $\dfrac{cx+x^2}{ca^2+a^2x}$ to its loweſt terms.

$$\begin{array}{l} cx+x^2)\,ca^2+a^2x \\ \textit{or}\ \ c+x\ )\,ca^2+a^2x\,(a^2 \\ \qquad\qquad ca^2+a^2x \\ \hline \qquad\qquad\quad * \end{array}$$

Therefore $c+x$ is the greateſt common meaſure,

and $c+x\Big)\dfrac{cx+x^2}{ca^2+a^2x}\Big(\dfrac{x}{a^2}=$ *fraction required.*

2. REDUCE $\dfrac{x^3-b^2x}{x^2+2bx+b^2}$ to its loweſt terms.

$$\begin{array}{l} x^2+2bx+b^2)\,x^3-b^2x\,(x \\ \qquad\qquad\quad x^3+2bx^2+b^2x \\ \hline \qquad\quad -2bx^2-2b^2x)\,x^2+2bx+b^2 \\ \qquad\textit{or}\quad x+b\ )\,x^2+2bx+b^2\,(x+b \\ \qquad\qquad\qquad\quad x^2+bx \\ \hline \qquad\qquad\qquad\qquad bx+b^2 \\ \qquad\qquad\qquad\qquad bx+b^2 \\ \hline \qquad\qquad\qquad\qquad\quad * \end{array}$$

Therefore $x+b$ is the greateſt common meaſure, and

$x+b\Big)\dfrac{x^3-b^2x}{x^2+2bx+b^2}\Big(\dfrac{x^2-bx}{x+b}=$ *fraction required.*

PROB. 6. *To add fractional quantities together.*

RULE. 1. Reduce the fractions to a common Denominator.

2. ADD all the numerators together, and under their ſum write the common denominator.

1. ADD $\dfrac{x}{2}$ and $\dfrac{x}{3}$.

$$\begin{array}{l} x\times3=3x \\ x\times2=2x \\ \hline 2\times3=6 \end{array}$$

$$\frac{3x}{6}+\frac{2x}{6}=\frac{5x}{6}=\textit{Sum.}$$

2. ADD $\dfrac{a}{b}$, $\dfrac{c}{d}$ and $\dfrac{e}{f}$.

$$\begin{array}{l} a\times d\times f=adf \\ c\times b\times f=cbf \\ e\times b\times d=ebd \\ \hline b\times d\times f=bdf \end{array} \qquad \frac{adf}{bdf}+\frac{cbf}{bdf}+\frac{ebd}{bdf}=\frac{adf+cbf+ebd}{bdf}=\textit{Sum.}$$

3. ADD

3. ADD $a-\frac{3x^2}{b}$ and $b+\frac{x-2b}{c}$.

$$3x^2\times c=3cx^2$$
$$\overline{x-2b}\times b=bx-2b^2$$
$$b\times c=bc$$

$$
\begin{array}{l}
a-\frac{3cx^2}{bc} \\
b+\frac{b-2b^2}{bc} \\
\hline
a+b+\frac{bx-3cx^2-2b^2}{bc}=\textit{Sum.}
\end{array}
$$

PROB. 7. *To subtract one fractional quantity from another.*

RULE. 1. Reduce the fractions to a common denominator.

2. SUBTRACT the numerators, and under their difference write the common denominator.

1. REQUIRED the difference of $\frac{x}{3}$ and $\frac{2x}{11}$?

$$
\begin{array}{r}
x\times 11=11x \\
2x\times 3=6x \\
\hline
3\times 11=33
\end{array}
\qquad \frac{11x}{33}-\frac{6x}{33}=\frac{5x}{33}=\textit{Difference.}
$$

2. WHAT is the difference of $\frac{x-a}{3b}$ and $\frac{2a-4x}{5c}$?

$$\overline{x-a}\times 5c=5cx-5ac$$
$$\overline{2a-4x}\times 3b=6ab-12bx$$
$$3b\times 5c=15bc$$

$$\frac{5cx-5ac}{15bc}-\frac{6ab-12bx}{15bc}=\frac{5cx-5ac-6ab+12bx}{15bc}=\textit{difference.}$$

PROB. 8. *To multiply fractional quantities.*

RULE. Multiply the numerators together for a new numerator, and the denominators, for a new denominator.

1. MULTIPLY $\frac{x}{6}$ and $\frac{2x}{9}$ together.

$$\left.\begin{array}{l} x\times 2x \\ 6\times 9 \end{array}\right\}=\frac{2x^2}{54}=\frac{x^2}{27}=\textit{Product.}$$

2. FIND the product of $\frac{x}{2}$, $\frac{4x}{5}$ *and* $\frac{10x}{21}$.

$$\left.\begin{array}{l} x\times 4x\times 10x \\ 2\times 5\times 21 \end{array}\right\}=\frac{40x^3}{210}=\frac{4x^3}{21}=\textit{Product.}$$

3. FIND

3. FIND the product of $\frac{x}{a}$ *and* $\frac{x+a}{a+c}$.

$$\left.\begin{matrix} x\times\overline{x+a} \\ a\times\overline{a+c}\end{matrix}\right\}=\frac{x^2+ax}{a^2+ac}=\textit{Product.}$$

PROB. 9. *To divide one fractional quantity by another.*

RULE. Invert the divisor, and proceed as in multiplication.

1. DIVIDE $\frac{x}{3}$ by $\frac{2x}{9}$. $\quad \frac{x}{3}\times\frac{9}{2x}=\frac{9x}{6x}=\frac{3}{2}=1\frac{1}{2}=$ *quot.*

2. DIVIDE $\frac{2a}{b}$ by $\frac{4c}{d}$. $\quad \frac{2a}{b}\times\frac{d}{4c}=\frac{2ad}{4bc}=\frac{ad}{2bc}=$ *quotient.*

3. DIVIDE $\frac{x+a}{2x-2b}$ by $\frac{x+b}{5x+a}$.

$$\frac{x+a}{2x-2b}\times\frac{5x+a}{x+b}=\frac{5x^2+6ax+a^2}{2x^2-2b^2}=\textit{quotient.}$$

INVOLUTION.

INVOLUTION is the raising of Powers from any proposed root; or the method of finding the square, cube, biquadrate, &c. of any given quantity.

RULE. Multiply the quantity into itself as often as is denoted by the Index, and the last product will be the power required. *Or,*

MULTIPLY the Index of the quantity by the index of the Power, and the result will be the same as before.

Note. When the sign of the root is +, all the powers of it will be +; and when the sign is —, all the odd powers will be —, and all the even powers +.

$$\textit{Root}=a\begin{cases}a^2=\textit{square.}\\ a^3=\textit{cube.}\\ a^4=\textit{4th power.}\\ a^5=\textit{5th power.}\end{cases}\qquad \textit{Root}=a^2\begin{cases}a^4=\textit{square.}\\ a^6=\textit{cube.}\\ a^8=\textit{4th power.}\\ a^{10}=\textit{5th power.}\end{cases}$$

$$\textit{Root}=-3a\begin{cases}+\ 9a^2=\textit{square.}\\ -\ 27a^3=\textit{cube.}\\ +\ 81a^4=\textit{4th power.}\\ -243a^5=\textit{5th power.}\end{cases}$$

$$\textit{Root}=-2ax^2\begin{cases}+\ 4a^2x^4=\textit{square.}\\ -\ 8a^3x^6=\textit{cube.}\\ +16a^4x^8=\textit{4th pow.}\\ -32a^5x^{10}=\textit{5th pow.}\end{cases}\qquad \textit{Root}=\frac{x}{a}\begin{cases}\frac{x^2}{a^2}=\textit{square.}\\ \frac{x^3}{a^3}=\textit{cube.}\\ \frac{x^4}{a^4}=\textit{biquadrate.}\end{cases}$$

Root =

$$Root = -\frac{2ax^2}{3b}\begin{cases} +\dfrac{4a^2x^4}{9b^2} = \textit{square.} \\ -\dfrac{8a^3x^6}{27b^3} = \textit{cube.} \\ +\dfrac{16a^4x^8}{81b^4} = \textit{biquadrate.} \end{cases}$$

$$\begin{array}{l} x+a \\ x+a \\ \hline x^2+ax \\ \quad +ax+a^2 \\ \hline x^2+2ax+a^2 = \textit{Square.} \\ x\ +a \\ \hline x^3+2ax^2+a^2x \\ \quad +\ ax^2+2a^2x+a^3 \\ \hline x^3+3ax^2+3a^2x+a^3 = \textit{Cube.} \\ x\ +\ a \\ \hline x^4+3ax^3+3a^2x^2+\ a^3x \\ \quad +\ ax^3+3a^2x^2+3a^3x+a^4 \\ \hline x^4+4ax^3+6a^2x^2+4a^3x+a^4 = \textit{Biquadrate.} \\ \hline \end{array}$$

Of the Compoſition and Reſolution of a Square raiſed from a Binomial Root.

A Binomial is a quantity conſiſting of two parts or members, connected together by the ſign + or —, as $x+a$, $x-a$, $x+\frac{b}{2}$, $x-\frac{b}{2}$, and a ſquare raiſed from a Binomial root is nothing elſe but the ſquare of ſuch a quantity: thus, the ſquare of $x+\frac{b}{2}$ is $x^2+bx+\frac{b^2}{4}$, and that of $x-\frac{b}{2}$ is $x^2-bx+\frac{b^2}{4}$.

$x+\frac{b}{2}$

$$x+\frac{b}{2}$$
$$x+\frac{b}{2}$$
$$x^2+\frac{bx}{2}+\frac{b^2}{4}$$
$$+\frac{bx}{2}$$
$$x^2+\frac{2bx}{2}+\frac{b^2}{4}=x^2+bx+\frac{b^2}{4}$$

$$x-\frac{b}{2}$$
$$x-\frac{b}{2}$$
$$x^2-\frac{bx}{2}+\frac{b^2}{4}$$
$$-\frac{bx}{2}$$
$$x^2-\frac{2bx}{2}+\frac{b^2}{4}=x^2-bx+\frac{b^2}{4}$$

THE difference between theſe two ſquares ariſes from the different ſign of b, and that only affects the ſecond member; for the third member $\frac{bb}{4}$ will be the ſame, whether the quantity b be affirmative or negative; therefore if thoſe caſes be thrown into one, it will ſtand thus: *The ſquare of* $x\pm\frac{b}{2}$; *viz.* $+bx$ *when the root is* $x+\frac{b}{2}$ and $-bx$ *when the root is* $x-\frac{b}{2}$. Now, of the three members, which compoſe this ſquare, the firſt, x^2 is the ſquare of x, the ſecond, $\pm bx$ is the root of that ſquare multiplied into the co-efficient $\pm b$, and the third member, $\frac{b^2}{4}$ is the ſquare of $\pm\frac{b}{2}$, that is, the ſquare of half the co-efficient of the ſecond member; whence may be deduced the following obſervations.

1. *Any quantity conſiſting of two members, as* $x^2\pm bx$, *whereof one, as* x^2 *is a ſquare, and the other* $\pm bx$ *is the root of that ſquare multiplied into ſome given co-efficient* $\pm b$, *it may be conſidered as an imperfect ſquare raiſed from a Binomial root, and may eaſily be completed by adding* $\frac{b^2}{4}$, *that is, by adding the ſquare of half the co-efficient of* x *in the ſecond term*; thus x^2+6x, when completed, is x^2+6x+9; x^2+3x becomes $x^2+3x+\frac{9}{4}$, becauſe half the co-efficient 3 is $\frac{3}{2}$. Again, $x^2+\frac{2x}{3}$ becomes $x^2+\frac{2x}{3}+\frac{1}{9}$, becauſe half the co-efficient is $\frac{1}{3}$, the ſquare of which is $\frac{1}{9}$: Laſtly, $x^2-\frac{bx}{a}$ becomes $x^2-\frac{bx}{a}+\frac{b^2}{4a^2}$: for the co-efficient is $-\frac{b}{a}$, its half $-\frac{b}{2a}$, & the ſquare $\frac{b^2}{4a^2}$.

2. *The*

2. *The Root of such a square, when completed, that is, the root of* $x^2 \pm bx + \frac{b^2}{4}$ *will always be* $x \pm \frac{b}{2}$, *that is, it will always be the square root of the first, together with half the co-efficient of the second:* thus the square root of x^2+6x+9 will be $x+3$, that of $x^2+3x+\frac{9}{4}$ will be $x+\frac{3}{2}$, that of $x^2+\frac{2x}{3}+\frac{1}{9}$ will be $x+\frac{1}{3}$, & lastly, that of $x^2-\frac{bx}{a}+\frac{b^2}{4a^2}$ will be $x-\frac{b}{2a}$.

SIR ISAAC NEWTON's RULE *for raising a binomial or residual quantity to any power whatever.*

1. *To find the Terms without the co-efficients.*

THE index of the first, or leading quantity, begins with that of the given power, and decreases continually by 1, in every term to the last; and in the following quantity the indices of the terms are, 0, 1, 2, 3, 4, &c.

2. *To find the Unciæ or co-efficients.*

THE first is always 1, and the second is the index of the power: and in general, if the co-efficient of any term be multiplied by the index of the leading quantity, and the product be divided by the number of terms to that place, it will give the co-efficient of the term next following.

Note. The whole number of terms will be one more than the index of the given power; and when both terms of the root are +, all the terms of the power will be +; but if the second term be —, then all the odd terms will be +, and the even terms —.

1. LET $a+x$ be involved to the fifth power.

The terms without the co-efficients will be $a^5, a^4x, a^3x^2, a^2x^3, ax^4, x^5$, *and the co-efficients will be* 1, 5, $\frac{5\times4}{2}$, $\frac{10\times3}{3}$, $\frac{10\times2}{4}$, $\frac{5\times1}{5}$, *or* 1,5,10 10,5,1, *and therefore the 5th power is* $a^5+5a^4x+10a^3x^2+10a^2x^3+5ax^4+x^5$.

2. LET $x-a$ be involved to the 6th power.

The terms without the co-efficients will be $x^6, x^5a, x^4a^2, x^3a^3, x^2a^4, xa^5, a^6$, *& the co-efficients will be* 1, 6, $\frac{6\times5}{2}$, $\frac{15\times4}{3}$, $\frac{20\times3}{4}$, $\frac{15\times2}{5}$, $\frac{6\times1}{6}$ *or* 1,6,15,20,15,6,1; *and therefore the 6th power of* $x-a$ *is* $x^6-6x^5a+15x^4a^2-20x^3a^3+15x^2a^4-6xa^5+a^6$.

EVOLUTION.

EVOLUTION is the reverse of involution, and teaches to find the roots of any given powers.

CASE

CASE 1. *To find the Roots of simple Quantities.*

RULE. Extract the root of the co-efficient, for the numerical part, and divide the index of the letters by the index of the power, and it will give the root required.

1. THE square root of $9x^2 = 3x^{\frac{2}{2}} = 3x$.

2. THE cube root of $8x^3 = 2x^{\frac{3}{3}} = 2x$.

3. THE square root of $3a^2x^6 = a^{\frac{2}{2}}x^{\frac{6}{2}}\sqrt{3} = ax^3\sqrt{3}$

4. THE cube root of $-125a^3x^6 = 5a^{\frac{3}{3}}x^{\frac{6}{3}} = 5ax^2$.

5. THE biquadrate root of $16a^4x^8 = 2a^{\frac{4}{4}}x^{\frac{8}{4}} = 2ax^2$.

CASE 2. *To find the Square Root of a compound Quantity.*

RULE. Range the quantities according to the dimensions of some letter, and set the root of the first term in the quotient.

2. SUBTRACT the square of the root, thus found, from the first term and bring down the two next terms to the remainder, for a dividend.

3. DIVIDE the dividend by double the root, and set the result in the quotient.

4. MULTIPLY the divisor and quotient by the term last put in the quotient, and subtract the product from the dividend, and so on, as in common Arithmetic.

1. EXTRACT the square root of $4a^4+12a^3x+13a^2x^2+6ax^3+x^4$.

$$
\begin{array}{r@{}l}
& 4a^4+12a^3x+13a^2x^2+6ax^3+x^4(2a^2+3ax+x^2 \\
& 4a^4 \\
\hline
4a^2+3ax) & 12a^3x+13a^2x^2 \\
& 12a^3x+9a^2x^2 \\
\hline
4a^2+6ax+x^2) & 4a^2x^2+6ax^3+x^4 \\
& 4a^2x^2+6ax^3+x^4 \\
\hline
& *
\end{array}
$$

2. EXTRACT the square root of $x^4-4x^3+6x^2-4x+1$.

$$
\begin{array}{r@{}l}
& x^4-4x^3+6x^2-4x+1 \\
& x^4 \\
\hline
2x^2-2x) & -4x^3+6x^2 \\
& -4x^3+4x^2 \\
\hline
2x^2-4x+1) & 2x^2-4x+1 \\
& 2x^2-4x+1 \\
\hline
& *
\end{array}
$$

CASE 3. *To find the Roots of Powers in general.*

RULE. 1. Find the root of the first term, and place it in the quotient.

2. SUBTRACT the power, and bring down the second term for a dividend.

3. INVOLVE the root, last found, to the next lowest power, and multiply it by the index of the given power, for a divisor.

4. DIVIDE the dividend by the divisor, and the quotient will be the next term of the root.

5. INVOLVE the whole root, and subtract and divide as before; and so on 'till the whole be finished.

1. REQUIRED the square root of $a^4-2a^3x+3a^2x^2-2ax^3+x^4$.

$$
\begin{array}{l}
a^4-2a^3x+3a^2x^2-2ax^3+x^4(a^2-ax+x^2 \\
a^4 \\
\hline
2a^2)-2a^3x \\
\hline
a^4-2a^3x+a^2x^2 \\
\hline
2a^2)2a^2x^2 \\
\hline
a^4-2a^3x+3a^2x^2-2ax^3+x^4 \\
\hline
*
\end{array}
$$

2. EXTRACT the cube root of $x^6+6x^5-40x^3+96x-64$

$$
\begin{array}{l}
x^6+6x^5-40x^3+96x-64(x^2+2x-4 \\
x^6 \\
\hline
3x^4)6x^5 \\
\hline
x^6+6x^5+12x^4+8x^3 \\
\hline
3x^4)-12x^4 \\
\hline
x^6+6x^5-40x^3+96x-64 \\
\hline
*
\end{array}
$$

INFINITE SERIES.

AN INFINITE SERIES is formed from a vulgar fraction, having a compound denominator, or by extracting the root of a surd quantity; and is such, as, being continued, would run on *ad infinitum*, in the manner of a decimal fraction. And, by obtaining a few of the first terms, the law of the progression will be manifest, so that the series may be continued, without actually performing the whole operation.

PROBLEM

Problem I. *To Reduce fractional quantities into Infinite Series.*

Rule. Divide the Numerator by the Denominator, and the operation continued, as far as shall be thought necessary, will give the operation required.

1. Let $\frac{1}{1+x}$ *be thrown into an infinite series.*

$$\begin{array}{l}
1+x)1 \;\ldots\; (1-x+x^2-x^3+x^4, \text{ \&c.}\\
\quad 1+x\\
\quad -x\\
\quad -x-x^2\\
\qquad +x^2\\
\qquad +x^2+x^3\\
\qquad\quad -x^3\\
\qquad\quad -x^3-x^4\\
\qquad\qquad +x^4, \text{ \&c.}
\end{array}$$

2. Let $\frac{1}{1-x}$ *be thrown into an infinite series.*

$$\begin{array}{l}
1-x)1 \;\ldots\; (1+x+x^2+x^3+x^4, \text{ \&c.}\\
\quad 1-x\\
\quad +x-x^2\\
\qquad +x^2\\
\qquad +x^2-x^3\\
\qquad\quad +x^3\\
\qquad\quad +x^3-x^4\\
\qquad\qquad +x^4, \text{ \&c.}
\end{array}$$

3. Let $\frac{ax}{a-x}$ be proposed.

$$\begin{array}{l}
a-x)ax \;\ldots\; (x+\frac{x^2}{a}+\frac{x^3}{a^2}+\frac{x^4}{a^3}, \text{ \&c.}\\
\quad ax-x^2\\
\quad x^2\\
\quad x^2-\frac{x^3}{a}\\
\qquad \frac{x^3}{a}\\
\qquad \frac{x^3}{a}-\frac{x^4}{a^2}\\
\qquad\quad \frac{x^4}{a^2}\\
\qquad\quad \frac{x^4}{a^2}-\frac{x^5}{a^3}\\
\qquad\qquad \frac{x^5}{a^3}, \text{ \&c.}
\end{array}$$

4. Let $\frac{a^2}{a^2+2ax+x^2}$ be proposed.

$$\begin{array}{l}
a^2+2ax+x^2)a^2 \;\ldots\; (1-\frac{2x}{a}+\frac{3x^2}{a^2}-\frac{4x^3}{a^3}\\
\quad a^2+2ax+x^2\\
\quad -2ax-x^2\\
\quad -2ax-4x^2-\frac{2x^3}{a}\\
\qquad 3x^2+\frac{2x^3}{a}\\
\qquad 3x^2+\frac{6x^3}{a}+\frac{3x^4}{a_2}\\
\qquad\quad -\frac{4x^3}{a}-\frac{3x^4}{a^2}\\
\qquad\quad \frac{4x^3}{a}-\frac{8x^4}{a^2}-\frac{4x^5}{a^3}\\
\qquad\qquad \frac{5x^4}{a^2}+\frac{4x^5}{a^3}\\
\qquad\qquad [\text{ \&c.}
\end{array}$$

Prob.

PROB. 2. *To reduce a Compound Surd into an Infinite Series.*

RULE. Extract the root to such degree of exactness as shall be thought necessary.

EXTRACT the square root of a^2+x^2 in an infinite series.

$$a^2+x^2)\,a+\frac{x^2}{2a}-\frac{x^4}{8a^3}+\frac{x^6}{16a^5}-\frac{5x^8}{128a^7},\ \&c.$$

$$\underline{a^2}$$

$$2a+\frac{x^2}{2a}\Big)\,x^2$$

$$\underline{x^2+\frac{x^4}{4a^2}}$$

$$2a+\frac{x^2}{a}-\frac{x^4}{8a^3}\Big)-\frac{x^4}{4a^2}$$

$$\underline{-\frac{x^4}{4a^2}-\frac{x^6}{8a^4}+\frac{x^8}{64a^6}}$$

$$2a+\frac{x^2}{a}-\frac{x^4}{8a^3},\ \&c.\Big)\,\frac{x^6}{8a^4}-\frac{x^8}{64a^6}$$

$$\underline{\frac{x}{8a^4}+\frac{x^8}{16a^6},\ \&c,}$$

$$\frac{5x^8}{64a^6},\ \&c.$$

ARITHMETICAL PROPORTION.

A SERIES IN ARITHMETICAL PROPORTION is thus expressed, $a, a+b, a+2b, a+3b, a+4b$, &c. Here the common difference is b. —See page 217, &c.

Note. The most useful part of Arithmetical proportion is contained in the 1st. 3d. and 4th. Theorems.

GEOMETRICAL PROPORTION.

A SERIES IN GEOMETRICAL PROPORTION is thus expressed, a, ar, ar^2, ar^3, ar^4, &c. Here r is the ratio. —See page 234, &c.

Note. The most useful part of Geom. Proportion is contained in the 1st, 3d, 5th and 8th, Theorems.

SIMPLE EQUATIONS.

AN EQUATION is the comparing of two equal quantities which are differently expressed, together, by means of the sign $=$ placed between them.

Thus $12-7=5$ *is an equation, expressing the equality of the quantities* $12-7$ and 5.

A SIMPLE EQUATION is that which contains only one unknown quantity, without including its power. Thus $x-a+b=c$ is a simple equation, containing only the unknown quantity x.

REDUCTION OF EQUATIONS is the method of finding the value of the unknown quantity; which is shewn in the following Rules.

RULE I. Any quantity may be transposed from one side of the equation to the other, by changing its sign.

Thus, if $x+3=7$, *then will* $x=7-3=4$. *And, if* $x-4+6=8$, *then will* $x=8+4-6=6$. *Also, if* $x-a+b=c-d$, *then will* $x=c-d+a-b$. *And, if* $4x-8=3x+20$, *then will* $4x-3x=20+8$, *or* $x=28$.

RULE 2. If the unknown term be multiplied by any quantity, it may be taken away by dividing all the other terms of the equation by it.

Thus, if $ax=ab-a$, *then will* $x=b-1$. *If* $2x+4=16$, *then will* $x+2=8$, *and* $x=8-2=6$. *Also, if* $ax+2ba=3c^2$, *then will* $x+2b=\frac{3c^2}{a}$, *and* $x=\frac{3c^2}{a}-2b$.

RULE 3. If the unknown term be divided by any quantity, it may be taken away by multiplying all the other terms of the equation by it.

Thus, if $\frac{x}{2}=5+3$, *then will* $x=10+6=16$. *If* $\frac{x}{a}=b+c-d$, *then will* $x=ab+ac-ad$. *Also, if* $\frac{2x}{3}-2=6+4$, *then will* $2x-6=18+12$, *and* $2x=18+12+6=36$, *or* $x=\frac{36}{2}=18$.

RULE 4. The unknown quantity in any equation may be made free from surds, by transposing the rest of the terms according to the rule, and then involving each side to such a power as is denoted by the Index of the said Surd.

Thus, if $\sqrt{x}-2=6$, *then will* $\sqrt{x}=6+2=8$, *and* $x=8^2=64$. *If* $\sqrt{4x+16}=12$, *then will* $4x+16=144$, *and* $4x=144-16=128$, *or* $x=\frac{128}{4}=32$. *Also, if* $\sqrt[3]{2x+3}+4=8$; *then will* $\sqrt[3]{2x+3}=8-4=4$, *and* $2x+3=4^3=64$, *and* $2x=64-3=61$, *or* $x=\frac{61}{2}=30\frac{1}{2}$.

RULE 5. If that side of the equation, which contains the unknown qnantity, be a complete power, it may be reduced by extracting the root of said Power from both sides of the equation.

Thus, if $x^2+6x+9=25$, *then will* $x+3=\sqrt{25}=5$, *or* $x=5-3=2$. *If* $3x^2-9=21+3$, *then will* $3x^2=21+3+9=33$, *and* $x^2=\frac{33}{3}=11$

or

or $x = \sqrt{11}$. *Also, if* $\frac{2x^2}{3} + 10 = 20$, *then will* $2x^2 + 30 = 60$, *and* $x^2 + 15 = 30$, *or* $x^2 = 30 - 15$, *or* $x = \sqrt{15}$.

RULE 6. Any analogy, or proportion, may be converted into an equation, by making the product of the two mean terms equal to that of the two extremes.

Thus, if $3x : 16 :: 5 : 10$, *then will* $3x \times 10 = 16 \times 5$, *and* $30x = 80$, *or* $x = \frac{80}{30} = 2\frac{2}{3}$. *If* $\frac{2x}{3} : a :: b : c$, *then will* $\frac{2cx}{3} = ab$, *and* $2cx = 3ab$, *or* $x = \frac{3ab}{2c}$. *Also, if* $12 - x : \frac{x}{2} :: 4 : 1$, *then will* $12 - x = \frac{4x}{2} = 2x$, *and* $2x + x = 12$, *or* $x = \frac{12}{3} = 4$.

RULE 7. If any quantity be found on both sides of the equation with the same sign, it may be taken away from them both; and if every term in an equation be multiplied or divided by the same quantity, it may be struck out of them all.

Thus, if $4x + a = b + a$, *then will* $4x = b$, *and* $x = \frac{b}{4}$. *If* $3ax + 5ab = 8ac$, *then will* $3x + 5b = 8c$, *and* $x = \frac{8c - 5b}{3}$. *Also, if* $\frac{2x}{3} - \frac{8}{3} = \frac{16}{3} - \frac{8}{3}$, *then will* $2x = 16$, *and* $x = 8$.

MISCELLANEOUS EXAMPLES.

1. GIVEN $5x - 15 = 2x + 6$, to find the value of x.

First, $5x - 2x = 6 + 15$, *then* $3x = 21$, *and* $x = \frac{21}{3} = 7$.

2. GIVEN $40 - 6x - 16 = 120 - 14x$, to find x.

First, $14x - 6x = 120 - 40 + 16$, *then* $8x = 96$, *therefore* $x = \frac{96}{8} = 12$.

3. GIVEN $5ax - 3b = 2dx + c$, to find x.

First, $5ax - 2dx = c + 3b$, *or* $\overline{5a - 2d} \times x = c + 3b$, *therefore* $x = \frac{c + 3b}{5a - 2d}$.

4. GIVEN $3x^2 - 10x = 8x + x^2$, to find x.

First, $3x - 10 = 8 + x$, *then* $3x - x = 8 + 10$, *therefore* $2x = 18$, *and* $x = \frac{18}{2} = 9$.

5. GIVEN $6ax^3 - 12abx^2 = 3ax^3 + 6ax^2$, to find x.

First, dividing the whole by $3ax^2$, *we shall have* $2x - 4b = x + 2$, *then* $2x - x = 2 + 4b$, *whence* $x = 2 + 4b$.

6. GIVEN

6. GIVEN $\frac{x}{2} - \frac{x}{3} + \frac{x}{4} = 10$, to find x.

First, $x - \frac{2x}{3} + \frac{2x}{4} = 20$, *then* $3x - 2x + \frac{6x}{4} = 60$, *and* $12x - 8x + 6x = 240$, *therefore* $10x = 240$, *and* $x = \frac{240}{10} = 24$.

7. GIVEN $\frac{x-3}{2} + \frac{x}{3} = 20 - \frac{x+19}{2}$, to find x.

First, $x - 3 + \frac{2x}{3} = 40 - x - 19$, *then* $3x - 9 + 2x = 120 - 3x - 57$, *therefore* $3x + 2x + 3x = 120 - 57 + 9$, *that is*, $8x = 72$, *or* $x = \frac{72}{8} = 9$.

8. GIVEN $\sqrt{\frac{2}{3}x + 5} = 7$, to find x.

First, $\sqrt{\frac{2}{3}x} = 7 - 5 = 2$, *then* $\frac{2}{3}x = 2^2 = 4$, & $2x = 12$, *or* $x = \frac{12}{2} = 6$.

9. GIVEN $x + \sqrt{a^2 + x^2} = \frac{2a^2}{\sqrt{a^2 + x^2}}$, to find x.

First, $x\sqrt{a^2 + x^2} + a^2 + x^2 = 2a^2$, *then* $x\sqrt{a^2 + x^2} = a^2 - x^2$, *and* $x^2 \times \overline{a^2 + x^2} = \overline{a^2 - x}\,|^2 = a^4 - 2a^2x^2 + x^4$, *or* $a^2x^2 + x^4 = a^4 - 2a^2x^2 + x^4$, *whence* $a^2x^2 + 2a^2x^2 = a^4$, *or* $3a^2x^2 = a^4$, *consequently* $x^2 = \frac{a^4}{3a^2}$, *and* $x = \sqrt{\frac{a^4}{3a^2}} = a\sqrt{\frac{1}{3}}$.

PROBLEM I. *To exterminate two unknown quantities, or to reduce the two simple equations, containing them, to a single one.*

RULE 1ſt. 1. Obſerve which of the unknown quantities is the leaſt involved, and find its value in each of the equations, by the methods already explained.

2. LET the two values thus found be made equal to each other, and there will ariſe a new equation with only one unknown quantity in it, whoſe value may be found as before.

1. GIVEN $\left\{\begin{array}{l} 2x + 3y = 23 \\ 5x - 2y = 10 \end{array}\right\}$ to find x and y.

From the first equation, $x = \frac{23 - 3y}{2}$, *and from the second*, $x = \frac{10 + 2y}{5}$ *and conſequently* $\frac{23 - 3y}{2} = \frac{10 + 2y}{5}$, *or* $115 - 15y = 20 + 4y$, *or* $19y = 115 - 20 = 95$, *and* $y = \frac{95}{19} = 5$, *whence* $x = \frac{23 - 15}{2} = 4$.

2. GIVEN

2. GIVEN $\left\{\begin{array}{l} x+y=a \\ x-y=b \end{array}\right\}$ to find x and y.

From the first equation, $x=a-y$, *and from the second,* $x=b+y$, *therefore* $a-y=b+y$, *or* $2y=a-b$, *consequently* $y=\frac{a-b}{2}$, *and* $x=a-y=a-\frac{a-b}{2}=\frac{a+b}{2}$.

3. GIVEN $\left\{\begin{array}{l} \frac{x}{2}+\frac{y}{3}=7 \\ \frac{x}{3}+\frac{y}{2}=8 \end{array}\right\}$ to find x and y.

From the first equation, $x=14-\frac{2y}{3}$, *and from the second,* $x=24-\frac{3y}{2}$, *therefore* $14-\frac{2y}{3}=24-\frac{3y}{2}$, *and* $42-2y=72-\frac{9y}{2}$, *or* $84-4y=144-9y$; *whence* $5y=144-84=60$, *and* $y=\frac{60}{5}=12$, *and* $x=14-\frac{2y}{3}=14-\frac{24}{3}=8$.

RULE 2d. 1. Consider which of the unknown quantities you would first exterminate, and let its value be found in that equation where it is the least involved.

2. SUBSTITUTE the value, thus found, for its equal in the other equation, and there will arise a new equation, with only one unknown quantity, whose value may be found as before.

1. GIVEN $\left\{\begin{array}{l} x+2y=17 \\ 3x-y=2 \end{array}\right\}$ to find x and y.

From the first equation, $x=17-2y$, *and this value, substituted for* x *in the second, gives* $\overline{17-2y}\times 3-y=2$, *or* $51-6y-y=2$, *or,* $51-7y=2$; *that is,* $7y=51-2=49$; *whence* $y=\frac{49}{7}=7$, *and* $x=17-2y=17-14=3$.

2. GIVEN $\left\{\begin{array}{l} a:b::x:y \\ x^2+y^2=c \end{array}\right\}$ to find x and y.

The first analogy, turned into an equation, is $bx=ay$, *or* $x=\frac{ay}{b}$, *and this value of* x, *substituted in the second, gives* $\overline{\frac{ay}{b}}\Big|^2+y^2=c$, *or* $\frac{a^2y^2}{b^2}+y^2=c$, *or* $a^2y^2+b^2y^2=cb^2$, *or* $y^2=\frac{cb^2}{a^2+b^2}$, *therefore* $y=\overline{\frac{cb^2}{a^2+b^2}}\Big|^{\frac{1}{2}}$, *and* $x=\overline{\frac{ca^2}{a^2+b^2}}\Big|^{\frac{1}{2}}$.

RULE

RULE 3d. Let the given equations be multiplied or divided by such numbers or quantities as will make the term, which contains one of the unknown quantities, to be the same in both equations, and then by adding or subtracting the equations, accordingly as is required, there will arise a new equation with only one unknown quantity, as before.

1. GIVEN $\left\{\begin{array}{l}3x+5y=40\\ x+2y=14\end{array}\right\}$ to find x and y.

First, multiply the 2d. Equation by 3, and we shall have $3x+6y=42$, *then, from this last equation subtract the first, and it will give* $6y-5y=42-40$, *or* $y=2$, *therefore* $x=14-2y=14-4=10$.

2. GIVEN $\left\{\begin{array}{l}5x-3y=9\\ 2x+5y=16\end{array}\right\}$ to find x and y.

Let the first Equation be multiplied by 2, and the 2d. by 5, and we shall have $\left\{\begin{array}{l}10x-6y=18\\ 10x+25y=80\end{array}\right\}$ *and if the former of these be subtracted from the latter, it will give* $31y=62$, *or* $y=\frac{62}{31}=2$, *consequently,* $x=\frac{9+6}{5}=\frac{15}{5}=3$.

Another Method.

Multiply the 1st. equation by 5, and the 2d by 3, and we shall have $\left\{\begin{array}{l}25x-15y=45\\ 6x+15y=48\end{array}\right\}$ *Now, add these two equations, and it will give* $31x=93$, *or* $x=\frac{93}{31}=3$, *consequently* $y=\frac{16-6}{5}=\frac{10}{5}=2$, *as before.*

PROB. 2. *To exterminate three unknown quantities, or to reduce the three simple equations, containing them, to a single one.*

RULE. 1. Let x, y, and z, be the three unknown quantities to be exterminated.

2. FIND the value of x, from each of the three given equations.

3. COMPARE the first value of x with the second, and an equation will arise, involving only y and z.

4. COMPARE the first value of x with the third, and another equation will arise, involving only y and z.

5. FIND the values of y and z from these two equations, according to the former rules, and x, y, and z will be exterminated as required.

Note. Any number of unknown quantities may be exterminated in nearly the same manner.

1. GIVEN $\left\{\begin{array}{l}x+y+z=29\\ x+2y+3z=62\\ \frac{x}{2}+\frac{y}{3}+\frac{z}{4}=10\end{array}\right\}$ to find x, y, and z.

 From

From the first equation, $x=29-y-z$. *From the 2d.* $x=62-2y-3z$. *From the 3d.* $x=20-\frac{2y}{3}-\frac{z}{2}$, *whence* $29-y-z=62-2y-3z$, *and* $29-y-z=20-\frac{2y}{3}-\frac{z}{2}$; *but from the first of these equations,* $y=62-29-2z=33-2z$; *and from the 2d.* $y=27-\frac{3z}{2}$, *therefore* $33-2z=27-\frac{3z}{2}$, *or* $z=12$, *and* $y=62-29-2z=62-29-24=9$, *and* $x=29-y-z=29-12-9=8$.

2. GIVEN $\left\{\begin{array}{l} \frac{x}{2}+\frac{y}{3}+\frac{z}{4}=62 \\ \frac{x}{3}+\frac{y}{4}+\frac{z}{5}=47 \\ \frac{x}{4}+\frac{y}{5}+\frac{z}{6}=38 \end{array}\right\}$ to find x, y, & z.

First, the given equations, cleared from fractions, become

$$12x+8y+6z=1488$$
$$20x+15y+12z=2820$$
$$30x+24y+20z=4560$$

Then, if the 2d. equation be subtracted from double the 1st. and 3 times the 3d. from 5 times the 2d. we shall have

$$4x+y=156$$
$$10x+3y=420$$

And again, if the second of these be subtracted from 3 times the first, it will give $12x-10x=468-420$, *or* $x=\frac{48}{2}=24$, *therefore* $y=156-4x=60$, *and* $z=\frac{1488-8y-12x}{6}=120$.

Questions producing simple Equations.

1. To find two such numbers, as that their sum shall be 40, and their difference 16.

Let x *denote the least of the two numbers required, then will* $x+16=$ *the greater,* $x+x+16=40$ *by the question, that is,* $2x=40-16=24$, *or* $x=\frac{24}{2}=12=$ *least number, and* $x+16=12+16=28=$ *greater number required.*

2. WHAT

2. What number is that, whoſe $\frac{1}{3}$ part exceeds its $\frac{1}{4}$ part by 16?

Let $x=$ *number required, then will its* $\frac{1}{3}$ *part be* $\frac{x}{3}$, *& its* $\frac{1}{4}$ *part* $\frac{x}{4}$; *therefore* $\frac{x}{3}-\frac{x}{4}=16$, *by the queſtion, that is* $x-\frac{3x}{4}=48$, *or* $4x-3x=192$; *whence* $x=192$ *the number required.*

3. Divide £1000 between A, B, and C, ſo that A ſhall have £72 more than B, and C, £100 more than A.

Let $x=$ *B's ſhare of the given ſum, then will* $x+72=$ *A's ſhare, and* $x+172=$ *C's ſhare; and the ſum of all theſe ſhares* $x+x+72+x+172$, *or* $3x+244=1000$, *by the queſtion, that is,* $3x=1000-244=756$, *or* $x=\frac{756}{3}=$ £$252=$ *B's ſhare, and* $x+72=252+72=$ £$324=$ *A's ſhare, and* $x+172=252+172=$ £$424=$ *C's ſhare.*

Proof $252+324+424=$ £1000.

4. A Prize of £1000 is to be divided between two perſons, whoſe ſhares therein are in the proportion of 7 to 9; Required the ſhare of each?

Let $x=$ *the firſt perſon's ſhare, then will* £$1000-x=$ *2d. perſon's ſhare, and* $x : 1000-x :: 7 : 9$, *by the queſtion, that is,* $9x=1000-x\times7=7000-7x$, *or* $16x=7000$, *whence* $x=\frac{7000}{16}=$ £437 10*ſ.* $=$ *1ſt. ſhare, and* $1000-x=1000-$ £437 10*ſ.* $=$ £562 10*ſ.* $=$ *2d. ſhare.*

5. The paving of a ſquare at 2*ſ. per* yard, coſt as much as the incloſing of it, at 5*ſ. per* yard; Required the ſide of the ſquare?

Let $x=$ *Side of the ſquare ſought, then* $4x=$ *yards of incloſure, and* $x^2=$ *yards of pavement; whence* $4x\times5=20x=$ *price of incloſing, and* $x^2\times\ =2x^2=$ *price of paving. But* $2x^2=20x$, *by the Queſtion, therefore* $x^2=10x$, *and* $x=10=$ *length of the Side required.*

6. A Labourer engaged to ſerve 40 days upon theſe conditions, that for every day he worked he ſhould receive 20*d.* but for every day he was abſent, he was to forfeit 8*d.*—Now, at the end of the time, there was due to him £1 11*ſ*8; How many days did he work, and how many was he abſent?

Let x *be the number of days he worked, then will* $40-x$ *be the number of days he was abſent; alſo,* $x\times20=20x=$ *ſum earned, and* $\overline{40-x}\times8=320-8x=$ *ſum forfeited; whence* $20x-\overline{320-8x}=380d.$ $(=$ £1 11*ſ*8$)$ *by the queſtion, that is,* $20x-320+8x=380$, *or* $28x=380+320=700$, *and* $x=\frac{700}{28}=25=$ *number of days he worked; and* $40-x=40-25=15=$ *number of days he was abſent.*

7. Out

7. OUT of a Cask of wine, which had leaked away $\frac{1}{3}$, 21 Gallons were drawn; and then, being gauged, it appeared to be half full; How much did it hold?

Let it be supposed to have held x gallons, then it would have leaked $\frac{x}{3}$ *gallons, and consequently there had been taken away* $21+\frac{x}{3}$ *gallons. But* $21+\frac{x}{3}=\frac{x}{2}$ *by the question, that is,* $63+x=\frac{3x}{2}$, *or* $126+2x=3x$, *hence* $3x-2x=126$, *or* $x=126$, *Answer.*

8. WHAT fraction is that, to the numerator of which if 1 be added, the value will be $\frac{1}{3}$; but if 1 be added to the denominator, its value will be $\frac{1}{4}$?

Let the fraction be represented by $\frac{x}{y}$ *then will* $\frac{x+1}{y}=\frac{1}{3}$, *and* $\frac{x}{y+1}=\frac{1}{4}$, *or* $3x+3=y$, *and* $4x=y+1$; *hence* $4x-3x-3=y+1-y$, *that is,* $x-3=1$, *or* $x=4$, *and* $y=3x+3=12+3=15$. *So that* $\frac{4}{15}$ $=$ *fraction required.*

9. A Market-woman bought a certain number of Eggs, at 2 for a Cent, and as many, at 3 for a Cent, and sold them all out again, at the rate of 5 for 2 Cents, and, by so doing, lost 4 Cents; What number of Eggs had she?

Let $x=$ *number of eggs of each sort, then will* $\frac{x}{2}=$ *price of the 1st. sort, and* $\frac{x}{3}=$ *price of the 2d. sort. But* $5:2::2x$ *(the whole number of eggs)* $:\frac{4x}{5}$; *therefore* $\frac{4x}{5}=$ *price of both sorts together, at 5 for 2 Cents, and* $\frac{x}{2}+\frac{x}{3}-\frac{4x}{5}=4$, *by the question; that is,* $x+\frac{2x}{3}-\frac{8x}{5}=8$; *or* $3x+2x-\frac{24x}{5}=24$; *or* $15x+10x-24x=120$; *whence* $x=120=$ *number of eggs of each sort required.*

10. A person in the afternoon being asked what o'clock it was, answered, that $\frac{3}{5}$ of the time from noon was equal to $\frac{5}{8}$ of the time to midnight; Required the Time?

Let $x=$ *the time sought from noon, then will* $12-x=$ *the time to midnight,* $\frac{3}{5}$ *of the time from noon* $=\frac{3x}{5}$, *and* $\frac{5}{8}$ *of the time to midnight* $=\frac{60-5x}{8}$, *therefore* $\frac{3x}{5}=\frac{60-5x}{8}$ *by the question; whence,* $3x=\frac{300-25x}{8}$ *and* $24x=300-25x$, *or* $24x+25x=300$, *or* $x=\frac{300}{49}$ $=6^h\ 7'\ 20''\frac{40}{49}$, *Answer.*

11. A

11. A Merchant ſhips Goods for South-Carolina to the amount of £700; What Sum, at 5 *per Cent.* ſhould he get inſured, to cover his adventure?

Let $x =$ *ſum to be inſured, then will* $x - \frac{5x}{100} = 700$, *whence* $100x - 5x = 70000$, *and* $x = \frac{70000}{95} =$ £736 $16/10\frac{10}{95}$, *Anſwer.*

12. A Man lays out 30 Cents for Apples and Pears, buying his Apples, at 4, and his Pears, at 5 for a Cent, and afterwards ſold $\frac{1}{2}$ of his Apples, and $\frac{1}{3}$ of his pears for 13 Cents, which was the prime coſt; I demand the number he bought of each?

Let $x =$ *the number of Apples, and* $z =$ *the number of Pears; then, if* 4 *apples coſt a Cent,* x *will coſt* $\frac{x}{4}$ *Cents, and for the ſame reaſon* z *will coſt* $\frac{z}{5}$ *Cents, and we ſhall have* $\frac{x}{4} + \frac{z}{5} = 30$, *for one fundamental equation. Again, the price of* $\frac{x}{2} = \frac{1}{2}$ *of his apples will be* $\frac{x}{8}$, *and the price of* $\frac{z}{3} = \frac{1}{3}$ *of his pears will be* $\frac{z}{15}$; *hence* $\frac{x}{8} + \frac{z}{15} = 13$, *for another fundamental equation: Now, croſs-multiplying* $\frac{x}{4} + \frac{z}{5} = 30$, *and then multiplying* 30 *by* 4 *and* 5 *we ſhall have the firſt equation* $= 5x + 4z = 600$; *and doing the ſame by* $\frac{x}{8} + \frac{z}{15} = 13$, *we have the* 2*d. equation* $= 15x + 8z = 1560$. *Subtract the* 2*d. equation from twice the* 1*ſt. and we ſhall have the* 3*d. equation* $= 4z = 240$, *therefore* 4*th. equation* $= z = 60 =$ *the number of pears: Now, ſubſtitute* 60 *for* z, *that is,* 240 *for* $4z$, *in the* 1*ſt. equation.* $5x + 4z = 600$, *we ſhall have* $5x + 240 = 600$, *whence, equation* 5*th.* $x = 72 =$ *the number of Apples.*

QUADRATIC EQUATIONS.

A SIMPLE QUADRATIC EQUATION is that which involves the ſquare of the unknown quantity only.

AN ADFECTED QUADRATIC EQUATION is that which involves the ſquare of the unknown quantity, together with the product, which ariſes from multiplying it by ſome known quantity.

Thus, $ax^2 = b$, *is a ſimle quadratic equation, and* $ax^2 + bx = c$ *is an adfected quadratic equation.*

ALL adfected quadratic equations fall under the three following forms.

1ſt. $x^2 + ax = b$. 2d. $x^2 - ax = b$. 3d. $x^2 - ax = -b$. *And the rule for finding the value of* x, *in each of theſe equations, is as follows:*

RULE

RULE.* 1. Transpose all the terms, which involve the unknown quantity, to one side of the equation, and the known terms to the other side, and let them be ranged according to their dimensions.

2. WHEN

* *The Square Root of any quantity may be either + or —, and therefore all quadratic equations admit of two Solutions. Thus the square root of* $+n^2$ *is* $+n$, *or* $-n$, *for* $+n\times+n$, *or* $-n\times-n$, *are each equal to* $+n^2$. *So, in the first form, where* $x+\frac{a}{2}$ *is found* $=\sqrt{b+\frac{a^2}{4}}$, *the root may be either* $+\sqrt{b+\frac{a^2}{4}}$, *or* $-\sqrt{b+\frac{a^2}{4}}$, *since either of them being multiplied by itself will produce* $\overline{b+\frac{a^2}{4}}$. *And this ambiguity is expressed by writing the uncertain sign* $\pm$ *before* $\sqrt{b+\frac{a^2}{4}}$; *thus* $x=\pm\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$. *In the first form, where* $x=\pm\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$ *the first value of* x, *viz.* $x=+\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$ *is always affirmative.*

The second value, viz. $x=-\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$, *will always be negative, because it is composed of two negative terms; therefore when* $x^2+ax=b$, *we shall have* $x=+\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$ *for the affirmative value of* x, *and* $x=-\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$ *for the negative value of* x.

In the second form, where $x=\pm\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$, *the first value, viz.* $x=+\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$ *is always affirmative, since it is composed of two affirmative terms, and the second value, viz.* $x=-\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$ *will always be negative, therefore when* $x^2-ax=b$, *we shall have* $x=+\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$, *for the affirmative value of* x, *and* $x=-\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$, *for the negative value of* x.

In

2. WHEN the ſquare of the unknown quantity has any co-efficient prefixed to it, let all the reſt of the terms be divided by that co-efficient.

3. ADD the ſquare of half the co-efficient of the ſecond term to both ſides of the equation, and that ſide, which involves the unknown quantity, will be a complete ſquare.

4. EXTRACT the ſquare root from both ſides of the equation, and the value of the unknown quantity will be determined.

Note. 1. The ſquare root of one ſide of the Equation is always equal to the unknown quantity, with half the co-efficient of the ſecond term ſubjoined to it.

2. ALL equations, wherein there are two terms involving the unknown quantity, and the index of one is juſt double that of the other, are ſolved like quadratics, by completing the ſquare.

Thus, $x^4 + ax^2 = b$, *or* $x^n + ax^{\frac{n}{2}} = b$, *are the ſame as quadratics, and the value of the unknown quantity may be determined accordingly.*

FROM this rule may be formed a general Theorem with which all particular equations may be compared, and by means whereof, they may be more readily reſolved.

SUPPOSE $ax^2 \doteq bx + c$ be the general quadratic equation propoſed to be reſolved; where a, b, and c denote known integral quantities, whether affirmative, or negative, and $x =$ the unknown quantity; to find the values of x in this equation.

HERE, tranſpoſing bx, we have $ax^2 - bx = c$, then dividing by a, in order to free x^2 the higheſt power of x from its co-efficient, we have $x^2 - \frac{bx}{a} = \frac{c}{a}$; this being done, the firſt ſide, $x^2 + \frac{bx}{a}$ may be conſidered as an imperfect ſquare raiſed from a binomial root, and accordingly we may complete that ſquare by adding the ſquare of

In the third form, where $x = \sqrt{\frac{a^2}{4} - b} + \frac{a}{2}$, *both the values of* x *will be poſitive, ſuppoſing* $\frac{a^2}{4}$ *to be greater than* b. *Therefore when* $x^2 - ax = -b$, *we ſhall have* $x = + \sqrt{\frac{a^2}{4} - b} + \frac{a}{2}$, *and* $- \sqrt{\frac{a^2}{4} - b} + \frac{a}{2}$, *both, for the affirmative value of* x.

But in this third form, if b *be greater than* $\frac{a^2}{4}$, *the ſolution of the propoſed queſtion will be impoſſible. For ſince the ſquare of any quantity is always affirmative, the ſquare root of a negative quantity is impoſſible.*

of half the co-efficient of the second term: But if $\frac{bb}{4a^2}$ must be added to the first side of the equation, to complete the square, it must be also added to the other side, to preserve the equality, otherwise by an unequal addition, the equation would be destroyed; this equal addition therefore being made, the equation will stand thus, $x^2 - \frac{bx}{a} + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} + \frac{c}{a}$; but the two fractions $\frac{b^2}{4a^2}$ and $\frac{c}{a}$ when added, give $\frac{ab^2 + 4a^2c}{4a^3}$, which divided by a, gives $\frac{b^2 + 4ac}{4a^2}$; therefore $x^2 = \frac{bx}{a} + \frac{4a^2}{b^2} = \frac{b^2 + 4ac}{4a^2}$; therefore the square root of one side will be equal to the square root of the other; But the square root of $\frac{b^2 + 4ac}{4a^2}$, as it here stands in letters, cannot be extracted, because, although the donominator $4a^2$ be a square, yet there is no literal quantity whatever, which, being multiplied into itself, will produce $b^2 + 4ac$, therefore, to put this numerator into the form of a square, let us suppose $b^2 + 4ac = ss$, and then the equation will be $x^2 - \frac{bx}{a} + \frac{b^2}{4a^2} = \frac{ss}{4a^2}$; but the square root of $x^2 - \frac{bx}{a} + \frac{b^2}{4a^2}$ is $x - \frac{b}{2a}$, and that of $\frac{ss}{4a^2}$ is $\pm \frac{s}{2a}$, therefore this equation will now be reduced to a simple one, and will stand thus, $x - \frac{b}{2a} = \pm \frac{s}{2a}$, therefore $x = \frac{b \pm s}{2a}$, that is, $x = \frac{b+s}{2a}$ and $x = \frac{b-s}{2a}$.

Note, When the quantity c (and consequently $4ac$) is negative, the quantity ss, or $b^2 + 4ac$ must be considered as the sum of the affirmative quantity b^2 and the negative one $4ac$, when added together according to the common rules of Addition.

Examples of the resolution of adfected equations with and without the general Theorem.

1. GIVEN $x^2 = 140 - 4x$, to find the values of x.

First, transposing $-4x$, *it is* $x^2 + 4x = 140$, *then*, $x^2 + 4x + 4 = 144$, *by completing the square; then* $\sqrt{x^2 + 4x + 4} = \sqrt{144}$, *by extracting the root; or* $x + 2 = \pm 12$, *that is*, $x = -2 \pm 12 = +10$, *or* -14.

By the general Theorem. a, *in the general theorem, answers to* 1 *in the particular one, that is, to the co-efficient of* x^2, b *answers to* 4, *and* c, *to* 140, *that is*, $a = 1$. $b = -4$, $c = 140$, *and* $4ac = 560$, *therefore* ss, *or* $b^2 + 4ac$ *will be the sum of* 16 *and* $560 = 576$, *therefore* $s = \pm 24$, $\frac{b+s}{2a}$

$\frac{b+s}{2a} = \frac{-4+24}{2} = +10$, *and* $\frac{b-s}{2a} = \frac{-4-24}{2} = -14$; *therefore the two roots of this equation are* 10 *and* -14.

2. GIVEN $x^2 + 8 = 6x + 80$, to find x.

First, $x^2 - 6x = 72$, *by transposition*; *then* $x^2 - 6x + 9 = 72 + 9 = 81$, *by completing the square, and* $x - 3 = \sqrt{81} = \pm 9$, *by extracting the root, therefore* $x = +3 \pm 9 = +12$, *or* -6.

By the Theorem. $a = 1$, $b = 6$, $c = 72$, *and* $4ac = 288$, *therefore* $ss = 36 + 288 = 324$, *therefore* $s = 18$, $\frac{b+s}{2a} = 12$, *and* $\frac{b-s}{2a} = -6$.

3. GIVEN $2x^2 - 20 = 70 - 8x$, to find x.

First, $2x^2 + 8x = 70 + 20 = 90$, *by transposition, then* $x^2 + 4x = 45$, *by dividing by the co-efficient* 2, *and* $x^2 + 4x + 4 = 45 + 4 = 49$, *by completing the square*; *whence* $x + 2 = \sqrt{49} = \pm 7$, *therefore*, $x = -2 \pm 7 = 5$ *or* -9.

By the Theorem. $a = 2$, $b = -8$, $c = 90$, $4ac = 720$, $ss = 64 + 720 = 784$, *therefore* $s = 28$, $\frac{b+s}{2a} = +5$; $\frac{b-s}{2a} = -9$, *so that* $+5$, *and* -9 *are the values of* x *.

As the General Theorem is sufficiently exemplified in the preceding Problems, the following equations will be solved by the Rule only.

4. GIVEN $3x^2 - 3x + 6 = 5\frac{1}{3}$, to find x.

Here $x^2 - x + 2 = 1\frac{7}{9}$ *by dividing by* 3, *and* $x^2 - x = 1\frac{7}{9} - 2$, *by transposition*; *Also* $x^2 - x + \frac{1}{4} = 1\frac{7}{9} - 2 + \frac{1}{4} = \frac{1}{36}$, *by completing the square*; *and* $x - \frac{1}{2} = \sqrt{\frac{1}{36}} = \pm\frac{1}{6}$, *by evolution*; *therefore* $x = +\frac{1}{2} \pm \frac{1}{6} = \frac{2}{3}$, *or* $\frac{1}{3}$.

5. GIVEN $\frac{x^2}{2} - \frac{x}{3} + 20\frac{1}{2} = 42\frac{2}{3}$, to find x.

Here $\frac{x^2}{2} - \frac{x}{3} = 42\frac{2}{3} - 20\frac{1}{2} = 22\frac{1}{6}$, *by transposition, and* $x^2 - \frac{2x}{3} = 44\frac{1}{3}$, *by dividing by* $\frac{1}{2}$, *whence* $x^2 - \frac{2x}{3} + \frac{1}{9} = 44\frac{1}{3} + \frac{1}{9} = 44\frac{4}{9}$, *by completing the square, and* $x - \frac{1}{3} = \sqrt{44\frac{4}{9}} = \pm 6\frac{2}{3}$, *therefore* $x = +\frac{1}{3} \pm 6\frac{2}{3} = 7$, *or* $-6\frac{1}{3}$.

* IN a quadratic equation of this form $ax^2 = bx + c$, the sum of the roots will always be $\frac{b}{a}$, and the product of their multiplication $\frac{-c}{a}$; therefore, if $a = 1$, that is, if the equation be $x^2 = bx + c$, the sum of the roots will be b, and their product $-c$, or the sum will be the co-efficient of the unknown quantity on the second side of the equation, and their product, what is called the absolute term, with its sign changed.

6. GIVEN $ax^2+bx=c$, to find x.

First, $x^2+\frac{b}{a}x=\frac{c}{a}$, *by division*; *then* $x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{c}{a}+\frac{b^2}{4a^2}$, *by completing the square*; *and* $x+\frac{b}{2a}=\sqrt{\frac{c}{a}+\frac{b^2}{4a^2}}=\sqrt{\frac{4ac+b^2}{4a^2}}$, *by evolution, therefore* $x=\pm\sqrt{\frac{4ac+b^2}{4a^2}}-\frac{b}{2a}$.

7. GIVEN $ax^2-bx+c=d$. to find x.

Here, $ax^2-bx=d-c$, *by transposition, and* $x^2-\frac{b}{a}x=\frac{d-c}{a}$ *by division.*

Also $x^2-\frac{b}{a}x+\frac{b^2}{4a^2}=\frac{d-c}{a}+\frac{b^2}{4a^2}$ *by completing the square; and* $x-\frac{b}{2a}=\pm\sqrt{\frac{d-c}{a}+\frac{b^2}{4a^2}}$, *by evolution; therefore* $x=\frac{b}{2a}\pm\sqrt{\frac{d-c}{a}+\frac{b^2}{4a^2}}$.

8. GIVEN $x^4+2ax^2=b$, to find x.

Here, $x^4+2ax^2+a^2=b+a$, *by completing the sq.* &, $x^2+a=\sqrt{b+a^2}$ *by evolution; whence* $x^2=\sqrt{b+a}-a$, & *consequently* $x=\sqrt{:\sqrt{b+a}-a}$.

9. GIVEN $ax^n-bx^{\frac{n}{2}}-c=-d$, to find x.

First, $ax^n-bx^{\frac{n}{2}}=c-d$, *by transposition, and* $x^n-\frac{b}{a}x^{\frac{n}{2}}=\frac{c-d}{a}$, *by division. Also* $x^n-\frac{b}{a}x^{\frac{n}{2}}+\frac{b^2}{4a^2}=\frac{c-d}{a}+\frac{b^2}{4a^2}$, *by completing the sq. and* $x^{\frac{n}{2}}-\frac{b}{2a}=\sqrt{\frac{c-d}{a}+\frac{b^2}{2a^2}}$, *by evolution; therefore* $x^{\frac{n}{2}}=\frac{b}{2a}\pm\sqrt{\frac{c-d}{a}+\frac{b^2}{4a^2}}$, *and consequently* $x=\frac{b}{2a}\pm\sqrt{\frac{c-d}{a}+\frac{b^2}{4a^2}}\Big|^{\frac{n}{2}}$.

QUESTIONS PRODUCING QUADRATIC EQUATIONS.

1. To find two numbers, whose difference is 8, and product 240.

Let $x=$ *the least number, then will* $x+8=$ *the greater, and* $x\times\overline{x+8}=x^2+8x=240$, *by the question; whence* $x^2+8x+16=240+16=256$ *by completing the square; also* $x+4=\sqrt{256}=16$, *by evolution, and therefore* $x=16-4=12=$ *the least number, and* $12+8=20=$ *the greater.*

2. To

2. To divide the number 60 into two such parts, as that their product may be 864.

Let $x =$ *greater part, then will* $60 - x =$ *the less, and* $x \times \overline{60 - x} = 60x - x^2 = 864$, *by the question, that is,* $x^2 - 60x = -864$; *whence* $x^2 - 60x + 900 = -864 + 900 = 36$, *by completing the square: Also* $x - 30 = \sqrt{36} = 6$, *by extracting the root, therefore* $x = 6 + 30 = 36$ $=$ *greater part, and* $60 - x = 60 - 36 = 24 =$ *the less part.*

3. Sold a piece of Cloth for £24, and gained as much *per Cent.* as the Cloth cost me; what was the price of it?

Let $x =$ *pounds the Cloth cost, then* $24 - x =$ *whole gain, but* $100 : x :: x : 24 - x$, *by the question, or* $x^2 = 100 \times \overline{24 - x} = 2400 - 100x$, *that is,* $x^2 + 100x = 2400$; *whence* $x^2 + 100x + 2500 = 2400 + 2500 = 4900$, *by completing the square, and* $x + 50 = \sqrt{4900} = 70$, *by extraction of roots, consequently* $x = 70 - 50 = 20 =$ *price of the cloth.*

4. A Person bought a number of Oxen for £80, and if he had bought 4 more for the same money, he would have paid £1 less for each; How many did he buy?

Suppose he bought x Oxen, Then $\frac{80}{x} =$ *price of each, and* $\frac{80}{x+4} =$ *price of each, if* $x + 4$ *had cost £80: But* $\frac{80}{x} = \frac{80}{x+4} + 1$, *by the question, or* $80 = \frac{80x}{x+4} + x$, *or* $80x + 320 = 80x + x^2 + 4x$, *that is,* $x^2 + 4x = 320$; *whence* $x^2 + 4x + 4 = 320 + 4 = 324$, *by completing the square, and* $x + 2 = \sqrt{324} = 18$, *by evolution, consequently* $x = 18 - 2 = 16 =$ *number of oxen required.*

5. What two numbers are those, whose sum, product, and difference of their squares are all equal to each other?

Let $x =$ *greater number, and* $y =$ *the less; then* $\left\{ \begin{array}{l} x + y = xy \\ x + y = x^2 - y^2 \end{array} \right\}$ *by the question, and* $1 = \frac{x^2 - y^2}{x+y} = x - y$, *or* $x = y + 1$, *from the 2d. equation: Also* $\overline{y + 1} + y = \overline{y + 1} \times y$, *from the first equation; or* $2y + 1 = y^2 + y$, *that is,* $y^2 - y = 1$; *whence* $y^2 - y + \frac{1}{4} = 1\frac{1}{4}$, *by completing the square: Also* $y - \frac{1}{2} = \sqrt{1\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$ *by evolution, consequently* $y = \frac{\sqrt{5}}{2} + \frac{1}{2} = \frac{\sqrt{5} + 1}{2}$, *and* $x = y + 1 = \frac{\sqrt{5} + 3}{2}$.

6. There are four numbers in Arithmetical Progression, whereof the product of the two extremes is 45, and that of the means 77; what are the numbers?

Let

Let $x =$ *less extreme, and* $y =$ *common difference, then* x, $x + y$, $x + 2y$, $x + 3y$ *will be the* 4 *numbers, and* $\left\{ \begin{array}{l} x \times \overline{x + 3y} = x^2 + 3xy = 45 \\ \overline{x+y} \times \overline{x + 2y} = x^2 + 3xy + 2y^2 \end{array} \right\}$ *by the question; whence* $2y^2 = 77 - 45 = 32$, *and* $y^2 = \frac{32}{2} = 16$, *by subtraction and division, or* $y = \sqrt{16} = 4$, *by evolution, therefore* $x^2 + 3xy = x^2 + 12x = 45$, *by the first equation; also* $x^2 + 12x + 36 = 45 + 36 = 81$, *by completing the square, and* $x + 6 = \sqrt{81} = 9$, *by the extraction of roots, consequently* $x = 9 - 6 = 3$, *and the numbers are* 3, 7, 11 *and* 15.

RECAPITULATION *of the* PRINCIPLES *of* ARITHMETIC *and* ALGEBRA.

Axiom 1. Since whole numbers increaſe in a decuple proportion, 10 is the univerſal ratio of any Series of numbers whatever; and the reaſon for carrying at 10 in Addition and Multiplication is ſelf-evident, ſince 10 in any place to the right is equal to 1 in the next place to the left. Hence alſo the reaſon for carrying according to the ſubdiviſions of any integer when ſeveral denominations are to be added.

Axiom 2. If two whole numbers be equally increaſed, their difference is always the ſame. Hence the reaſon of borrowing 10 in one place to the right, and paying it back by carrying one to the next place. Hence likewiſe the reaſon will be evident, for placing the firſt figure to the right of the product of every particular multiplier directly below its own multiplier.

Axiom 3. The multiplicand will be increaſed or diminiſhed in proportion to the multiplier, when the ſame multiplicand is uſed. Hence the reaſon why the multiplicand is increaſed, when it is multiplied by any thing greater than unity, and decreaſed, when it is multiplied by a fraction.

Axiom 4. The Dividend will be increaſed or diminiſhed in proportion to the diviſor, when the ſame dividend is uſed. Hence, to divide by any thing greater than unity, will quote a number leſs than the dividend; and, on the contrary, to divide by any thing leſs than unity, will quote a number greater than the dividend.

Axiom 5. THE whole is equal to all its parts taken together. Hence one ſum may be made equal to ſeveral by Addition, and ſubtraction may be proved by adding the difference to the leaſt given ſum.

Axiom 6. If equal quantities be added to, taken from, multiplied or divided by, equal quantities, the ſums, remainders, products, and quotients, will reſpectively be equal. Hence the reaſon of reducing equations by Addition, Subtraction, Multiplication, and Diviſion, and of abridging commenſurable Terms, and cancelling equal quantities and numbers.

Axiom

Axiom 7. To multiply, or divide, any quantity or number by other quantities or numbers continually, is the ſame as to multiply by the product of thoſe other numbers. Hence the reaſon of multiplying, or dividing by component parts.

Axiom 8. If four numbers or quantities be proportional, the rectangle or product of the extremes will be equal to the product of the means; and *vice verſa*, if the product of the extremes be equal to that of the means, the numbers or quantities are proportional.

Axiom 9. The Quotient of any two ſucceeding Powers, when the next higher is divided by the next lower, exhibits the root of theſe powers. On the contrary, if any power be multiplied by the root of that power, the product will be the next higher power of the root: and if a higher power be divided by the root, the quotient will exhibit the next lower power. Again, if a proportional part of a higher power be divided by a proportional part of the next lower power, the quotient will exhibit a proportional part of the root. Hence the firſt figure or figures in the root of any power being raiſed to the power next lower than that whoſe root is wanted, and that power multiplied by a number expreſſing the proportion, which the given power bears to its root, produces a proportional diviſor, whoſe Ratio, compared with the dividual, is a proportional part of the root, which being annexed to the former part of the root, and raiſed to the full power of the given number, will be either the whole or a proportional part of the given power, diſcoverable by Subtraction, &c. Hence we have a general Rule for extracting the root of any power whatever.

An

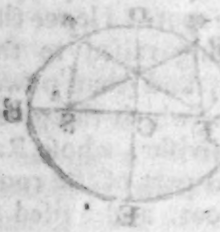
CONIC SECTIONS

SECTION I.

Of the ELLIPSIS.

Definition 1.

Fig. 1.

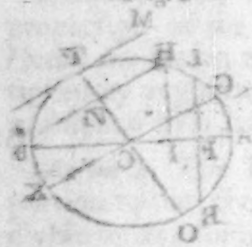


IF two Pins be fixed at the points F and S, and a thread PSFP put about them, and knotted at P; then if the thread be drawn tight, and the point P and the thread be moved about the fixed centres F, S, the point P will describe the curve PDASEB, called an *Ellipsis*.—See Fig. 1.

Def. 2. The points or centres F, S, are called the *Foci*.

Def. 3. The line A B, drawn through the Foci to the curve, is called the *transverse Axis*.

Fig. 2.



Def. 4. The point C in the middle of the Axis AB, is the *Centre*.—See Fig. 2.

Def. 5. The Line DE, (drawn through the centre C) perpendicular to the transverse AB, is called the *conjugate Axis*. See Fig. 2.

Def. 6. Any line TO, drawn through the Centre C to the curve, is called a *diameter*. And the Extremity T (or O) its *vertex*.

Def. 7. If T O be a diameter, then the diameter G K, drawn parallel to the Tangent at its vertex T, is called its *conjugate*. And the two diameters TO, GK, are said to be *conjugates* to one another.

Def. 8. The Line LR (drawn through the focus F, perpendicular to the transverse axis AB,) is called the *parameter* or *latus rectum*.

Def.

AN

INTRODUCTION

TO

CONIC SECTIONS.

SECTION I.

Of the ELLIPSIS.

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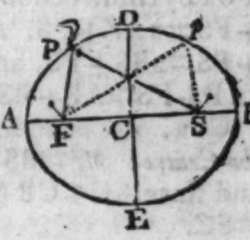

IF two Pins be fixed at the points F, S; and a thread PSFP, put about them and knotted at P; then if the thread be drawn tight, and the point P and the thread be moved about the fixed centres F, S; the point P will describe the curve PD*p*BEAP, called an *Ellipsis*.—See Fig. 1.

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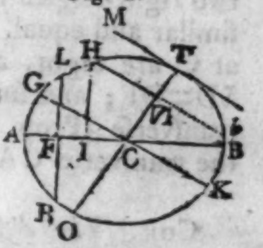

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Def.

Def. 9. A line drawn from any point of the curve (as HI) perpendicular to the transverse axis, is called an *Ordinate* to the transverse. And, in general, any line drawn from the curve to any diameter TO, parallel to its conjugate GK, (as HN,) is an *ordinate* to that principal diameter TO. If it go quite through the figure, as H*h*, it is called a *double Ordinate.*

Def. 10. A right line meeting the ellipsis in one point M, but does not cut it, is called a *Tangent* to it in that point, as TM.

Def. 11. The part of the Diameter between the vertex and the ordinate, is called the *abscissa,* TN, AI. And the *vertex* is the extremity of any diameter.

PROPOSITION 1. *The Sum of the lines* FP, SP, *drawn from the foci, to any point of the curve, is equal to the transverse axis* AB.

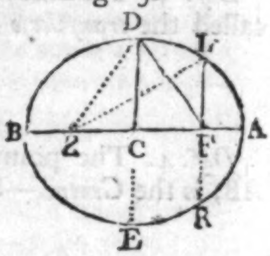
See Fig. 1.

FOR by construction, $PF+PS=AF+AS=AF+AF+FS=AF+AF+FS=2AF+FS$. And the same $PF+PS=2BS+FS$; therefore $2AF+FS=2BS+FS$, and $2AF=2BS$, or $AF=BS$. Whence $PF+PS=2AF+FS=AF+BS+FS=AB$.

COR. *The two foci are equally distant from the vertices, and also from the Centre*: $AF=BS$; and $FC=SC$. For it is proved that $AF=BS$; and since $AC=CB$ (def. 4) therefore $AC-AF=CB-BS$, or $FC=SC$.

PROP. 2. *A line drawn from the end of the conjugate axis, to the focus, is equal to half the transverse*; $DF=CA$. See Fig. 3.

DRAW DS to the other focus. Then the two right-angled triangles CDF and CDS are similar and equal. For $SC=CF$, the angles at C are right, and CD common; therefore $SD=DF$; and since the sum $SD+DF=$ the transverse (Prop. 1,) one of them $DF=$ half the transverse CA.



COR. *The distance of the foci is a mean proportional between the sum and difference of the transverse and conjugate axis,* $SF^2=\overline{BA+DE}\times\overline{BA-DE}$: For $CA^2=DF^2=DC^2+CF^2$; and $CF^2==CA^2-CD^2=\overline{CA+CD}\times\overline{CA-CD}$; and $4CF^2$ or $SF^2=\overline{2CA+2CD}\times\overline{2CA-2CD}$. $AF\times FB=DC^2$.

PROP. 3. *The rectangle of the focal distances, from either vertex, is equal to the square of the semi-conjugate:* $AF\times FB=DC^2$.

See Fig. 3.

FOR $DC^2=DF^2-CF^2=$ (Prop. 2.) $CA^2-CF^2=\overline{CA+CF}\times\overline{CA-CF}=\overline{BC+CF}\times CA-CF=BF\times FA$.

Prop. 4. *As the transverse axis to the conjugate, so the conjugate to the latus rectum of the transverse:* $AB : DE :: DE : LR$. See Fig. 3.

For $SL+LF=BA=2CA$ (Prop. 1.); and $SL=2CA-LF$, and by squaring $SL^2=4CA^2-4CA\times LF+LF^2$. And in the right-angled Triangle SLF, $SL^2=SF^2+LF^2$; whence $4AC^2-4CA\times LF+LF^2=SF^2+LF^2$, and $4AC^2-4CA\times LF=SF^2=4CF^2$, and $4AC^2=4CA\times LF+4CF^2=4CA\times LF+4DF^2-4DC^2$, and $4AC^2+4DC^2=4CA\times LF+4DF^2$; but $CA^2=DF^2$ (Prop. 2.); therefore $4DC^2=4CA\times LF=2CA\times 2LF$; that is, $DE^2=BA\times LR$.

Cor. 1. *As the semi-transverse is to the semi-conjugate, so the semi-conjugate to half the latus rectum*; $CA : DC :: DC : LF$.

Cor. 2. *As the semi-transverse, to the distance of the focus from the centre; so is the same distance, to the difference between the semi-transverse and half the latus rectum*: $FC^2=CA\times\overline{CA-LF}$.

For $CF^2=DF^2-DC^2=CA^2-CD^2=CA^2-CA\times LF$.

Cor. 3. *The Rectangle* $BFA=$ *half the transverse* $\times$ *half the latus rectum* $=CA\times FL$. By Cor. 1. and Prop. 3.—See Fig. 3.

Scholium. Since the transverse axis is to the conjugate, so the conjugate to the latus rectum, of the transverse axis. Therefore in any other diameters, the third proportional, to the diameter and its conjugate, is called the *latus rectum* of that diameter.

Prop. 5. *From any point* M *in the curve, drawing the lines* MF, MS, *to the two foci; and the ordinate* MP *perpendicular to the transverse axis* BA; *it will be,*

As the semi-transverse, CA :
To the distance of the focus from the centre, CF ::
So the distance of the ordinate from the centre, CP :
To half the difference of the lines MF, MS, *or* $\frac{MS-MF}{2}$.

Fig. 4.

For, make $SD=CA$, then $SM=CA+DM$, and $FM=2CA-SM=CA-DM$. In the right-angled Triangle SMP, SM^2 or $CA^2+2CA\times DM+DM^2=SP^2+PM^2=\overline{CF+CP}^2+PM^2=CF^2+2CF\times CP+CP^2+PM^2$, and in the right-angled Triangle FMP, FM^2 or $CA^2-2CA\times DM+DM^2=FP^2+PM^2=\overline{CF-CP}^2+PM^2=CF^2-2CF\times CP+CP^2+PM^2$; then subtracting the latter equation from the former, $SM^2-FM^2=4CA\times DM=4CF\times CP$, and $CF\times CP=CA\times DM$. But since $SM=CA+DM$, and $FM=CA-DM$; therefore $SM-FM=2DM$: therefore $CF\times CP=CA\times\frac{SM-FM}{2}$.

Cor. 1. *If* F, S, *be the foci,* MP *an ordinate; then it is* $CA : CF :: CP : \overline{CA-MF}$ *or* $\overline{SM-CA}$. See Fig 4.

FOR $CF \times CP = CA \times DM$, and $DM = SM - CA = CA - FM$.

COR. 2. *If* F, S, *be the foci,* MP *an ordinate; then the difference of the squares of the lines* SM, FM; *that is,* $SM^2 - FM^2 = 4CF \times CP$.

COR. 3. *If* F, S, *be the foci,* MP *an ordinate; then* $CA \times \overline{SM - FM} = 2CF \times CP$.

FOR $SM^2 - FM^2 = \overline{SM + FM} \times \overline{SM - FM} = 2CA \times \overline{SM - FM} = 4CF \times CP$, and $CA \times \overline{SM - FM} = 2CF \times CP$.

SCHOLIUM. If PM fall on the other side of F, as *pm*, then $pF = Cp - CF$, and its square the same as before, and the rest of the demonstration the same.

PROP. 6. *If an ordinate* MP *be drawn to the transverse axis; it will be,*

As the square of the transverse, BA^2 :
To the square of the Conjugate, NE^2 ::
So the rectangle of the Segments of the transverse BPA :
To the square of the ordinate, PM^2. See Fig. 4.

FOR make $SD = CA$, then DM is half the difference of SM and MF; therefore by Prop. 5. $CA : CF :: CP : DM$, and $CA : CA + CF$ or $BF :: CP : CP + DM$, and $CA : CP :: BF : CP + DM$, and $CA : CA + CP$ or $BP :: BF : BF + CP + DM$. But $BF = BC + CF = SD + CF$; and $BF + CP + DM = SD + CF + CP + DM = SM + CS + CP = SM + SP$; whence $CA : BP :: BF : SM + SP$. Again, since $CA : CF :: CP : DM$; then $CA : (CA - CF)\ AF :: CP : CP - DM$; and $CA : CP :: AF : CP - DM$. And $CA : (CA - CP)\ PA :: AF : AF - CP + DM$. But $AF = CA - CF = SD - SC$; therefore $AF - CP + DM = SD - SC - CP + DM = SM - SP$; therefore $CA : PA :: AF : SM - SP$, and we had before, $CA : BP :: BF : SM + SP$; then multiplying these proportions together, we have $CA^2 : BP \times PA :: BF \times FA : SM^2 - SP^2$.

BUT (Prop. 3.) $BF \times FA = CN^2$; and $SM^2 - SP^2 = PM^2$; therefore $CA^2 : BPA :: CN^2 : PM^2$, or alternately, $CA^2 : CN^2 :: BPA : PM^2$, or $BA^2\ (4CA^2) : NE^2\ (4CN^2) :: BPA : PM^2$.

COR. 1. $CA^2 : CN^2 :: BFA : PM^2$.

COR. 2. *As the transverse* BA : *to its latus rectum* :: *So the rectangle* BPA : *to square of the ordinate* PM^2.

FOR (Prop. 4) latus rectum $= \frac{NE^2}{AB}$, whence, since $BA^2 : EN^2 :: BFA : PM^2$, therefore, $BA : \frac{NE^2}{BA}$ or latus rectum $:: BFA : PM^2$.

COR. 3. *The rectangles of the segments of the transverse are as the squares of the ordinates.*

FOR every rectangle is to the square of its ordinate, in the given ratio of CA^2 to CN^2, or of BA to the latus rectum.

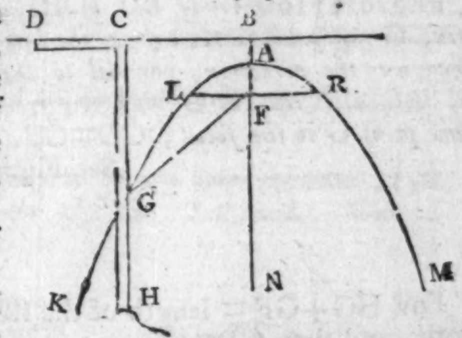
COR

COR. 4. As the ſquare of the ſemi-tranſverſe CA^2 :
Rectangle of the focal diſtances from vertex BFA ::
So rectangle of the Segments BPA :
To ſquare of the ordinate PM^2.

SECTION II.

Of the PARABOLA.

Fig. 5.



Definition 1. IF one end of a thread, equal in length to CH, be fixed at the point F, and the other end fixed at H, the end of the ſquare DCH. And if the ſide CD of the ſquare be moved along the right line BD, and always coincide with it, then, if the ſtring FGH be always kept tight, and cloſe to the ſide GH of the ſquare, the point or pin G (where it leaves the ſquare) will deſcribe a curve MRALGK called a *Parabola*. See Fig. 5.

Def. 2. The fixed point F is called the *Focus*.

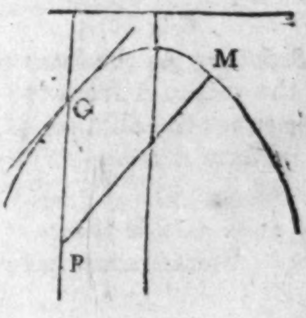
Def. 3. The right line BD is called the *Directrix*.

Def. 4. IF the line BN be drawn through the Focus F, perpendicular to BD; then AN is called the *Axis* of the Parabola, and A the *Vertex*.

Def. 5. A line drawn through the focus F, perpendicular to the Axis, as LR, is called the *Parameter* or *Latus rectum*.

Def. 6. Any line drawn within the curve, parallel to the axis, as GH, is called a *diameter*. And the point G, where it cuts the curve, is the *vertex*.

Fig. 6.



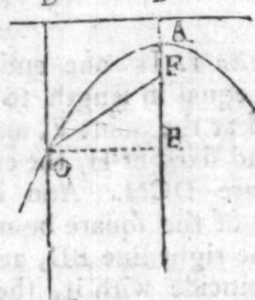
Def. 7. A right line drawn from any diameter to the curve, and parallel to the Tangent at the vertex, as PM, is called an *ordinate*. If it go quite through the curve, it is called a *double ordinate*.

See Fig. 6.

Def. 8. The part of the Diameter between the vertex and ordinate, as GP, is called the *Abscissa*.

Def 9. A right line meeting the curve in one point G, but does not cut it, is called a *Tangent* in that point.

Fig. 7.



PROPOSITION I. *If* BD *be the directrix,* G *any point in the curve, the line* GD *drawn to the directrix, parallel to the axis, is equal to the line* GF *drawn from the same point* G *to the focus* ; GD=GF. See Fig. 7.

FOR HG+GF = length of the string=HD ; take away GH from both, and then GD=GF.

COR. 1. *The distances of the focus, and of the directrix from the vertex are equal.* AB=AF. For when D is at B, G will be at A ; consequently AB=AF.

COR. 2. *If* GP *be an ordinate to the Axis* ; *then* AP+AF=FG ; *For* AP+AF=BP=GD.

COR. 3. FG—FP= *half the latus rectum.*

PROP. 2. *The Distance of the focus from the vertex is* $\frac{1}{4}$ *the latus rectum* : AF=$\frac{1}{4}$LR=$\frac{1}{2}$LF. See Fig. 5.

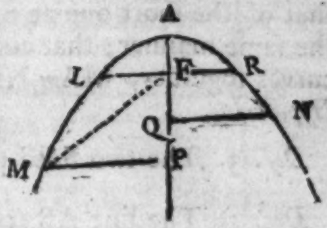
FOR when the pin G comes to L, then LF=FB (Prop. 1. Cor. 1.) =2FA, and AF=$\frac{1}{2}$ FL. For the same reason FA=$\frac{1}{2}$FR, therefore FA=$\frac{1}{4}$LR.

Scholium. As the latus rectum to the axis is four times the distance of the vertex A from the focus F ; So in any other diameter GH, four times the distance of its vertex from the focus, or AFG is called its *latus rectum.*

PROP. 3. *The square of any ordinate to the axis is equal to the rectangle of the latus rectum and abscissa* : PM^2=LR×AP. See Fig. 8.

FOR

Fig. 8.



For $MF = AF + AP =$ (Prop. 2.) $AP + \frac{1}{4} LR$, and $FP = AP - AF = AP - \frac{1}{4} LR$. And in the right angled Triangle MFP, $MP^2 = MF^2 - FP^2 = \overline{MF + FP} \times \overline{MF - FP} = 2AP \times \frac{1}{2} LR = AP \times LR$.

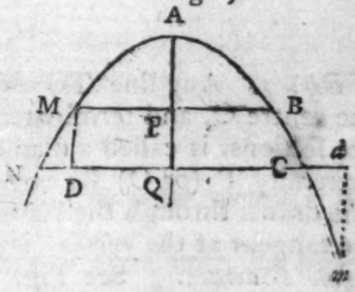
Cor. 1. *If* F *be the focus,* $MP^2 = AP \times 4AF$.

Cor. 2. *The abscissas are as the squares of their ordinates.* $AP : AQ :: PM^2 : QN^2$. For $AP : AQ :: AP \times LR : AQ \times LR :: PM^2 : QM^2$.

Cor. 3. *The latus rectum is a third proportional to the abscissa and ordinate.* $AP : PM : LR \div$.

Prop. 4. *As the latus rectum to the sum of any two ordinates* :: *so their difference* : *to the difference of the abscissæ.* *Lat. rect.* : CD :: ND : PQ. See Fig. 9.

Fig. 9.



Let L = latus rectum, then (Prop. 3.) $L \times AP = PM^2$; and $L \times AQ = NQ^2$. And by subtraction, $L \times AQ - L \times AP = NQ^2 - PM^2$; therefore $L : NQ + PM :: NQ - PM : AQ - AP$; that is, $L : DC :: ND : PQ$.

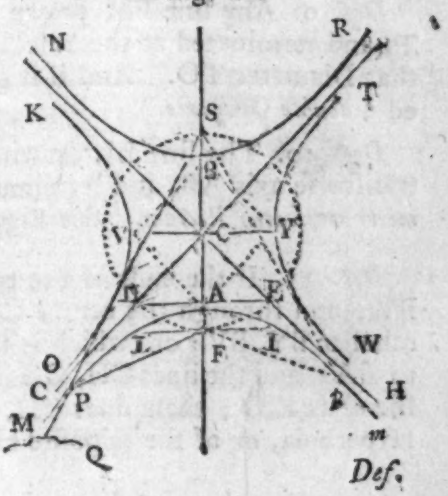
Cor. 1. If MD be the axis, NC an ordinate to it; then the rectangle NDC = MD × *parameter*.

Cor. 2. The rectangle NDC is every where as MD.

SECTION III.

Of the Hyperbola.

Fig. 10.



Definition 1. If the ends of two threads SPQ, FPQ, be fastened at the points S, F, and be made to pass through a small bead, or pin P, and knotted together at Q; then taking hold of Q, and drawing the threads tight; if the bead be moved along the threads, the point P will describe the curve *m p* APM, called an *Hyperbola.*——See Fig. 10.

Def.

Def. 2. And if the end of the long thread be fixed at F, and that of the short one at S; and the curve NBR be described after the same manner; that curve is called the *opposite Hyperbola*; and both curves together, MA*m*,NBR, are called *opposite Sections*, or *opposite Hyperbolas*.

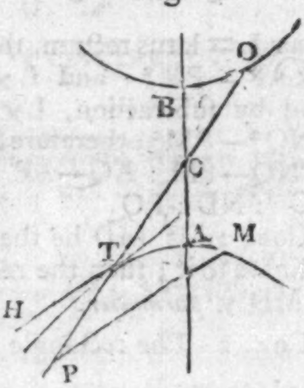
Def. 3. The two fixed points F,S, are called the *foci*.

Def. 4. The line AB (passing through the foci, when continued) contained between the two parts of the curve, is called the *transverse axis*,

Def. 5. The middle point of AB, that is, C, is called the *Centre* of the Hyperbola, or of the opposite sections.

Def. 6. If VY be drawn through the centre C perp. to AB; and with radius CF, and centre A, an arch be described, cutting VY in V, and Y; then VY is called the *conjugate Axis*.

Fig. 11.



Def. 7. Any line TO drawn through the centre C, and terminated at the opposite sections, is called a diameter; and the extremity T (or O) its *vertex*. And the line drawn through the centre, parallel to the tangent at the vertex, is called its *conjugate Diameter*. See Fig. 10.

Def. 8. If any diameter OT be continued within the curve, the part within, TP, is called the *Abscissa*.

Def. 9. Any line PM, drawn parallel to the tangent at the vertex T, and terminated at the abscissa and curve, is called an *Ordinate* to that Diameter TO. And if it go quite through the curve, it is called a *double Ordinate*.

Def. 10. The line LI, drawn through the focus F, perp. to the transverse axis AB, and terminating at the curve, is called the *Parameter* or *Latus Rectum*. See Fig. 10.

Def. 11. If the ends of the two axes be joined by the lines BY, BV; and through the centre C, two lines CH, CG, be drawn parallel to BY, BV; or (which is the same) if VY be placed at A. perp. to BA; and the lines CH, CG, be drawn from the centre C, through the ends E,D; these lines CH, CG, are called the *Assymptotes* of the Hyperbola, or of the opposite Hyperbolas.

Def.

Def. 12. When the transverse and conjugate axes are equal, AC $=$CV or AD, the curve is called an *equilateral Hyperbola*, or *right angled Hyperbola*.

Def. 13. A right line, which meets the hyperbola in one point T, but does not cut it, as TH, is called a *Tangent* to it, in that point T. See Fig 11.

Def. 14. If two opposite Hyperbolas, KO, TW, be in like manner described to the transverse VY ($=$DE), and conjugate AB; these are called *conjugate Hyperbolas*, with regard to the former.

Proposition 1. *The difference of the lines* SP, FP, *drawn from the foci, to any point* P *of the curve, is equal to the transverse axis* AB. See Fig. 10.

For by construction $PS - PF = AS - AF = AB + BS - AF =$ (because $BS = AF$) AB.

Cor. *Hence* $CF = CS$, *or the foci are equally distant from the centre.*

Prop. 2. *The square of the distance of the focus from the centre is equal to the sum of the squares of the semitransverse and semiconjugate.* $CF^2 = CA^2 + CY^2$.

For, make AE equal and parallel to CY, then the radius $CE = CF$; and in the right angled triangle CAE, $CE^2 = CA^2 + AE^2$; that is, $CF^2 = CA^2 + AE^2 = CA^2 + CY^2$.

Cor. $CF^2 - AE^2 = CA^2$; and $CF^2 - CA^2 = AE^2 = CY^2$.

Prop. 3. *The rectangle of the focal distances from either vertex is equal to the square of the semiconjugate*, $FA \times SA = CY^2$.

For, making $AE = CY$; by the property of the Circle, $FA \times AS = AE^2 = CY^2$.

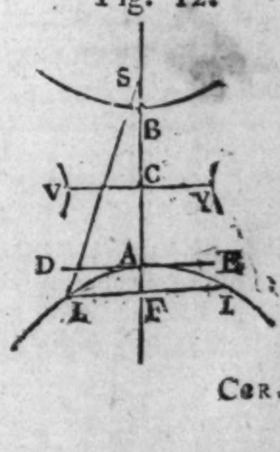
Cor. *The rectangle if the distance of either focus from the two vertices is equal to the square of the semicongate*, $FA \times FB = AE^2 = CY^2$.

For $SB = FA$ and $SA = FB$, whence $FA \times FB = FA \times SA = AE^2$.

Prop. 4. *As the transverse axis is to the conjugate; so the conjugate, to the latus rectum of the transverse*; AB : VY : : VY : LI.

See Fig. 12

Fig. 12.



For (Prop. 1.) $SL - LF = BA = 2CA$; and $SL = 2CA + LF$; and $SL^2 = 4CA^2 + 4CA \times LF + LF^2$: and in the right angled triangle SLF, $SL^2 = SF^2 + LF^2$, and subtracting LF^2 from these two values of SL^2; then $4CA^2 + 4CA \times LF = SF^2 = 4CF^2$; and $CF^2 = CA^2 + CA \times LF$. But (Prop. 2.) $CF^2 = CA^2 + CY^2 = CA^2 + CA \times LF$; therefore $CY^2 = CA \times LF$, and multiplying by 4, $VY^2 = BA \times LI$.

Cor. 1. *As the ſemi-tranſverſe, to the ſemi-conjugate; ſo the ſemi-conjugate to half the latus rectum,* CA : CY : LF.

Cor. 2. *As the ſemi-tranſverſe to the diſtance of the focus from the centre; ſo is the ſame diſtance, to the ſum of the ſemi-tranſverſe and half the latus rectum,* CA : CF : : CF : CA + LF.

For, (Prop. 2.) $CF^2 = CA^2 + CY^2 = CA^2 + CA \times LF = CA \times \overline{CA \times FL}$.

Cor. 3. *The rectangle* BFA $= \frac{1}{2}$ *tranſverſe* $\times \frac{1}{2}$ *latus rectum* $= CA \times FL$. By Cor. 1. and Prop. 3.

Scholium. Since the tranſverſe axis is to the conjugate, as the conjugate to the latus rectum of the tranſverſe axis; Therefore in any other Diameters, the third Proportional, to any diameter and its conjugate, is called the *Latus Rectum of that diameter.* Therefore in a right-angled hyperbola, every diameter is equal to its latus rectum.

FINIS.

In a few of the first printed Copies of this Work, the following errors escaped notice.

Page 470, Line 22. for $12ax-{}^2$ read $12ax-x^2$. l. 31. for $5x2$, r. $5x^2$. Page 471. l. 6. for $-xy$, r. x^2y. P. 473. l. 12. for $\frac{a^2+a^2}{x}$, r. $\frac{ax+a^2}{x}$. l. 13, for $\frac{x^2}{a-x}$, r. $\frac{2x^2}{a-x}$. P. 474, l. 13, for $\frac{b^3-b^2x}{x^2+2bx+b}$, r. $\frac{x^3-b^2x}{x^2+2bx+b^2}$. l. 14, for $x+2bx+b^2$, r. $x^2+2bx+b^2$. P. 475, l. 8, for $x+2bx+b^2$, r. $x^2+2bx+b^2$. P. 477 last l. for $\frac{x^4}{a}$, r. $\frac{x^4}{a^4}$. P. 481, Ex. 2. root $=x^2-2x+1$. P. 483, quotient of Ex. 4. for $-\frac{4x^3}{3}$, r. $\frac{4x^3}{a^3}$. P. 484, l. 12, for $\frac{x}{8a^4}$, r. $\frac{x^6}{8a^4}$. P. 495, l. 26, for $x^2+\frac{bx}{a}$, r. $x^2-\frac{bx}{a}$. P. 496, l. 8. for $x^2=\frac{bx}{a}+\frac{4a^2}{b^2}$, r. $x^2-\frac{bx}{a}+\frac{4a^2}{b^2}$. P. 498, l. 12, for $b+a$. r. $b+a^2$, and l. 15, for x^n r. $x^{\frac{n}{2}}$. P. 500, l. 3, r. $x^2+3xy+2y^2=77$. P. 504, l. 15 and 16, dele AF+AF+ES: and l. 36, dele $AF\times FB=DC^2$. Last line, draw a Viculum over CA—CF, P. 506. l. 37, for $\frac{NE^{11}}{AB}$, r. $\frac{NE^2}{AB}$.

Rule to calc Interest

Call the pounds Integers or whole numbers then multiply the Shillings by 5. and bring in the pence at the rate of 5 for 12 — call the months Integers also, & take one third of the odd days for a decimal fraction.

After thus preparing the question place the pounds and decimal parts of a pound as the multiplicand, & the months and decimal parts of a month as a multiplier, & always cut off at the right hand of the product one more decimal figure than both the multiplicand and multiplier contain, which will be in shillings & decimal parts of a shilling

Ex: What is the Interest of £336.13.8 for one year, seven months & 15 days

336..13.8
5 for the 8d bring in 3.
336,68
19.5
168340
303012
33668
20 / 656,5260
32 12
6,3120
4
1,2480

To find (to one penny, by one operation) the Interest (at £6 p C. p. an.) of any Sum of pounds shillings & pence, for any time given in Months or Days.

Table. 1. Decimals of a Pound.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	00	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
1	00	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
2	01	06	11	16	21	26	31	36	41	46	51	56	61	66	71	76	81	86	91	96
3	01	06	11	16	21	26	31	36	41	46	51	56	61	66	71	76	81	86	91	96
4	02	07	12	17	22	27	32	37	42	47	52	57	62	67	72	77	82	87	92	97
5	02	07	12	17	22	27	32	37	42	47	52	57	62	67	72	77	82	87	92	97
6	02	07	12	17	22	27	32	37	42	47	52	57	62	67	72	77	82	87	92	97
7	03	08	13	18	23	28	33	38	43	48	53	58	63	68	73	78	83	88	93	98
8	03	08	13	18	23	28	33	38	43	48	53	58	63	68	73	78	83	88	93	98
9	04	09	14	19	24	29	34	39	44	49	54	59	64	69	74	79	84	89	94	99
10	04	09	14	19	24	29	34	39	44	49	54	59	64	69	74	79	84	89	94	99
11	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100

Table 2

Reverted Decimals of a Month.

1	90
2	70
3	01
4	31
5	71
6	02
7	32
8	72
9	03
10	33
11	73
12	04
13	34
14	74
15	05
16	35
17	75
18	06
19	36
20	76
21	07
22	37
23	77
24	08
25	38
26	78
27	09
28	39
29	79

Explanation

Table. 1. The figures at the top, are Shillings, those in the first column pence, all the other figures are Decimals. The Decimals of any sum of Shillings & pence are found by looking in the column under the former, & on the line against the latter. Thus the decimals of 9/4 is 47 — of 16/7 is 83 &c —

Use

Rule. 1. To the figures expressing the pounds annex the Decimals of the shillings & pence. 2. Under the Units & Tens of the pounds, or the places thereof, set down the Decimals of the days; & to these annex the figures

there be more than one, to place them in a reverted order.

3 – Multiply, according to the Rule of contracted multiplication* – the pounds & Decimals by the months and Decimals so set down.

4 – From the product cut off two figures at the right hand, which divided by 8 will give the pence, & the left hand figures will be the shillings of the Interest required.

Note – If there be no pounds in the sum, or months in the time given, their places may be supplied by cyphers, or nots for cyphers.

Ex: Principal £16..12..7 Time M.68 D.19.

16.63
36.86
4972
1330
100
5
114.13

Int. £5..14..2

2.° £363.4.9. Time M.7.D26

363,49
78.7.
25444
2907
254
286,05

Int. £14.6.1

3.° Prin. £6..12..7 Time M.246.D.14.

6.49
74.642
13260
2652
398
24
5
163.41

Int. £8..3..5.

4.th £0.17.6 M.68.D.19

.87
3686
522
70
5
5.97

Int. 6/–

5.th £47.14.2 M.0.D.7.

47.71
75
238
33
271

Int. 2/9.

6.th £3.17.9 M.86.D.0

3.89
68
3112
233
33,45

Int. £1.13.6.

Rule 1. To turn shillings & pence into Decimals of a pound. Multiply the shillings by 5, & to the product add as many units as there are 2½ in the pence. Thus the Decimal of 15/3 is 77 – of 12/9 nearly 64.

Rule 2.d To turn days into Decimals of a month. Divide the number of days by 3; if there be no remainder, annex a cypher to the Quotient: if there be a remainder of one, annex 3. If the remainder be two annex 7. Revert the places of the figures thus found, and you

recurring decimals of 18 [illegible], will be found to have 106. of 13 [illegible]. 34 — of 26 [illegible]. 78. —

* Then, reject all the figures in the multiplicand that stand at the right hand of the figure you multiply by, and set down them so directly, so that the right hand figures of each line stand directly under each other, drawing to each such right hand figure what it would arise, by carrying one for every ten, or nearest tenth, that is, 2 for 16 or more, under 26: 3 for 26 or more; 4 for 36 or more, under 40 &c —